11

Belt, Rope and Chain Drives

11.1. Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors :

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

It may be noted that

(*a*) The shafts should be properly in line to insure uniform tension across the belt section.

(*b*) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(*c*) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(*d*) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(*e*) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(*f*) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 metres and the minimum should not be less than 3.5 times the diameter of the larger pulley.

11.2. Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

11.3. Types of Belt Drives

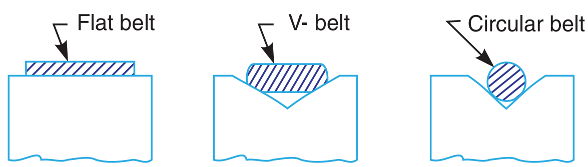
The belt drives are usually classified into the following three groups :

1. ***Light drives***. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. ***Medium drives***. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives***. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

11.4. Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Fig. 11.1. Types of belts.

Though there are many types of belts used these days, yet the following are important from the subject point of view :

1. ***Flat belt.*** The flat belt, as shown in Fig. 11.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. ***V-belt.*** The V-belt, as shown in Fig. 11.1 (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

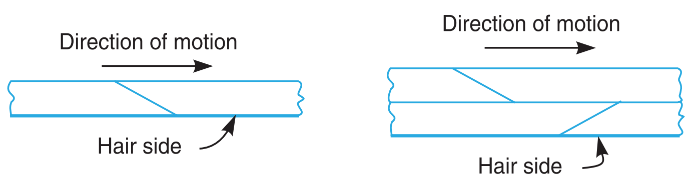
3. ***Circular belt or rope***. The circular belt or rope, as shown in Fig. 11.1 (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

11.5. Material used for Belts

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows :

1. *Leather belts*. The most important material for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. 11.2. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.



(*a*) Single layer belt. (*b*) Double layer belt.

Fig. 11.2. Leather belts.

The leather may be either oak-tanned or mineral salt tanned *e.g*. chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers *e.g.* single, double or triple ply and according to the thickness of hides used *e.g.* light, medium or heavy.

The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neats foot or other suitable oils so that the belt will remain soft and flexible.

2. *Cotton or fabric belts*. Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

3. *Rubber belt*. The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantage of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

4. *Balata belts*. These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected by animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

11.6. Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives:

1. *Open belt drive*. The open belt drive, as shown in Fig. 11.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (*i.e.* lower side RQ) and delivers it to the other side (*i.e.* upper side LM). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig. 11.3.

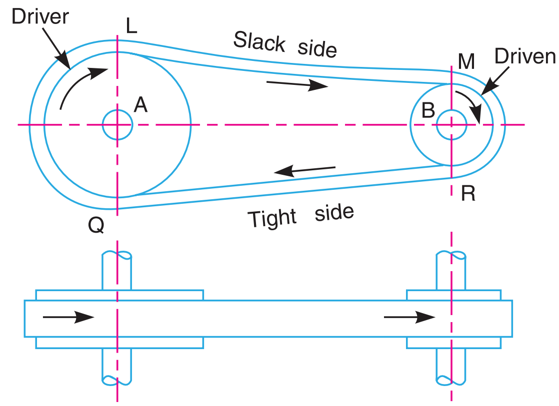


Fig. 11.3. Open belt drive.

2. *Crossed or twist belt drive*. The crossed or twist belt drive, as shown in Fig. 11.4, is used with shafts arranged parallel and rotating in the opposite directions.

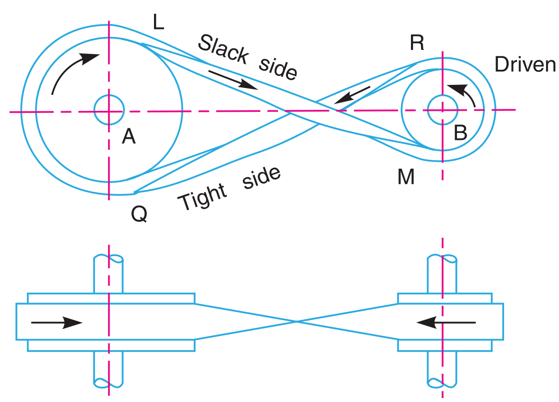


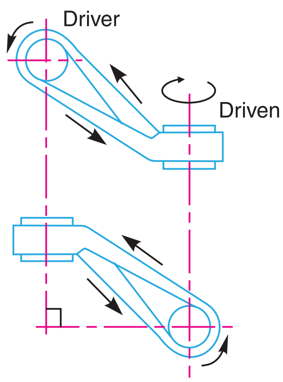
Fig. 11.4. Crossed or twist belt drive.

In this case, the driver pulls the belt from one side (*i.e.* RQ) and delivers it to the other side (*i.e.* LM). Thus the tension in the belt RQ will be more than that in the belt LM. The belt RQ (because of more tension) is known as ***tight side,*** whereas the belt LM (because of less tension) is known as ***slack side***, as shown in Fig. 11.4.

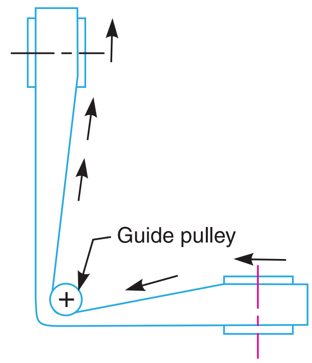
A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.

3. *Quarter turn belt drive.* The quarter turn belt drive also known as right angle belt drive, as shown in Fig. 11.5 (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is the width of belt.

In case the pulleys cannot be arranged, as shown in Fig. 11.5 (*a*), or when the reversible motion is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. 11.5 (*b*), may be used.



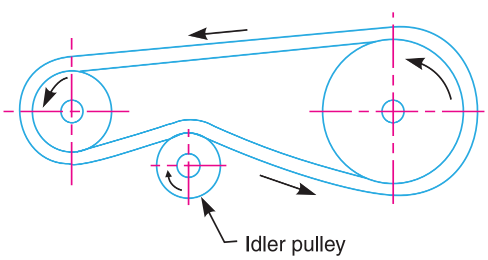
(*a*) Quarter turn belt drive.



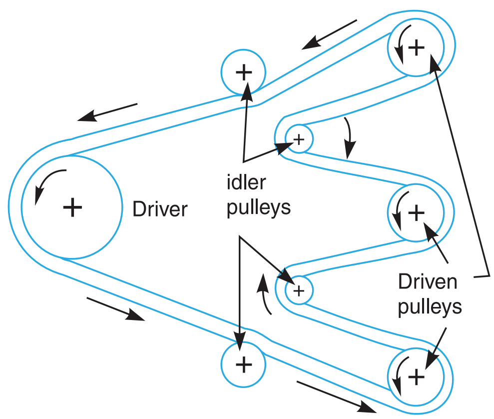
(*b*) Quarter turn belt drive with guide pulley.

Fig. 11.5

4. *Belt drive with idler pulleys*. A belt drive with an idler pulley, as shown in Fig. 11.6 (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.



(*a*) Belt drive with single idler pulley.



(*b*) Belt drive with many idler pulleys.

Fig. 11.6

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 11.6 (*b*), may be employed.

5. *Compound belt drive*. A compound belt drive, as shown in Fig. 11.7, is used when power is transmitted from one shaft to another through a number of pulleys.

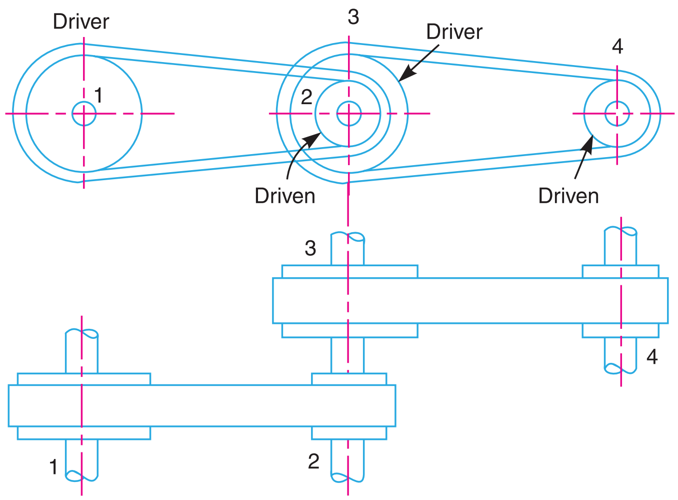


Fig. 11.7. Compound belt brive.

6. *Stepped or cone pulley drive*. A stepped or cone pulley drive, as shown in Fig. 11.8, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. *Fast and loose pulley drive*. A fast and loose pulley drive, as shown in Fig. 11.9, is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

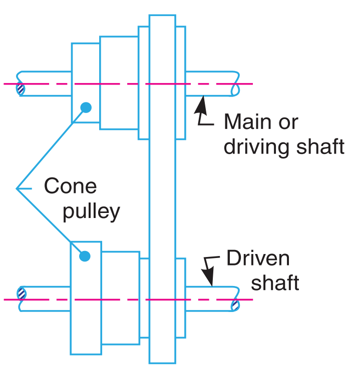


Fig. 11.8. Stepped or cone pulley drive.

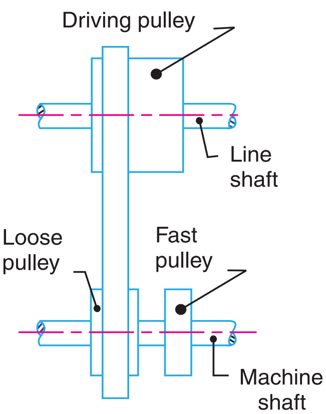


Fig. 11.9. Fast and loose pulley drive.

11.7. Velocity Ratio of Belt Drive

It is the **ratio between the velocities of the driver and the follower or driven.** It may be expressed, mathematically, as discussed below :

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m., and

N_2 = Speed of the follower in r.p.m.

∴ Length of the belt that passes over the driver, in one minute

$$= \pi\, d_1 . N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi\, d_2 \,.\, N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\pi\, d_1 \,.\, N_1 = \pi\, d_2 \,.\, N_2$$

∴ Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Note: The velocity ratio of a belt drive may also be obtained as discussed below :

We know that peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi\, d_1 . N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven or follower pulley,

$$v_2 = \frac{\pi\, d_2 . N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \qquad \frac{\pi\, d_1 . N_1}{60} = \frac{\pi\, d_2 . N_2}{60} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

11.8. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1,

N_1 = Speed of the pulley 1 in r.p.m.,

d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \qquad \text{...}(i)$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \qquad \text{...}(ii)$$

Multiplying equations (*i*) and (*ii*),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

or $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{...}(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

or $$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

11.9. Slip of Belt

In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let s_1 % = Slip between the driver and the belt, and

s_2 % = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad \text{...}(i)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2\, N_2}{60} = \frac{\pi d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \text{...}\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Example 11.1. *An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when* ***1.*** *there is no slip, and* ***2.*** *there is a slip of 2% at each drive.*

Solution. Given : N_1 = 150 r.p.m. ; d_1 = 750 mm ; d_2 = 450 mm ; d_3 = 900 mm ; d_4 = 150 mm

The arrangement of belt drive is shown in Fig. 11.10.

Let N_4 = Speed of the dynamo shaft .

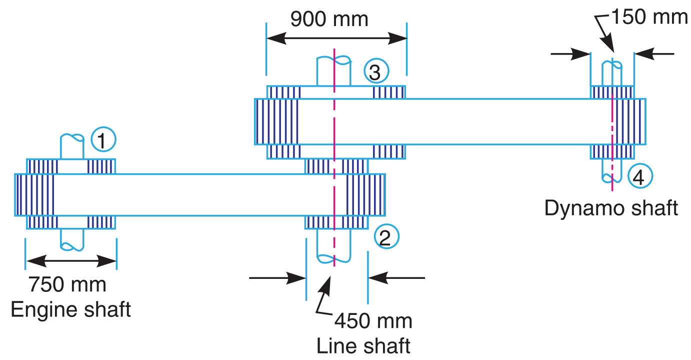


Fig. 11.10

1. ***When there is no slip***

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$\therefore$ N_4 = 150 × 10 = 1500 r.p.m. **Ans.**

2. ***When there is a slip of 2% at each drive***

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$\therefore$ N_4 = 150 × 9.6 = 1440 r.p.m. **Ans.**

11.10. Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as ***creep***. The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Example 11.2. *The power is transmitted from a pulley 1 m diameter running at 200 r.p.m. to a pulley 2.25 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The Young's modulus for the material of the belt is 100 MPa.*

Solution. Given : $d_1 = 1$ m ; $N_1 = 200$ r.p.m. ; $d_2 = 2.25$ m ; $\sigma_1 = 1.4$ MPa $= 1.4 \times 10^6$ N/m^2; $\sigma_2 = 0.5$ MPa $= 0.5 \times 10^6$ N/m^2 ; $E = 100$ MPa $= 100 \times 10^6$ N/m^2

Let N_2 = Speed of the driven pulley.

Neglecting creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad N_2 = N_1 \times \frac{d_1}{d_2} = 200 \times \frac{1}{2.25} = 88.9 \text{ r.p.m.}$$

Considering creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

or
$$N_2 = 200 \times \frac{1}{2.25} \times \frac{100 \times 10^6 + \sqrt{0.5 \times 10^6}}{100 \times 10^6 + \sqrt{1.4 \times 10^6}} = 88.7 \text{ r.p.m.}$$

∴ Speed lost by driven pulley due to creep

$= 88.9 - 88.7 = 0.2$ r.p.m. **Ans.**

11.11. Length of an Open Belt Drive

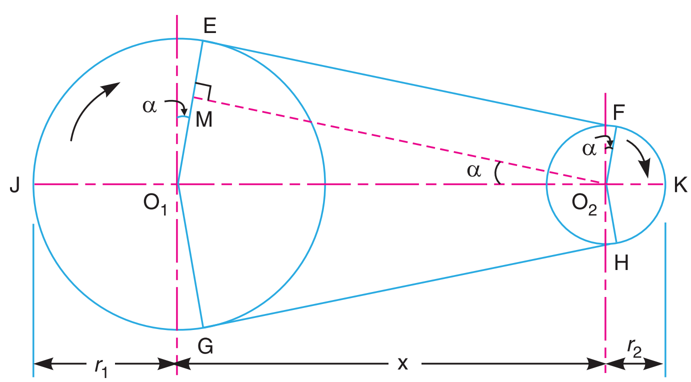


Fig. 11.11. Length of an open belt drive.

We have already discussed in Art. 11.6 that in an open belt drive, both the pulleys rotate in the ***same*** direction as shown in Fig. 11.11.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 11.11. Through O_2, draw $O_2 M$ parallel to FE.

From the geometry of the figure, we find that $O_2 M$ will be perpendicular to $O_1 E$.

Let the angle $MO_2 O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \qquad \text{...}(ii)$$

$\therefore$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iii)$$

Similarly

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \qquad \text{...}(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 - r_2)^2}{2x} \qquad \text{...}(v)$$

Substituting the values of arc *JE* from equation ***(iii)***, arc *FK* from equation ***(iv)*** and *EF* from equation ***(v)*** in equation ***(i)***, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$ from equation ***(ii)***,

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

11.12. Length of a Cross Belt Drive

We have already discussed in Art. 11.6 that in a cross belt drive, both the pulleys rotate in ***opposite*** directions as shown in Fig. 11.12.

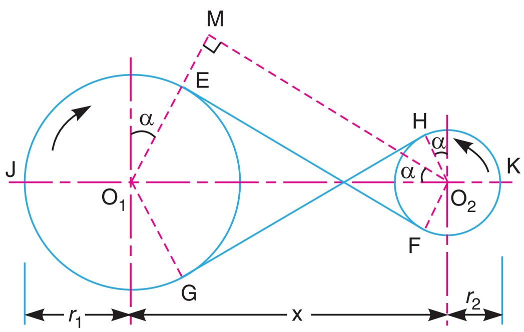


Fig. 11.12. Length of a cross belt drive.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1\, O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. 11.12. Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2\, O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \qquad \text{...}(ii)$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iii)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iv)$$

and $$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 + r_2)^2}{2\,x} \qquad ...(v)$$

Substituting the values of arc *JE* from equation ***(iii)***, arc *FK* from equation ***(iv)*** and *EF* from equation ***(v)*** in equation ***(i)***, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 . \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2 . \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$ from equation ***(ii)***,

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus obvious that if sum of the radii of the two pulleys be constant, then length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

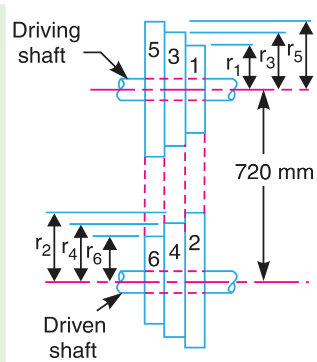


Fig. 11.13.

Example 11.3. *A shaft which rotates at a constant speed of 160 r.p.m. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for **1.** a crossed belt, and **2.** an open belt. Neglect belt thickness and slip.*

Solution. Given : $N_1 = N_3 = N_5 = 160$ r.p.m. ; $x = 720$ mm ; $N_2 = 60$ r.p.m.; $N_4 = 80$ r.p.m.; $N_6 = 100$ r.p.m. ; $r_1 = 40$ mm

Let r_2, r_3, r_4, r_5 and r_6 be the radii of the pulleys 2, 3, 4, 5, and 6 respectively, as shown in Fig. 11.13.

1. *For a crossed belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

or $$r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2\,r_3$$

We know that for a crossed belt drive,

$$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 = 146.7 \text{ mm} \quad \text{...}(i)$$

$\therefore$ $$r_3 + 2\,r_3 = 146.7 \quad \text{or} \quad r_3 = 146.7/3 = 48.9 \text{ mm} \quad \textbf{Ans.}$$

and $$r_4 = 2\,r_3 = 2 \times 48.9 = 97.8 \text{ mm} \quad \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or} \quad r_6 = r_5 \times \frac{N_5}{N_6} = r_5 \times \frac{160}{100} = 1.6\,r_5$$

From equation **(*i*)**,

$$r_5 + 1.6\,r_5 = 146.7 \quad \text{or} \quad r_5 = 146.7/2.6 = 56.4 \text{ mm} \quad \textbf{Ans.}$$

and $$r_6 = 1.6\,r_5 = 1.6 \times 56.4 = 90.2 \text{ mm} \quad \textbf{Ans.}$$

2. *For an open belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \quad \text{or} \quad r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2\,r_3$$

We know that length of belt for an open belt drive,

$$L = \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x$$

$$= \pi(40 + 106.7) + \frac{(106.7 - 40)^2}{720} + 2 \times 720 = 1907 \text{ mm}$$

Since the length of the belt in an open belt drive is constant, therefore for pulleys 3 and 4, length of the belt (L),

$$1907 = \pi(r_3 + r_4) + \frac{(r_4 - r_3)^2}{x} + 2x$$

$$= \pi(r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 \times 720$$

$$= 9.426\, r_3 + 0.0014\, (r_3)^2 + 1440$$

or $\quad 0.0014\, (r_3)^2 + 9.426\, r_3 - 467 = 0$

$$\therefore \quad r_3 = \frac{-9.426 \pm \sqrt{(9.426)^2 + 4 \times 0.0014 \times 467}}{2 \times 0.0014}$$

$$= \frac{-9.426 \pm 9.564}{0.0028} = 49.3 \text{ mm} \quad \textbf{Ans.}$$

...(Taking +ve sign)

and $\quad r_4 = 2\, r_3 = 2 \times 49.3 = 98.6$ mm **Ans.**

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or}$$

$$r_6 = \frac{N_5}{N_6} \times r_5 = \frac{160}{100} \times r_5 = 1.6\, r_5$$

and length of the belt (L),

$$1907 = \pi(r_5 + r_6) + \frac{(r_6 - r_5)^2}{x} + 2x$$

$$= \pi(r_5 + 1.6 r_5) + \frac{(1.6 r_5 - r_5)^2}{720} + 2 \times 720$$

$$= 8.17\, r_5 + 0.0005\, (r_5)^2 + 1440$$

or $\quad 0.0005\, (r_5)^2 + 8.17\, r_5 - 467 = 0$

$$\therefore \quad r_5 = \frac{-8.17 \pm \sqrt{(8.17)^2 + 4 \times 0.0005 \times 467}}{2 \times 0.0005}$$

$$= \frac{-8.17 \pm 8.23}{0.001} = 60 \text{ mm} \quad \textbf{Ans.}$$

...(Taking +ve sign)

and $\quad r_6 = 1.6\, r_5 = 1.6 \times 60 = 96$ mm **Ans.**

Milling machine is used for dressing surfaces by rotary cutters.

Note : This picture is given as additional information and is not a direct example of the current chapter.

11.13. Power Transmitted by a Belt

Fig. 11.14 shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig. 11.14.

Let $\quad T_1$ and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.

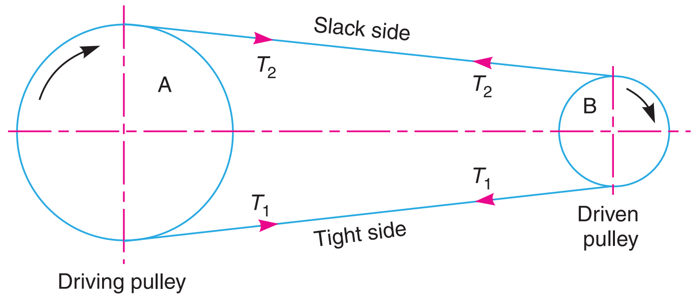


Fig. 11.14. Power transmitted by a belt.

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

$\therefore$ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\, v$ W ...($\because$ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley *i.e.* follower is $(T_1 - T_2)\, r_2$.

11.14. Ratio of Driving Tensions For Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. 11.15.

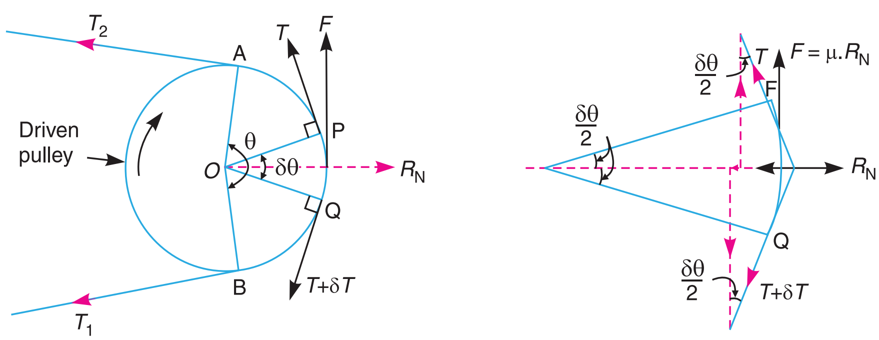


Fig. 11.15. Ratio of driving tensions for flat belt.

Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc AB, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 11.15. The belt PQ is in equilibrium under the following forces :

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T) \sin \frac{\delta\theta}{2} + T \sin \frac{\delta\theta}{2} \quad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta / 2 = \delta\theta / 2$ in equation (*i*),

$$R_N = (T + \delta T) \frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \quad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T) \cos \frac{\delta\theta}{2} - T \cos \frac{\delta\theta}{2} \quad ...(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta / 2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \text{ or } R_N = \frac{\delta T}{\mu} \quad ...(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \text{ or } \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

i.e. $$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \quad \text{or} \quad \log_e \left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta} \quad ...(v)$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

11.15. Determination of Angle of Contact

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig. 11.16 (*a*), then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).

From Fig. 11.16 (*a*),

$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \quad ...(\because ME = O_2 F = r_2)$$

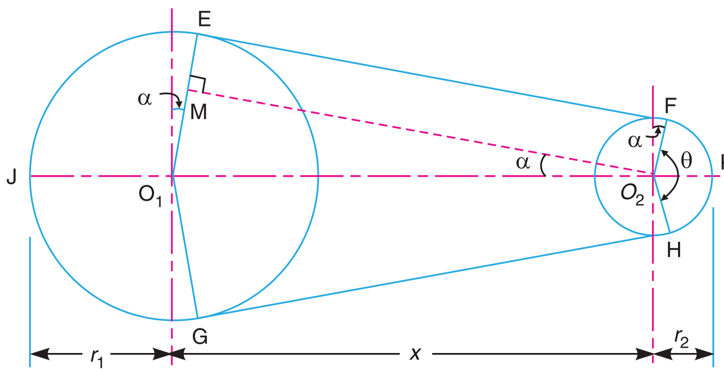
$\therefore$ Angle of contact or lap,

$$\theta = (180° - 2\alpha) \frac{\pi}{180} \text{ rad}$$

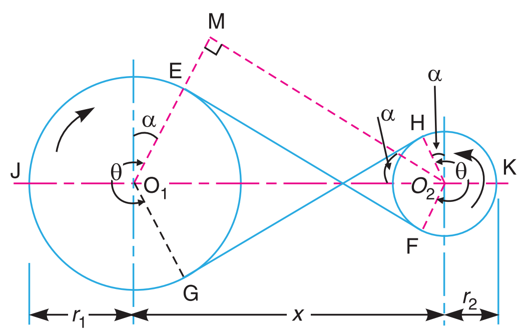
A little consideration will show that when the two pulleys are connected by means of a crossed belt as shown in Fig. 11.16 (*b*), then the angle of contact or lap (θ) on both the pulleys is same. From Fig. 11.16 (*b*),

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

$\therefore$ Angle of contact or lap, $\theta = (180° + 2\alpha)\dfrac{\pi}{180}$ rad



(*a*) Open belt drive.



(*b*) Crossed belt drive.

Fig. 11.16

Example 11.4. *Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2500 N.*

Solution. Given : d = 600 mm = 0.6 m ; N = 200 r.p.m. ; μ = 0.25 ; θ = 160° = 160 × π / 180 = 2.793 rad ; T_1 = 2500 N

We know that velocity of the belt,

$$v = \frac{\pi d . N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284 \text{ m/s}$$

Let T_2 = Tension in the slack side of the belt.

We know that $2.3 \log\left(\dfrac{T_1}{T_2}\right) = \mu . \theta = 0.25 \times 2.793 = 0.6982$

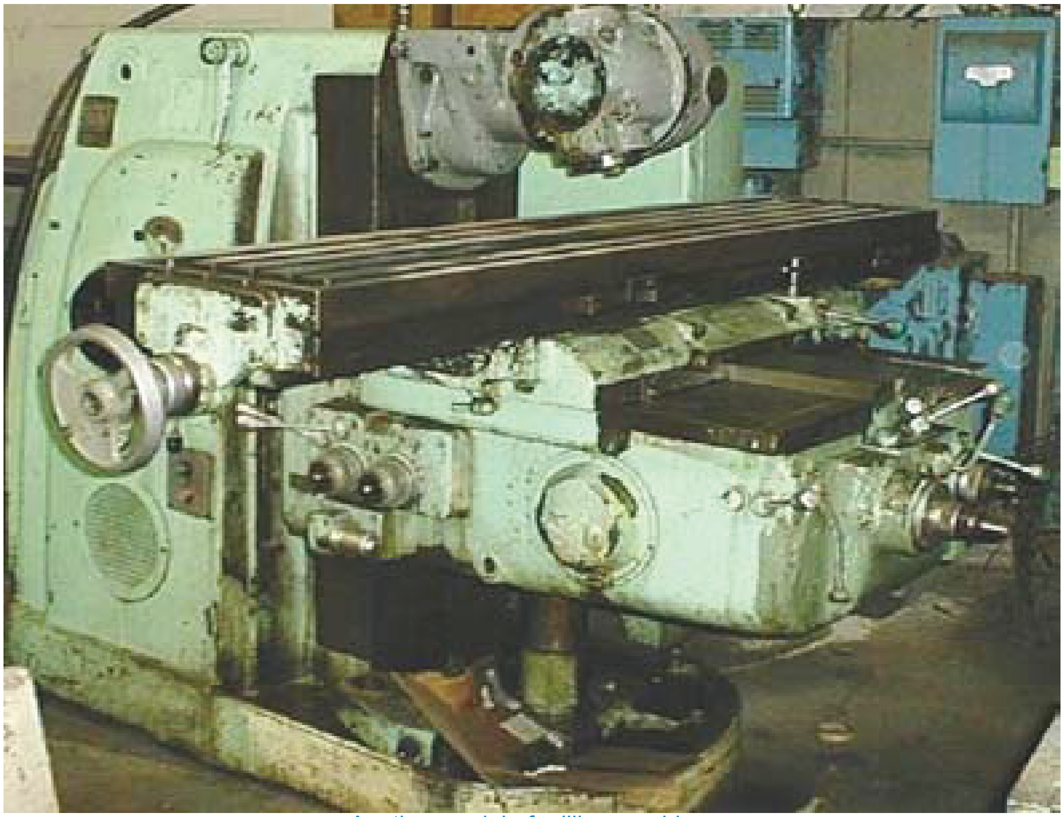
$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6982}{2.3} = 0.3036$$

$\therefore$ $$\frac{T_1}{T_2} = 2.01 \quad \text{...(Taking antilog of 0.3036)}$$

and $$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

We know that power transmitted by the belt,

$$P = (T_1 - T_2)\, v = (2500 - 1244)\, 6.284 = 7890 \text{ W}$$
$$= 7.89 \text{ kW} \quad \textbf{Ans.}$$



Another model of milling machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 11.5. *A casting weighing 9 kN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20 r.p.m. The other end of the rope is pulled by a man. The coefficient of friction is 0.25. Determine* ***1.*** *The force required by the man, and* ***2.*** *The power to raise the casting.*

Solution. Given : $W = T_1 = 9$ kN = 9000 N ; d = 300 mm = 0.3 m ; N = 20 r.p.m. ; $\mu = 0.25$

1. ***Force required by the man***

Let T_2 = Force required by the man.

Since the rope makes 2.5 turns round the drum, therefore angle of contact,

$$\theta = 2.5 \times 2\,\pi = 5\,\pi \text{ rad}$$

We know that $$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 5\pi = 3.9275$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{3.9275}{2.3} = 1.71 \quad \text{or} \quad \frac{T_1}{T_2} = 51$$

...(Taking antilog of 1.71)

$\therefore$ $$T_2 = \frac{T_1}{51} = \frac{9000}{51} = 176.47 \text{ N} \quad \textbf{Ans.}$$

2. ***Power to raise the casting***

We know that velocity of the rope,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 0.3 \times 20}{60} = 0.3142 \text{ m/s}$$

$\therefore$ Power to raise the casting,

$$P = (T_1 - T_2)\, v = (9000 - 176.47)\, 0.3142 = 2772 \text{ W}$$
$$= 2.772 \text{ kW} \quad \textbf{Ans.}$$

Example 11.6. *Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.*

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution. Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

We know that speed of the belt,

$$v = \frac{\pi d_1.N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{ m} \quad \textbf{Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6°$$

$\therefore$ $$\theta = 180° + 2\,\alpha = 180° + 2 \times 9.6° = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad} \quad \textbf{Ans.}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \quad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.714 = 2740 \text{ W} = 2.74 \text{ kW} \quad \textbf{Ans.}$$

11.16. Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

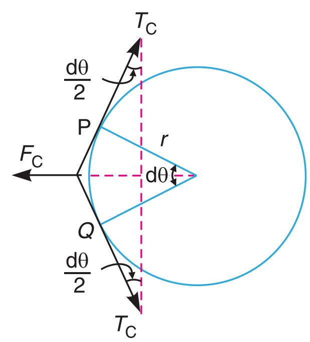


Fig. 11.17. Centrifugal tension.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ the centre of the pulley as shown in Fig. 11.17.

Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at *P* and *Q* in newtons.

We know that length of the belt *PQ*

$$= r.\, d\theta$$

and mass of the belt *PQ* $= m.\, r.\, d\theta$

$\therefore$ Centrifugal force acting on the belt *PQ*,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at *P* and *Q* keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m \,.\, v^2$$

Notes : 1. When the centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

2. Power transmitted,

$$P = (T_{t1} - T_{t2})\, v \quad \text{...(in watts)}$$

$$= [(T_1 + T_C) - (T_2 + T_C)]\, v = (T_1 - T_2)\, v \quad \text{...(same as before)}$$

Thus we see that centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as

$$2.3\log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

11.17. Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = $\sigma.\, b.\, t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

11.18. Condition For the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\, v \quad \text{...}(i)$$

where T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \quad \text{...}(ii)$$

Substituting the value of T_2 in equation ***(i)***,

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C \quad \text{...}(iii)$$

where $$C = 1 - \frac{1}{e^{\mu.\theta}}$$

We know that $T_1 = T - T_C$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation ***(iii)***,

$$P = (T - T_C)\, v.C$$

$$= (T - m.v^2)\, v.C = (T.v - m\, v^3)\, C \qquad \text{... (Substituting } T_C = m.\, v^2)$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \qquad \text{or} \qquad \frac{d}{dv}(T.v - mv^3)\, C = 0$$

$$\therefore \qquad T - 3\, m\, .\, v^2 = 0$$

or $$T - 3\, T_C = 0 \quad \text{or} \quad T = 3\, T_C \qquad \text{...}(iv)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Notes : **1.** We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$\therefore \qquad T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

2. From equation ***(iv)***, the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Example. 11.7. *A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is* ***1.*** *an open belt drive, and* ***2.*** *a cross belt drive. Take μ = 0.3.*

Solution. Given : N_1 = 200 r.p.m. ; N_2 = 300 r.p.m. ; P = 6 kW = 6 × 10^3 W ; b = 100 mm ; t = 10 mm ; x = 4 m ; d_2 = 0.5 m ; μ = 0.3

Let σ = Stress in the belt.

1. *Stress in the belt for an open belt drive*

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2.d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75\,\text{m}$$

and velocity of the belt, $$v = \frac{\pi\, d_2.N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \quad \text{or} \quad \alpha = 1.8°$$

$\therefore$ Angle of contact, $\theta = 180° - 2\,\alpha = 180 - 2 \times 1.8 = 176.4°$

$= 176.4 \times \pi / 180 = 3.08$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.924}{2.3} = 0.4017 \quad \text{or} \quad \frac{T_1}{T_2} = 2.52 \qquad ...(i)$$

...(Taking antilog of 0.4017)

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2)\,v = (T_1 - T_2)\,7.855$$

$$\therefore \quad T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N} \qquad ...(ii)$$

From equations (*i*) and (*ii*),

$$T_1 = 1267 \text{ N, and } T_2 = 503 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1267 = \sigma\,.\,b.\,t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\therefore \quad \sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa} \quad \textbf{Ans.}$$

...[$\because$ 1 MPa = 1 MN/m^2 = 1 N/mm^2]

Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \quad \text{or} \quad \alpha = 9°$$

$\therefore$ Angle of contact, $\theta = 180° + 2\alpha = 180 + 2 \times 9 = 198°$

$= 198 \times \pi / 180 = 3.456$ rad

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.456 = 1.0368$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.0368}{2.3} = 0.4508 \quad \text{or} \quad \frac{T_1}{T_2} = 2.82 \qquad ...(iii)$$

...(Taking antilog of 0.4508)

From equations (*ii*) and (*iii*),

$$T_1 = 1184 \text{ N and } T_2 = 420 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1184 = \sigma.\,b.\,t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\therefore \quad \sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa} \quad \textbf{Ans.}$$

Example 11.8. *A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the coefficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather 1 Mg/m^3 and thickness of belt 10 mm, determine the width of the belt taking centrifugal tension into account.*

Solution. Given : $P = 7.5$ kW = 7500 W ; $d = 1.2$ m ; $N = 250$ r.p.m. ; $\theta = 165° = 165 \times \pi / 180 = 2.88$ rad ; $\mu = 0.3$; $\sigma = 1.5$ MPa $= 1.5 \times 10^6$ * N/m^2 ; $\rho = 1$ Mg/m^3 $= 1 \times 10^6$ g/m^3 = 1000 kg/m^3; $t = 10$ mm = 0.01 m

Let b = Width of belt in metres,

T_1 = Tension in the tight side of the belt in N, and

T_2 = Tension in the slack side of the belt in N.

We know that velocity of the belt,

$$v = \pi\, d \,.\, N / 60 = \pi \times 1.2 \times 250/60 = 15.71 \text{ m/s}$$

and power transmitted (P),

$$7500 = (T_1 - T_2)\, v = (T_1 - T_2)\, 15.71$$

$$\therefore \quad T_1 - T_2 = 7500 / 15.71 = 477.4 \text{ N} \qquad \text{...}(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu . \theta = 0.3 \times 2.88 = 0.864$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{0.864}{2.3} = 0.3756 \quad \text{or} \quad \frac{T_1}{T_2} = 2.375 \qquad \text{...}(ii)$$

...(Taking antilog of 0.3756)

From equations (*i*) and (*ii*),

$$T_1 = 824.6 \text{ N}, \quad \text{and} \quad T_2 = 347.2 \text{ N}$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$

$$= b \times 0.01 \times 1 \times 1000 = 10\, b \text{ kg}$$

$\therefore$ Centrifugal tension,

$$T_C = m \,.\, v^2 = 10\, b\, (15.71)^2 = 2468\, b \text{ N}$$

and maximum tension in the belt,

$$T = \sigma .\, b .\, t = 1.5 \times 10^6 \times b \times 0.01 = 15\,000\, b \text{ N}$$

We know that $T = T_1 + T_C$ or $15000\, b = 824.6 + 2468\, b$

$$15\,000\, b - 2468\, b = 824.6 \quad \text{or} \quad 12\,532\, b = 824.6$$

$$\therefore \quad b = 824.6 / 12532 = 0.0658 \text{ m} = 65.8 \text{ mm}$$ **Ans.**

Example. 11.9. *Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 r.p.m. and the distance between the centre of two pulleys is 3 metres. The density of the leather is 1000 kg/m^3. The maximum allowable stress in the leather is 2.5 MPa. The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.*

Solution. Given : $t = 9.75$ mm $= 9.75 \times 10^{-3}$ m ; $P = 15$ kW $= 15 \times 10^3$ W ; $N_1 = 900$ r.p.m. ; $d_1 = 300$ mm = 0.3 m ; $N_2 = 300$ r.p.m. ; $x = 3$m ; $\rho = 1000$ kg/m^3 ; $\sigma = 2.5$ MPa $= 2.5 \times 10^6$ N/m^2 ; $\mu = 0.3$

* 1 MPa = 1×10^6 N/m^2

First of all, let us find out the diameter of the driven pulley (d_2). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_2 = \frac{N_1 \times d_1}{N_2} = \frac{900 \times 0.3}{300} = 0.9 \text{ m}$$

and velocity of the belt, $$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.3 \times 900}{60} = 14.14 \text{ m/s}$$

For an open belt drive,

$$\sin \alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.9 - 0.3}{2 \times 3} = 0.1 \qquad ...(\because d_2 > d_1)$$

or $$\alpha = 5.74°$$

$\therefore$ Angle of lap, $\theta = 180° - 2\,\alpha = 180 - 2 \times 5.74 = 168.52°$

$= 168.52 \times \pi / 180 = 2.94$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.94 = 0.882$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.882}{2.3} = 0.3835 \quad \text{or} \quad \frac{T_1}{T_2} = 2.42 \qquad ...(i)$$

... (Taking antilog of 0.3835)

We also know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.14$$

$$\therefore \quad T_1 - T_2 = 15 \times 10^3 / 14.14 = 1060 \text{ N} \qquad .... (ii)$$

From equations ***(i)*** and ***(ii)***,

$$T_1 = 1806 \text{ N}$$

Let b = Width of the belt in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$

$$= b \times 9.75 \times 10^{-3} \times 1 \times 1000 = 9.75\, b \text{ kg}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 9.75\, b\, (14.14)^2 = 1950\ b \text{ N}$$

Maximum tension in the belt,

$$T = \sigma.\, b.\, t = 2.5 \times 10^6 \times b \times 9.75 \times 10^{-3} = 24\,400\, b \text{ N}$$

We know that $T = T_1 + T_C$ or $T - T_C = T_1$

$$24\,400\, b - 1950\, b = 1806 \quad \text{or} \quad 22\,450\, b = 1806$$

$\therefore$ $b = 1806 / 22\,450 = 0.080$ m $= 80$ mm **Ans.**

Example. 11.10. *A pulley is driven by a flat belt, the angle of lap being 120°. The belt is 100 mm wide by 6 mm thick and density1000 kg/m³. If the coefficient of friction is 0.3 and the maximum stress in the belt is not to exceed 2 MPa, find the greatest power which the belt can transmit and the corresponding speed of the belt.*

Solution. Given : $\theta = 120° = 120 \times \pi / 180 = 2.1$ rad ; $b = 100$ mm $= 0.1$ m ; $t = 6$ mm $= 0.006$ m ; $\rho = 1000$ kg / m^3 ; $\mu = 0.3$; $\sigma = 2$ MPa $= 2 \times 10^6$ N/m^2

Speed of the belt for greatest power

We know that maximum tension in the belt,

$$T = \sigma.\, b.\, t = 2 \times 10^6 \times 0.1 \times 0.006 = 1200 \text{ N}$$

and mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.\, t.\, l.\, \rho$$
$$= 0.1 \times 0.006 \times 1 \times 1000 = 0.6 \text{ kg/m}$$

∴ Speed of the belt for greatest power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1200}{3 \times 0.6}} = 25.82 \text{ m/s} \qquad \textbf{Ans.}$$

Greatest power which the belt can transmit

We know that for maximum power to be transmitted, centrifugal tension,

$$T_C = T/3 = 1200/3 = 400 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 1200 - 400 = 800 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.1 = 0.63$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.63}{2.3} = 0.2739 \quad \text{or} \quad \frac{T_1}{T_2} = 1.88 \qquad \text{...(Taking antilog of 0.2739)}$$

and
$$T_2 = \frac{T_1}{1.88} = \frac{800}{1.88} = 425.5 \text{ N}$$

∴ Greatest power which the belt can transmit,

$$P = (T_1 - T_2)\, v = (800 - 425.5)\, 25.82 = 9670 \text{ W} = 9.67 \text{ kW} \quad \textbf{Ans.}$$

Example 11.11. *An open belt drive connects two pulleys 1.2 m and 0.5 m diameter, on parallel shafts 4 metres apart. The mass of the belt is 0.9 kg per metre length and the maximum tension is not to exceed 2000 N. The coefficient of friction is 0.3. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Due to belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, and power lost in friction. What is the efficiency of the drive ?*

Solution. Given : $d_1 = 1.2$ m or $r_1 = 0.6$ m ; $d_2 = 0.5$ m or $r_2 = 0.25$ m ; $x = 4$ m ; $m = 0.9$ kg/m; $T = 2000$ N ; $\mu = 0.3$; $N_1 = 200$ r.p.m. ; $N_2 = 450$ r.p.m.

We know that velocity of the belt,

$$v = \frac{\pi\, d_1 . N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s}$$

and centrifugal tension, $T_C = m.v^2 = 0.9\,(12.57)^2 = 142$ N

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 2000 - 142 = 1858 \text{ N}$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} = 0.0875 \quad \text{or} \quad \alpha = 5.02°$$

∴ Angle of lap on the smaller pulley,

$$\theta = 180° - 2\,\alpha = 180° - 2 \times 5.02° = 169.96°$$
$$= 169.96 \times \pi / 180 = 2.967 \text{ rad}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.967 = 0.8901$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8901}{2.3} = 0.387 \quad \text{or} \quad \frac{T_1}{T_2} = 2.438$$

...(Taking antilog of 0.387)

$$\therefore \quad T_2 = \frac{T_1}{2.438} = \frac{1858}{2.438} = 762 \text{ N}$$

Torque on the shaft of larger pulley

We know that torque on the shaft of larger pulley,

$$T_L = (T_1 - T_2)\,r_1 = (1858 - 762)\,0.6 = 657.6 \text{ N-m}$$ **Ans.**

Torque on the shaft of smaller pulley

We know that torque on the shaft of smaller pulley,

$$T_S = (T_1 - T_2)\,r_2 = (1858 - 762)\,0.25 = 274 \text{ N-m}$$ **Ans.**

Power transmitted

We know that the power transmitted,

$$P = (T_1 - T_2)\,v = (1858 - 762)\,12.57 = 13780 \text{ W}$$
$$= 13.78 \text{ kW}$$ **Ans.**

Power lost in friction

We know that input power,

$$P_1 = \frac{T_L \times 2\pi N_1}{60} = \frac{657.6 \times 2\pi \times 200}{60} = 13\,780 \text{ W} = 13.78 \text{ kW}$$

and output power,

$$P_2 = \frac{T_S \times 2\pi N_2}{60} = \frac{274 \times 2\pi \times 450}{60} = 12\,910 \text{ W} = 12.91 \text{ kW}$$

∴ Power lost in friction $= P_1 - P_2 = 13.78 - 12.91 = 0.87$ kW **Ans.**

Efficiency of the drive

We know that efficiency of the drive,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{12.91}{13.78} = 0.937 \text{ or } 93.7\%$$ **Ans.**

11.19. Initial Tension in the Belt

When a belt is wound round the two pulleys (*i.e.* driver and follower), its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called ***initial tension***.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers it to the other side (decreasing the tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side of the belt is called tension in the slack side.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha\,(T_1 - T_0) \quad \text{...}(i)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha\,(T_0 - T_2) \quad \text{...}(ii)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations **(*i*)** and **(*ii*)**,

$$\alpha\,(T_1 - T_0) = \alpha\,(T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

$$\therefore \quad T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2\,T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

Example. 11.12. *In a flat belt drive the initial tension is 2000 N. The coefficient of friction between the belt and the pulley is 0.3 and the angle of lap on the smaller pulley is 150°. The smaller pulley has a radius of 200 mm and rotates at 500 r.p.m. Find the power in kW transmitted by the belt.*

Solution. Given : $T_0 = 2000$ N ; $\mu_0 = 0.3$; $\theta = 150° = 150° \times \pi / 180 = 2.618$ rad ; $r_2 = 200$ mm or $d_2 = 400$ mm $= 0.4$ m ; $N_2 = 500$ r.p.m.

We know that velocity of the belt,

$$v = \frac{\pi\, d_2 . N_2}{60} = \frac{\pi \times 0.4 \times 500}{60} = 10.47 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that initial tension (T_0),

$$2000 = \frac{T_1 + T_2}{2} \quad \text{or} \quad T_1 + T_2 = 4000 \text{ N} \quad \ldots(i)$$

We also know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta = 0.3 \times 2.618 = 0.7854$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{0.7854}{2.3} = 0.3415$$

or $$\frac{T_1}{T_2} = 2.2 \quad \ldots(ii)$$

...(Taking antilog of 0.3415)

A military tank uses chain, belt and gear drives for its movement and operation.

From equations (*i*) and (*ii*),

$$T_1 = 2750 \text{ N};$$

and $$T_2 = 1250 \text{ N}$$

∴ Power transmitted, $P = (T_1 - T_2)\, v$

$$= (2750 - 1250)\ 10.47$$

$$= 15\ 700 \text{ W} = 15.7 \text{ kW} \quad \textbf{Ans.}$$

Example 11.13. *Two parallel shafts whose centre lines are 4.8 m apart, are connected by open belt drive. The diameter of the larger pulley is 1.5 m and that of smaller pulley 1 m. The initial tension in the belt when stationary is 3 kN. The mass of the belt is 1.5 kg / m length. The coefficient of friction between the belt and the pulley is 0.3. Taking centrifugal tension into account, calculate the power transmitted, when the smaller pulley rotates at 400 r.p.m.*

Solution. Given : $x = 4.8$ m ; $d_1 = 1.5$ m ; $d_2 = 1$ m ; $T_0 = 3$ kN = 3000 N ; $m = 1.5$ kg / m ; $\mu = 0.3$; $N_2 = 400$ r.p.m.

We know that velocity of the belt,

$$v = \frac{\pi d_2 . N_2}{60} = \frac{\pi \times 1 \times 400}{60} = 21 \text{ m/s}$$

and centrifugal tension, $T_C = m.v^2 = 1.5\,(21)^2 = 661.5$ N

Let T_1 = Tension in the tight side, and

T_2 = Tension in the slack side.

We know that initial tension (T_0),

$$3000 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{T_1 + T_2 + 2 \times 661.5}{2}$$

$$\therefore \quad T_1 + T_2 = 3000 \times 2 - 2 \times 661.5 = 4677 \text{ N} \quad \ldots(i)$$

For an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{1.5 - 1}{2 \times 4.8} = 0.0521 \quad \text{or} \quad \alpha = 3°$$

∴ Angle of lap on the smaller pulley,

$$\theta = 180° - 2\,\alpha = 180° - 2 \times 3° = 174°$$

$$= 174° \times \pi / 180 = 3.04 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.04 = 0.912$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.912}{2.3} = 0.3965 \quad \text{or} \quad \frac{T_1}{T_2} = 2.5 \qquad \text{...}(ii)$$

...(Taking antilog of 0.3965)

From equations **(*i*)** and **(*ii*),**

T_1 = 3341 N ; and T_2 = 1336 N

∴ Power transmitted,

$P = (T_1 - T_2)\, v$ = (3341 – 1336) 21 = 42 100 W = 42.1 kW **Ans.**

Example 11.14. *An open flat belt drive connects two parallel shafts 1.2 metres apart. The driving and the driven shafts rotate at 350 r.p.m. and 140 r.p.m. respectively and the driven pulley is 400 mm in diameter. The belt is 5 mm thick and 80 mm wide. The coefficient of friction between the belt and pulley is 0.3 and the maximum permissible tension in the belting is 1.4 MN/m². Determine*:

1. *diameter of the driving pulley,* ***2.*** *maximum power that may be transmitted by the belting, and* ***3.*** *required initial belt tension.*

Solution. Given : x = 1.2 m ; N_1 = 350 r.p.m. ; N_2 = 140 r.p.m. ; d_2 = 400 mm = 0.4 m ; t = 5 mm = 0.005 m ; b = 80 mm = 0.08 m ; μ = 0.3 ; σ = 1.4 MN/m² = 1.4 × 10⁶ N/m²

1. *Diameter of the driving pulley*

Let d_1 = Diameter of the driving pulley.

We know that
$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2 . d_2}{N_1} = \frac{140 \times 0.4}{350} = 0.16 \text{ m} \quad \textbf{Ans.}$$

2. *Maximum power transmitted by the belting*

First of all, let us find the angle of contact of the belt on the smaller pulley (or driving pulley).

Let θ = Angle of contact of the belt on the driving pulley.

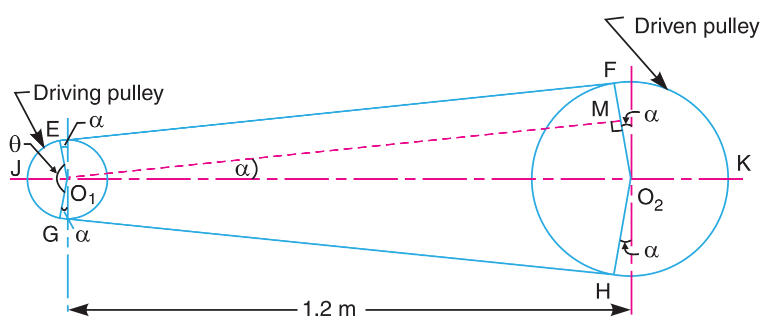


Fig. 11.18

From Fig. 11.18, we find that

$$\sin\alpha = \frac{O_2 M}{O_1 O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.4 - 0.16}{2 \times 1.2} = 0.1$$

or $\alpha = 5.74°$

$\therefore$ $\theta = 180° - 2\alpha = 180° - 2 \times 5.74° = 168.52°$

$= 168.52 \times \pi / 180 = 2.94$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.94 = 0.882$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.882}{2.3} = 0.3835 \quad \text{or} \quad \frac{T_1}{T_2} = 2.42 \qquad ...(i)$$

...(Taking antilog of 0.3835)

We know that maximum tension to which the belt can be subjected,

$$T_1 = \sigma \times b \times t = 1.4 \times 10^6 \times 0.08 \times 0.005 = 560 \text{ N}$$

$\therefore$
$$T_2 = \frac{T_1}{2.42} = \frac{560}{2.42} = 231.4 \text{ N} \qquad ...[\text{From equation } (i)]$$

Velocity of the belt,
$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 0.16 \times 350}{60} = 2.93 \text{ m/s}$$

$\therefore$ Power transmitted, $P = (T_1 - T_2)\, v = (560 - 231.4)\, 2.93 = 963$ W $= 0.963$ kW **Ans.**

3. *Required initial belt tension*

We know that the initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{560 + 231.4}{2} = 395.7 \text{ N} \quad \textbf{Ans.}$$

Example 11.15. *An open belt running over two pulleys 240 mm and 600 mm diameter connects two parallel shafts 3 metres apart and transmits 4 kW from the smaller pulley that rotates at 300 r.p.m. Coefficient of friction between the belt and the pulley is 0.3 and the safe working tension is 10N per mm width. Determine :* **1.** *minimum width of the belt,* **2.** *initial belt tension, and* **3.** *length of the belt required.*

Solution. Given : $d_2 = 240$ mm $= 0.24$ m ; $d_1 = 600$ mm $= 0.6$ m ; $x = 3$ m ; $P = 4$ kW $= 4000$ W; $N_2 = 300$ r.p.m. ; $\mu = 0.3$; $T_1 = 10$ N/mm width

1. *Minimum width of belt*

We know that velocity of the belt,

$$v = \frac{\pi d_2 . N_2}{60} = \frac{\pi \times 0.24 \times 300}{60} = 3.77 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$\therefore$ Power transmitted (P),

$$4000 = (T_1 - T_2)\, v = (T_1 - T_2)\, 3.77$$

or
$$T_1 - T_2 = 4000 / 3.77 = 1061 \text{ N} \qquad ...(i)$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.6 - 0.24}{2 \times 3} = 0.06 \quad \text{or} \quad \alpha = 3.44°$$

and angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2 \times 3.44° = 173.12°$$

$$= 173.12 \times \pi / 180 = 3.022 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.022 = 0.9066$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.9066}{2.3} = 0.3942 \quad \text{or} \quad \frac{T_1}{T_2} = 2.478 \qquad \text{...}(ii)$$

...(Taking antilog of 0.3942)

From equations **(*i*)** and **(*ii*)**,

$$T_1 = 1779 \text{ N}, \text{ and } T_2 = 718 \text{ N}$$

Since the safe working tension is 10 N per mm width, therefore minimum width of the belt,

$$b = \frac{T_1}{10} = \frac{1779}{10} = 177.9 \text{ mm} \quad \textbf{Ans.}$$

2. *Initial belt tension*

We know that initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1779 + 718}{2} = 1248.5 \text{ N} \quad \textbf{Ans.}$$

3. *Length of the belt required*

We know that length of the belt required,

$$L = \frac{\pi}{2}(d_1 - d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.6 + 0.24) + 2 \times 3 + \frac{(0.6 - 0.24)^2}{4 \times 3}$$

$$= 1.32 + 6 + 0.01 = 7.33 \text{ m} \quad \textbf{Ans.}$$

Example 11.16. *The following data refer to an open belt drive* :

Diameter of larger pulley = 400 mm ; Diameter of smaller pulley = 250 mm ; Distance between two pulleys = 2 m ; Coefficient of friction between smaller pulley surface and belt = 0.4 ; Maximum tension when the belt is on the point of slipping = 1200 N.

Find the power transmitted at speed of 10 m/s. It is desired to increase the power. Which of the following two methods you will select ?

1. Increasing the initial tension in the belt by 10 per cent.

2. Increasing the coefficient of friction between the smaller pulley surface and belt by 10 per cent by the application of suitable dressing on the belt.

Find, also, the percentage increase in power possible in each case.

Solution. Given : d_1 = 400 mm = 0.4 m ; d_2 = 250 mm = 0.25 m ; x = 2 m ; μ = 0.4 ; T = 1200 N ; v = 10 m/s

Power transmitted

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.4 - 0.25}{2 \times 2} = 0.0375 \quad \text{or} \quad \alpha = 2.15°$$

$\therefore$ Angle of contact,

$$\theta = 180° - 2\alpha = 180° - 2 \times 2.15° = 175.7°$$

$$= 175.7 \times \pi / 180 = 3.067 \text{ rad}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

Neglecting centrifugal tension,

$$T_1 = T = 1200 \text{ N} \qquad \text{...(Given)}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.4 \times 3.067 = 1.2268$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.2268}{2.3} = 0.5334 \quad \text{or} \quad \frac{T_1}{T_2} = 3.41$$

...(Taking antilog of 0.5334)

and

$$T_2 = \frac{T_1}{3.41} = \frac{1200}{3.41} = 352 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1200 - 352)\, 10 = 8480 \text{ W} = 8.48 \text{ kW} \quad \textbf{Ans.}$$

Power transmitted when initial tension is increased by 10%

We know that initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1200 + 352}{2} = 776 \text{ N}$$

$\therefore$ Increased initial tension,

$$T_0' = 776 + \frac{776 \times 10}{100} = 853.6 \text{ N}$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side of the belt respectively.

$$\therefore \quad T_0' = \frac{T_1 + T_2}{2}$$

or

$$T_1 + T_2 = 2\, T'_0 = 2 \times 853.6 = 1707.2 \text{ N} \qquad \text{...}(i)$$

Since the ratio of tensions is constant, therefore

$$\frac{T_1}{T_2} = 3.41 \qquad \text{...}(ii)$$

From equations ***(i)*** and ***(ii)*,**

$$T_1 = 1320.2 \text{ N} \text{ ; and } T_2 = 387 \text{ N}$$

∴ Power transmitted, $P = (T_1 - T_2)\, v = (1320.2 - 387)\, 10 = 9332 \text{ W} = 9.332 \text{ kW}$

Power transmitted when coefficient of friction is increased by 10%

We know that coefficient of friction,

$$\mu = 0.4$$

∴ Increased coefficient of friction,

$$\mu' = 0.4 + 0.4 \times \frac{10}{100} = 0.44$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side respectively.

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu' . \theta = 0.44 \times 3.067 = 1.3495$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{1.3495}{2.3} = 0.5867 \quad \text{or} \quad \frac{T_1}{T_2} = 3.86 \qquad \text{...}(iii)$$

... (Taking antilog of 0.5867)

Here the initial tension is constant, *i.e.*

$$T_0 = \frac{T_1 + T_2}{2} \quad \text{or} \quad T_1 + T_2 = 2\, T_0 = 2 \times 776 = 1552 \text{ N} \qquad \text{...}(iv)$$

From equations ***(iii)*** and ***(iv)*,**

$$T = 1232.7 \text{ N and } T_2 = 319.3 \text{ N}$$

∴ Power transmitted,

$$P = (T_1 - T_2)\, v = (1232.7 - 319.3)\, 10 = 9134 \text{ W} = 9.134 \text{ kW}$$

Since the power transmitted by increasing the initial tension is more, therefore in order to increase the power transmitted we shall adopt the method of increasing the initial tension. **Ans.**

Percentage increase in power

We know that percentage increase in power when the initial tension is increased

$$= \frac{9.332 - 8.48}{8.48} \times 100 = 10.05\% \quad \textbf{Ans.}$$

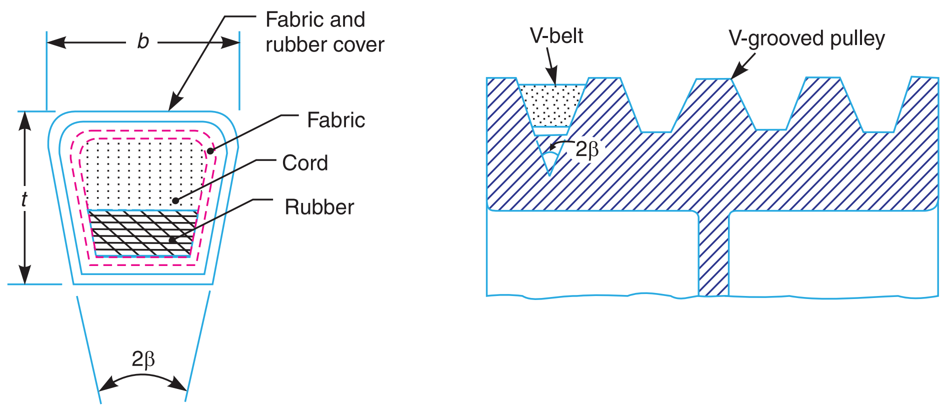
and percentage increase in power when coefficient of friction is increased

$$= \frac{9.134 - 8.48}{8.48} \times 100 = 7.7\% \quad \textbf{Ans.}$$

11.20. V-belt drive

We have already discussed that a V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

The V-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber, as shown in Fig. 11.19 (*a*). These belts are moulded to a trapezoidal shape and are made endless. These are particularly suitable for short drives *i.e.* when the shafts are at a short distance apart. The included angle for the V-belt is usually from 30° – 40°. In case of flat belt drive, the belt runs over the pulleys whereas in case of V-belt drive, the rim of the pulley is grooved in which the V-belt runs. The effect of the groove is to increase the frictional grip of the V-belt on the pulley and thus to reduce the tendency of slipping. In order to have a good grip on the pulley, the V-belt is in contact with the side faces of the groove and not at the bottom. The power is transmitted by the *wedging action between the belt and the V-groove in the pulley.



(*a*) Cross-section of a V-belt. (*b*) Cross-section of a V-grooved pulley.

Fig. 11.19. V-belt and V-grooved pulley.

A clearance must be provided at the bottom of the groove, as shown in Fig. 11.19 (*b*), in order to prevent touching to the bottom as it becomes narrower from wear. The V-belt drive, may be inclined at any angle with tight side either at top or bottom. In order to increase the power output, several V- belts may be operated side by side. It may be noted that in multiple V-belt drive, all the belts should stretch at the same rate so that the load is equally divided between them. When one of the set of belts break, the entire set should be replaced at the same time. If only one belt is replaced, the new unworn and unstressed belt will be more tightly stretched and will move with different velocity.

11.21. Advantages and Disadvantages of V-belt Drive Over Flat Belt Drive

Following are the advantages and disadvantages of the V-belt drive over flat belt drive.

Advantages

1. The V-belt drive gives compactness due to the small distance between the centres of pulleys.
2. The drive is positive, because the slip between the belt and the pulley groove is negligible.
3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.
4. It provides longer life, 3 to 5 years.

* The wedging action of the V-belt in the groove of the pulley results in higher forces of friction. A little consideration will show that the wedging action and the transmitted torque will be more if the groove angle of the pulley is small. But a smaller groove angle will require more force to pull the belt out of the groove which will result in loss of power and excessive belt wear due to friction and heat. Hence a selective groove angle is a compromise between the two. Usually the groove angles of 32° to 38° are used.

5. It can be easily installed and removed.

6. The operation of the belt and pulley is quiet.

7. The belts have the ability to cushion the shock when machines are started.

8. The high velocity ratio (maximum 10) may be obtained.

9. The wedging action of the belt in the groove gives high value of limiting ratio of tensions. Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.

10. The V-belt may be operated in either direction with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages

1. The V-belt drive cannot be used with large centre distances.

2. The V-belts are not so durable as flat belts.

3. The construction of pulleys for V-belts is more complicated than pulleys for flat belts.

4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed application such as synchronous machines, and timing devices.

5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.

6. The centrifugal tension prevents the use of V-belts at speeds below 5 m/s and above 50m/s.

11.22. Ratio of Driving Tensions for V-belt

A V-belt with a grooved pulley is shown in Fig. 11.20.

Let R_1 = Normal reaction between the belt and sides of the groove.

R = Total reaction in the plane of the groove.

$2\,\beta$ = Angle of the groove.

μ = Coefficient of friction between the belt and sides of the groove.

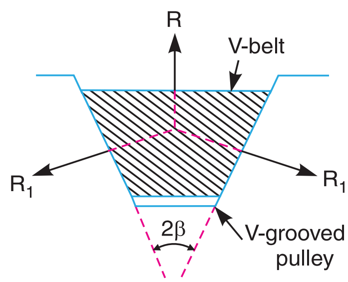


Fig. 11.20.

Resolving the reactions vertically to the groove,

$$R = R_1 \sin\beta + R_1 \sin\beta = 2\,R_1 \sin\beta$$

or

$$R_1 = \frac{R}{2\sin\beta}$$

We know that the frictional force

$$= 2\mu . R_1 = 2\mu \times \frac{R}{2\sin\beta} = \frac{\mu . R}{\sin\beta} = \mu . R\ \mathrm{cosec}\,\beta$$

Consider a small portion of the belt, as in Art. 11.14, subtending an angle $\delta\theta$ at the centre. The tension on one side will be T and on the other side $T + \delta T$. Now proceeding as in Art. 11.14, we get the frictional resistance equal to $\mu . R$ cosec β instead of $\mu . R$. Thus the relation between T_1 and T_2 for the V-belt drive will be

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta\, \mathrm{cosec}\,\beta$$

Example 11.17. *A belt drive consists of two V-belts in parallel, on grooved pulleys of the same size. The angle of the groove is 30°. The cross-sectional area of each belt is 750 mm^2 and μ. = 0.12. The density of the belt material is 1.2 Mg/m^3 and the maximum safe stress in the material is 7 MPa. Calculate the power that can be transmitted between pulleys 300 mm diameter rotating at 1500 r.p.m. Find also the shaft speed in r.p.m. at which the power transmitted would be maximum.*

Solution. Given : $2\beta = 30°$ or $\beta = 15°$; $\alpha = 750$ mm^2 = 750×10^{-6} m^2 ; $\mu = 0.12$; $\rho = 1.2$ Mg/m^3 = 1200 kg/m^3 ; $\sigma = 7$ MPa = 7×10^6 N/m^2 ; $d = 300$ mm = 0.3 m ; $N = 1500$ r.p.m.

Power transmitted

We know that velocity of the belt,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 \text{ m/s}$$

and mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 750 \times 10^{-6} \times 1 \times 1200 = 0.9 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.9\,(23.56)^2 = 500 \text{ N}$$

We know that maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{cross-sectional area of belt} = \sigma \times a$$
$$= 7 \times 10^6 \times 750 \times 10^{-6} = 5250 \text{ N}$$

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 5250 - 500 = 4750 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

Since the pulleys are of the same size, therefore angle of contact, $\theta = 180° = \pi$ rad.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.12 \times \pi \times \operatorname{cosec} 15° = 1.457$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.457}{2.3} = 0.6334 \quad \text{or} \quad \frac{T_1}{T_2} = 4.3$$

...(Taking antilog of 0.6334)

and
$$T_2 = \frac{T_1}{4.3} = \frac{4750}{4.3} = 1105 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v \times 2 \qquad \text{...(∵ No. of belts = 2)}$$
$$= (4750 - 1105)\, 23.56 \times 2 = 171\ 752 \text{ W} = 171.752 \text{ kW}$$ **Ans.**

Shaft speed

Let N_1 = Shaft speed in r.p.m., and

v_1 = Belt speed in m/s.

We know that for maximum power, centrifugal tension,

$$T_C = T/3 \quad \text{or} \quad m\,(v_1)^2 = T/3 \quad \text{or} \quad 0.9\,(v_1)^2 = 5250/3 = 1750$$

$$\therefore \quad (v_1)^2 = 1750/0.9 = 1944.4 \quad \text{or} \quad v_1 = 44.1 \text{ m/s}$$

We know that belt speed (v_1),

$$44.1 = \frac{\pi d \,.\, N_1}{60} = \frac{\pi \times 0.3 \times N_1}{60} = 0.0157\,\mathrm{N}_1$$

$\therefore$ $N_1 = 44.1 / 0.0157 = 2809$ r.p.m. **Ans.**

Example 11.18. *Power is transmitted using a V-belt drive. The included angle of V-groove is 30°. The belt is 20 mm deep and maximum width is 20 mm. If the mass of the belt is 0.35 kg per metre length and maximum allowable stress is 1.4 MPa, determine the maximum power transmitted when the angle of lap is 140°. μ = 0.15.*

Solution. Given : $2\beta = 30°$ or $\beta = 15°$; $t = 20$ mm $= 0.02$ m ; $b = 20$ mm $= 0.02$ m ; $m = 0.35$ kg/m ; $\sigma = 1.4$ MPa $= 1.4 \times 10^6$ N/m^2 ; $\theta = 140° = 140° \times \pi / 180 = 2.444$ rad ; $\mu = 0.15$

We know that maximum tension in the belt,

$$T = \sigma . b . t = 1.4 \times 10^6 \times 0.02 \times 0.02 = 560 \text{ N}$$

and for maximum power to be transmitted, velocity of the belt,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{560}{3 \times 0.35}} = 23.1 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta \operatorname{cosec} \beta = 0.15 \times 2.444 \times \operatorname{cosec} 15° = 1.416$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.416}{2.3} = 0.616 \quad \text{or} \quad \frac{T_1}{T_2} = 4.13 \qquad \ldots(i)$$

...(Taking antilog of 0.616)

Centrifugal tension, $T_C = \dfrac{T}{3} = \dfrac{560}{3} = 187$ N

and $T_1 = T - T_C = 560 - 187 = 373$ N

$$T_2 = \frac{T_1}{4.13} = \frac{373}{4.13} = 90.3 \text{ N} \qquad \ldots[\text{From equation } (i)]$$

We know that maximum power transmitted,

$$P = (T_1 - T_2)\, v = (373 - 90.3)\, 23.1 = 6530 \text{ W} = 6.53 \text{ kW}$$ **Ans.**

Example 11.19. *A compressor, requiring 90 kW is to run at about 250 r.p.m. The drive is by V-belts from an electric motor running at 750 r.p.m. The diameter of the pulley on the compressor shaft must not be greater than 1 metre while the centre distance between the pulleys is limited to 1.75 metre. The belt speed should not exceed 1600 m/min.*

Determine the number of V-belts required to transmit the power if each belt has a cross-sectional area of 375 mm², density 1000 kg/m³ and an allowable tensile stress of 2.5 MPa. The groove angle of the pulley is 35°. The coefficient of friction between the belt and the pulley is 0.25. Calculate also the length required of each belt.

Solution. Given : $P = 90$ kW ; $N_2 = 250$ r.p.m. ; $N_1 = 750$ r.p.m. ; $d_2 = 1$ m ; $x = 1.75$ m ; $v = 1600$ m/min $= 26.67$ m/s ; $a = 375$ mm^2 $= 375 \times 10^{-6}$ m^2 ; $\rho = 1000$ kg/m^3 ; $\sigma = 2.5$ MPa $= 2.5 \times 10^6$ N/m^2 ; $2\beta = 35°$ or $\beta = 17.5°$; $\mu = 0.25$

First of all, let us find the diameter of pulley on the motor shaft (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2 . d_2}{N_1} = \frac{250 \times 1}{750} = 0.33 \text{ m}$$

We know that the mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density}$$
$$= 375 \times 10^{-6} \times 1 \times 1000 = 0.375 \text{ kg}$$

∴ Centrifugal tension, $T_C = m.v^2 = 0.375\,(26.67)^2 = 267$ N

and maximum tension in the belt,

$$T = \sigma .\, a = 2.5 \times 10^6 \times 375 \times 10^{-6} = 937.5 \text{ N}$$

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 937.5 - 267 = 670.5 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

For an open belt drive, as shown in Fig. 11.21,

$$\sin \alpha = \frac{O_2 M}{O_1 O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{1 - 0.33}{2 \times 1.75} = 0.1914$$

∴ $\alpha = 11°$

and angle of lap on smaller pulley (*i.e.* pulley on motor shaft),

$$\theta = 180° - 2\alpha = 180° - 2 \times 11° = 158°$$
$$= 158 \times \pi / 180 = 2.76 \text{ rad}$$

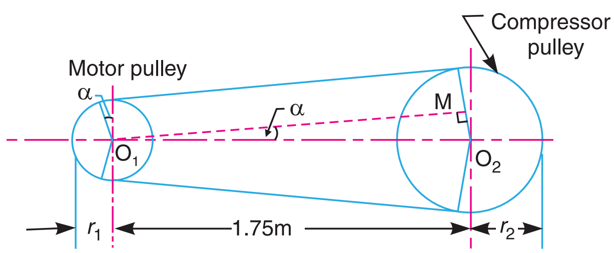


Fig. 11.21

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu . \theta \operatorname{cosec} \beta = 0.25 \times 2.76 \times \operatorname{cosec} 17.5° = 2.295$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{2.295}{2.3} = 0.998 \quad \text{or} \quad \frac{T_1}{T_2} = 9.954 \qquad \text{...(Taking antilog of 0.998)}$$

and $$T_2 = \frac{T_1}{9.954} = \frac{670.5}{9.954} = 67.36 \text{ N}$$

Number of V-belts

We know that power transmitted per belt

$$= (T_1 - T_2)\, v = (670.5 - 67.36)\, 26.67 = 16\ 086 \text{ W}$$
$$= 16.086 \text{ kW}$$

∴ $$\text{Number of V-belts} = \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{90}{16.086} = 5.6 \text{ or } 6$$ **Ans.**

Length of each belt

We know that length of belt for an open belt drive,

$$L = \frac{\pi}{2}(d_2 + d_1) + 2x + \frac{(d_2 - d_1)^2}{4x}$$

$$= \frac{\pi}{2}(1 + 0.33) + 2 \times 1.75 + \frac{(1 - 0.33)^2}{4 \times 1.75}$$

$$= 2.1 + 3.5 + 0.064 = 5.664 \text{ m}$$ **Ans.**

11.23. Rope Drive

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted by the flat belt, then it would result in excessive belt cross-section. It may be noted that frictional grip in case of rope drives is more than that in V-drive. One of the main advantage of rope drives is that a number of separate drives may be taken from the one driving pulley. For example, in many spinning mills, the line shaft on each floor is driven by ropes passing directly from the main engine pulley on the ground floor.

The rope drives use the following two types of ropes :

1. Fibre ropes, and **2.** Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

11.24. Fibre Ropes

The ropes for transmitting power are usually made from fibrous materials such as hemp, manila and cotton. Since the hemp and manila fibres are rough, therefore the ropes made from these fibres are not very flexible and possesses poor mechanical properties. The hemp ropes have less strength as compared to manila ropes. When the hemp and manila ropes are bent over the sheave (or pulley), there is some sliding of fibres, causing the rope to wear and chafe internally. In order to minimise this defect, the rope fibres are lubricated with a tar, tallow or graphite. The lubrication also makes the rope moisture proof. The hemp ropes are suitable only for hand operated hoisting machinery and as tie ropes for lifting tackle, hooks etc.

The cotton ropes are very soft and smooth. The lubrication of cotton ropes is not necessary. But if it is done, it reduces the external wear between the rope and the grooves of its sheaves. It may be noted that manila ropes are more durable and stronger than cotton ropes. The cotton ropes are costlier than manila ropes.

Note : The diameter of manila and cotton ropes usually ranges from 38 mm to 50 mm. The size of the rope is usually designated by its circumference or **'girth'.**

11.25. Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.

2. They are little affected by out door conditions.

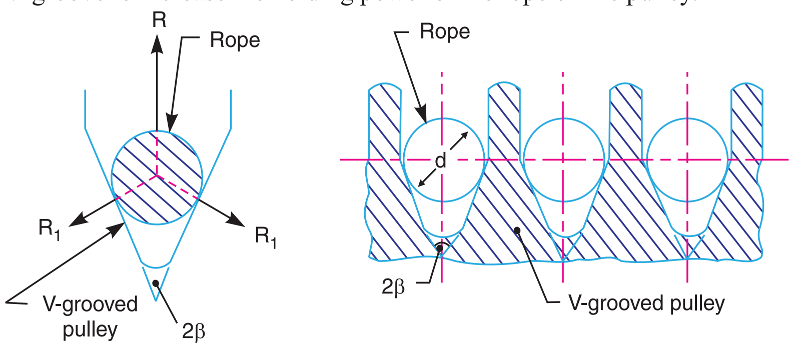
3. The shafts may be out of strict alignment.

4. The power may be taken off in any direction and in fractional parts of the whole amount.

5. They give high mechanical efficiency.

11.26. Sheave for Fibre Ropes

The fibre ropes are usually circular in cross-section as shown in Fig. 11.22 (*a*). The sheave for the fibre ropes is shown in Fig. 11.22 (*b*). The groove angle of the pulley for rope drives is usually 45°. The grooves in the pulleys are made narrow at the bottom and the rope is pinched between the edges of the V-groove to increase the holding power of the rope on the pulley.



(*a*) Cross-section of a rope. (*b*) Sheave (Grooved pulley) for ropes.

Fig. 11.22. Rope and sheave.

11.27. Wire Ropes

When a large amount of power is to be transmitted over long distances from one pulley to another (*i.e.* when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are

This electric hoist uses wire ropes.

widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves. The wire ropes have the following advantage over cotton ropes.

* The fibre ropes do not rest at the bottom of the groove.

1. These are lighter in weight, **2.** These offer silent operation, **3.** These can withstand shock loads, **4.** These are more reliable, **5.** They do not fail suddenly, **6.** These are more durable, **7.** The efficiency is high, and **8.** The cost is low.

11.28. Ratio of Driving Tensions for Rope Drive

The ratio of driving tensions for the rope drive may be obtained in the similar way as V-belts. We have discussed in Art. 11.22, that the ratio of driving tensions is

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta$$

where, μ, θ and β have usual meanings.

Example 11.20. *A rope drive transmits 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160° ; the angle of groove 45° ; the coefficient of friction 0.28 ; the mass of rope 1.5 kg / m and the allowable tension in each rope 2400 N. Find the number of ropes required.*

Solution. Given : $P = 600$ kW ; $d = 4$ m ; $N = 90$ r.p.m. ; $\theta = 160° = 160 \times \pi / 180 = 2.8$ rad; $2\beta = 45°$ or $\beta = 22.5°$; $\mu = 0.28$; $m = 1.5$ kg / m ; $T = 2400$ N

We know that velocity of the rope,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 4 \times 90}{60} = 18.85 \text{ m/s}$$

∴ Centrifugal tension, $T_C = m.v^2 = 1.5\,(18.85)^2 = 533$ N

and tension in the tight side of the rope,

$$T_1 = T - T_C = 2400 - 533 = 1867 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.28 \times 2.8 \times \operatorname{cosec} 22.5° = 2.05$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{2.05}{2.3} = 0.8913 \quad \text{or} \quad \frac{T_1}{T_2} = 7.786$$

...(Taking antilog of 0.8913)

and
$$T_2 = \frac{T_1}{7.786} = \frac{1867}{7.786} = 240 \text{ N}$$

We know that power transmitted per rope

$$= (T_1 - T_2)\,v = (1867 - 240)\,18.85 = 30\ 670 \text{ W} = 30.67 \text{ kW}$$

∴
$$\text{Number of ropes} = \frac{\text{Total power transmitted}}{\text{Power transmitted per rope}} = \frac{600}{30.67} = 19.56 \text{ or } 20 \quad \textbf{Ans.}$$

Example 11.21. *A pulley used to transmit power by means of ropes has a diameter of 3.6 metres and has 15 grooves of 45° angle. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 960 N and the mass of the rope is 1.5 kg per metre length. What is the speed of pulley in r.p.m. and the power transmitted if the condition of maximum power prevail ?*

Solution. Given : $d = 3.6$ m ; No. of grooves = 15 ; $2\beta = 45°$ or $\beta = 22.5°$; $\theta = 170° = 170\,\pi \times 180 = 2.967$ rad ; $\mu = 0.28$; $T = 960$ N ; $m = 1.5$ kg/m

Speed of the pulley

Let N = Speed of the pulley in r.p.m.

We know that for maximum power, velocity of the rope or pulley,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{960}{3 \times 1.5}} = 14.6 \text{ m/s}$$

$$\therefore \quad N = \frac{v \times 60}{\pi d} = \frac{14.6 \times 60}{\pi \times 3.6} = 77.5 \text{ r.p.m. } \textbf{Ans.} \quad \ldots\left(\because v = \frac{\pi d N}{60}\right)$$

Power transmitted

We know that for maximum power, centrifugal tension,

$$T_C = T/3 = 960/3 = 320 \text{ N}$$

$\therefore$ Tension in the tight side of the rope,

$$T_1 = T - T_C = 960 - 320 = 640 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that $$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.28 \times 2.967 \times \operatorname{cosec} 22.5° = 2.17$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{2.17}{2.3} = 0.9438 \quad \text{or} \quad \frac{T_1}{T_2} = 8.78$$

...(Taking antilog of 0.9438)

and $$T_2 = \frac{T_1}{8.78} = \frac{640}{8.78} = 73 \text{ N}$$

$\therefore$ Power transmitted per rope $= (T_1 - T_2)\, v = (640 - 73)\, 14.6 = 8278$ W = 8.278 kW

Since the number of grooves are 15, therefore total power transmitted

$= 8.278 \times 15 = 124.17$ kW **Ans.**

Example 11.22. *Following data is given for a rope pulley transmitting 24 kW :*

Diameter of pulley = 400 mm ; Speed = 110 r.p.m.; angle of groove = 45° ; Angle of lap on smaller pulley = 160° ; Coefficient of friction = 0.28 ; Number of ropes = 10 ; Mass in kg/m length of ropes = 53 C^2 ; and working tension is limited to 122 C^2 kN, where C is girth of rope in metres.

Find initial tension and diameter of each rope.

Solution. Given : $P_T = 24$ kW ; $d = 400$ mm = 0.4 m ; $N = 110$ r.p.m. ; $2\beta = 45°$ or $\beta = 22.5°$; $\theta = 160° = 160 \times \pi / 180 = 2.8$ rad ; $n = 0.28$; $n = 10$; $m = 53\, C^2$ kg/m ; $T = 122\, C^2$ kN $= 122 \times 10^3\, C^2$ N

Initial tension

We know that power transmitted per rope,

$$P = \frac{\text{Total power transmitted}}{\text{No. of ropes}} = \frac{P_T}{n} = \frac{24}{10} = 2.4 \text{ kW} = 2400 \text{ W}$$

and velocity of the rope, $$v = \frac{\pi d . N}{60} = \frac{\pi \times 0.4 \times 110}{60} = 2.3 \text{ m/s}$$

Let T = Tension in the tight side of the rope, and

T_2 = Tension in the slack side of the rope.

We know that power transmitted per rope (P)

$$2400 = (T_1 - T_2)\,v = (T_1 - T_2)\,2.3$$

$$\therefore \quad T_1 - T_2 = 2400 / 2.3 = 1043.5 \text{ N} \qquad ...(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.28 \times 2.8 \times \operatorname{cosec} 22.5° = 2.05$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{2.05}{2.3} = 0.8913 \quad \text{or} \quad \frac{T_1}{T_2} = 7.786 \qquad ...(ii)$$

...(Taking antilog of 0.8913)

From equations ***(i)*** and ***(ii)***,

$$T_1 = 1197.3 \text{ N, and } T_2 = 153.8 \text{ N}$$

We know that initial tension in each rope,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1197.3 + 153.8}{2} = 675.55 \text{ N} \quad \textbf{Ans.}$$

Diameter of each rope

Let d_1 = Diameter of each rope,

We know that centrifugal tension,

$$T_C = m.v^2 = 53\, C^2\, (2.3)^2 = 280.4\, C^2 \text{ N}$$

and working tension (T),

$$122 \times 10^3\, C^2 = T_1 + T_C = 1197.3 + 280.4\, C^2$$

$$122 \times 10^3\, C^2 - 280.4\, C^2 = 1197.3$$

$$\therefore \quad C^2 = 9.836 \times 10^{-3} \quad \text{or} \quad C = 0.0992 \text{ m} = 99.2 \text{ mm}$$

We know that girth (*i.e.* circumference) of rope (C),

$$99.2 = \pi\, d_1 \quad \text{or} \quad d_1 = 99.2 / \pi = 31.57 \text{ mm} \quad \textbf{Ans.}$$

11.29. Chain Drives

We have seen in belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for warping around the driving and driven wheels. The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain as shown in Fig. 11.23. The wheels and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio. The toothed wheels are known as ***sprocket wheels*** or simply ***sprockets***. These wheels resemble to spur gears.

The chains are mostly used to transmit motion and power from one shaft to another, when the distance between the centres of the shafts is short such as in bicycles, motor cycles, agricultural machinery, road rollers, etc.

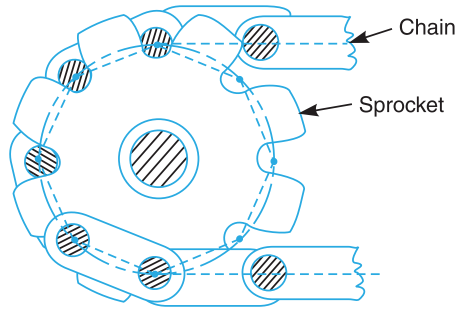


Fig. 11.23. Sprocket and chain.

11.30. Advantages and Disadvantages of Chain Drive Over Belt or Rope Drive

Following are the advantages and disadvantages of chain drive over belt or rope drive :

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. The chain drives may be used when the distance between the shafts is less.
4. The chain drive gives a high transmission efficiency (upto 98 per cent).
5. The chain drive gives less load on the shafts.
6. The chain drive has the ability of transmitting motion to several shafts by one chain only.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance.
3. The chain drive has velocity fluctuations especially when unduly stretched.

11.31. Terms Used in Chain Drive

The following terms are frequently used in chain drive.

1. ***Pitch of the chain*** **:** It is the distance between the hinge centre of a link and the corresponding hinge centre of the adjacent link as shown in Fig. 11.24. It is usually denoted by p.

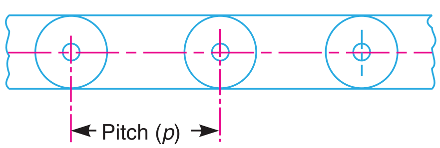


Fig. 11.24. Pitch of the chain.

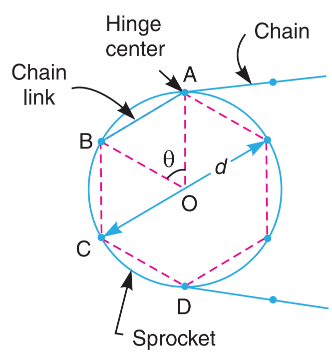


Fig. 11.25. Pitch circle diameter of the chain sprocket.

2. ***Pitch circle diameter of the chain sprocket.*** It is the diameter of the circle on which the hinge centres of the chain lie, when the chain is wrapped round a sprocket as shown in Fig. 11.25. The points A, B, C, and D are the hinge centres of the chain and the circle drawn through these centres is called pitch circle and its diameter (d) is known as pitch circle diameter.

11.32. Relation Between Pitch and Pitch Circle Diameter

A chain wrapped round the sprocket is shown in Fig. 11.25. Since the links of the chain are rigid, therefore pitch of the chain does not lie on the arc of the pitch circle. The pitch length becomes a chord. Consider one pitch length *A B* of the chain subtending an angle θ at the centre of sprocket (or pitch circle).

Let d = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

From Fig. 11.25, we find that pitch of the chain,

$$p = AB = 2\,AO \sin\left(\frac{\theta}{2}\right) = 2 \times \frac{d}{2} \sin\left(\frac{\theta}{2}\right) = d \sin\left(\frac{\theta}{2}\right)$$

We know that

$$\theta = \frac{360°}{T}$$

$$\therefore \qquad p = d \sin\left(\frac{360°}{2T}\right) = d \sin\left(\frac{180°}{T}\right)$$

or

$$d = p \operatorname{cosec}\left(\frac{180°}{T}\right)$$

11.33. Relation Between Chain Speed and Angular Velocity of Sprocket

Since the links of the chain are rigid, therefore they will have different positions on the sprocket at different instants. The relation between the chain speed (v) and angular velocity of the sprocket (ω) also varies with the angular position of the sprocket. The extreme positions are shown in Fig. 11.26 (*a*) and (*b*).

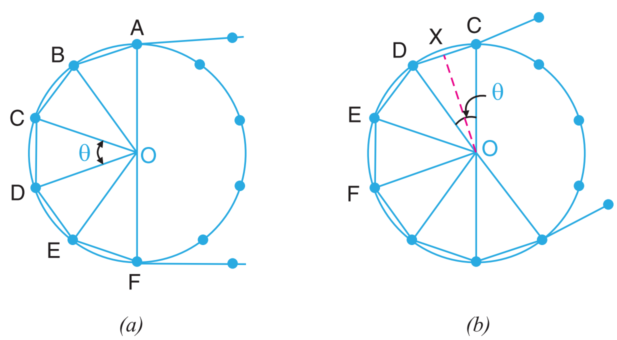


Fig. 11.26. Relation between chain speed and angular velocity of sprocket.

For the angular position of the sprocket as shown in Fig. 11.26 (*a*),

$$v = \omega \times OA$$

and for the angular position of the sprocket as shown in Fig. 11.26 (*b*),

$$v = \omega \times OX = \omega \times OC \cos\left(\frac{\theta}{2}\right) = \omega \times OA \cos\left(\frac{\theta}{2}\right) \quad \ldots (\because OC = OA)$$

11.34. Kinematic of Chain Drive

Fig. 11.27 shows an arrangement of a chain drive in which the smaller or driving sprocket has 6 teeth and the larger or driven sprocket has 9 teeth. Though this is an impracticable case, but this is considered to bring out clearly the kinematic conditions of a chain drive. Let both the sprockets rotate anticlockwise and the angle subtended by the chain pitch at the centre of the driving and driven sprockets be α and ϕ respectively. The lines AB and A_1B_1 show the positions of chain having minimum and maximum inclination respectively with the line of centres O_1O_2 of the sprockets. The points A, B_2 and B are in one straight line and the points A_1, C and B_1 are in one straight line. It may be noted that the straight length of the chain between the two sprockets must be equal to exact number of pitches.

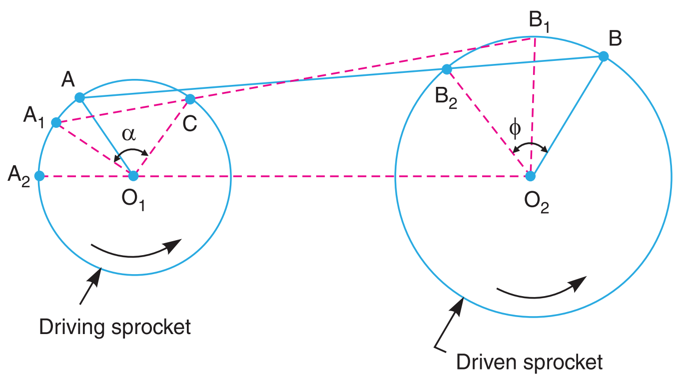


Fig. 11.27. Kinematic of chain drive.

Let us now consider the pin centre on the driving sprocket in position A. The length of the chain AB will remain straight as the sprockets rotate, until A reaches A_1 and B reaches B_1. As the driving sprocket continues to turn, the link A_1C of the chain turns about the pin centre C and the straight length of the chain between the two sprockets reduces to CB_1. When the pin centre C moves to the position A_1, the pin centre A_1 moves to the position A_2. During this time, each of the sprockets rotate from its original position by an angle corresponding to one chain pitch. During the first part of the angular displacement, the radius O_1A moves to O_1A_1 and the radius O_2B moves to O_2B_1. This arrangement is kinematically equivalent to the four bar chain O_1ABO_2.

During the second part of the angular displacement, the radius O_1A_1 moves to O_1A_2 and the radius O_2B_1 moves to O_2B_2. This arrangement is kinematically equivalent to the four bar chain $O_1CB_1O_2$. The ratio of the angular velocities, under these circumstances, cannot be constant. This may be easily shown as discussed below :

First of all, let us find the instantaneous centre for the two links O_1A and O_2B. This lies at point I which is the intersection of BA and O_2O_1 produced as shown in Fig. 11.28. If ω_1 is the angular velocity of the driving sprocket and ω_2 is the angular velocity of the driven sprocket, then

$$\omega_1 \times O_1I = \omega_2 \times O_2I$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2I}{O_1I} = \frac{O_2O_1 + O_1I}{O_1I} = 1 + \frac{O_2O_1}{O_1I}.$$

The distance between the centres of two sprockets $O_1 O_2$ is constant for a given chain drive, but the distance $O_1 I$ varies periodically as the two sprockets rotate. This period corresponds to a rotation of the driving sprocket by an angle α. It is clear from the figure that the line $A B$ has minimum inclination with line $O_1 O_2$. Therefore the distance $O_1 I$ is maximum and thus velocity ratio (ω_1 / ω_2) is minimum. When the chain occupies the position $A_1 B_1$, the inclination of line $A_1 B_1$ is maximum with the line $O_1 O_2$. Therefore the distance $O_1 I_1$ is minimum and thus the velocity ratio (ω_1 / ω_2) is maximum.

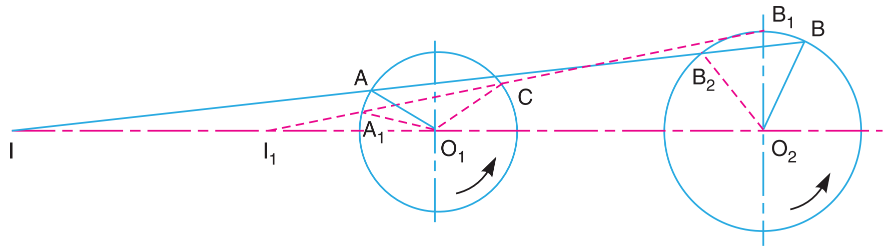


Fig. 11.28. Angular velocities of the two sprockets.

In actual practice, the smaller sprocket have a minimum of 18 teeth and hence the actual variation of velocity ratio (ω_1/ω_2) from the mean value is very small.

11.35. Classification of Chains

The chains, on the basis of their use, are classified into the following three groups :

1. Hoisting and hauling (or crane) chains,

2. Conveyor (or tractive) chains, and

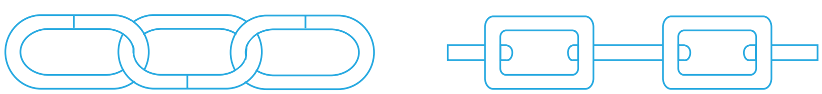
3. Power transmitting (or driving) chains.

These chains are discussed, in detail, in the following pages.

11.36. Hoisting and Hauling Chains

These chains are used for hoisting and hauling purposes. The hoisting and hauling chains are of the following two types :

1. ***Chain with oval links.*** The links of this type of chain are of oval shape, as shown in Fig. 11.29 (*a*). The joint of each link is welded. The sprockets which are used for this type of chain have receptacles to receive the links. Such type of chains are used only at low speeds such as in chain hoists and in anchors for marine works.



(*a*) Chain with oval links. (*b*) Chain with square links.

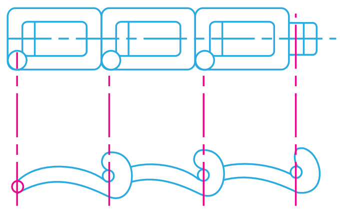
Fig. 11.29. Hoisting and hauling chains.

2. ***Chain with square links.*** The links of this type of chain are of square shape, as shown in Fig. 11.29 (*b*). Such type of chains are used in hoists, cranes, dredges. The manufacturing cost of this type of chain is less than that of chain with oval links, but in these chains, the kinking occurs easily on overloading.

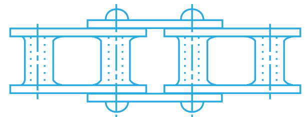
11.37. Conveyor Chains

These chains are used for elevating and conveying the materials continuously. The conveyor chains are of the following two types :

1. Detachable or hook joint type chain, as shown in Fig. 11.30 (*a*), and
2. Closed joint type chain, as shown in Fig. 11.30 (*b*).



(*a*) Detachable or hook joint type chain.



(*b*) Closed joint type chain.

Fig. 11.30. Conveyor chains.

The conveyor chains are usually made of malleable cast iron. These chains do not have smooth running qualities. The conveyor chains run at slow speeds of about 3 to 12 km.p.h.

11.38. Power Transmitting Chains

These chains are used for transmission of power, when the distance between the centres of shafts is short. These chains have provision for efficient lubrication. The power transmitting chains are of the following three types.

1. ***Block chain.*** A block chain, as shown in Fig. 11.31, is also known as **bush chain.** This type of chain was used in the early stages of development in the power transmission.

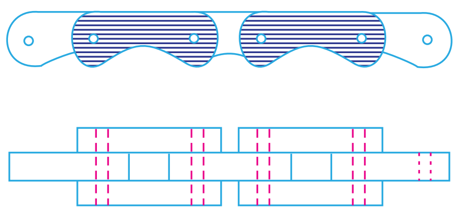


Fig. 11.31. Block chain.

It produces noise when approaching or leaving the teeth of the sprocket because of rubbing between the teeth and the links. Such type of chains are used to some extent as conveyor chain at small speed.

2. ***Bush roller chain.*** A bush roller chain, as shown in Fig. 11.32, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the sprocket wheel teeth against wear.

A bush roller chain is extremely strong and simple in construction. It gives good service under severe conditions. There is a little noise with this chain which is due to impact of the rollers on the sprocket wheel teeth. This chain may be used where there is a little lubrication. When one of these chains elongates slightly due to wear and stretching of the parts, then the extended chain is of greater pitch than the pitch of the sprocket wheel teeth. The rollers then fit unequally into the cavities of the wheel. The result is that the total load falls on one teeth or on a few teeth. The stretching of the parts increase wear of the surfaces of the roller and of the sprocket wheel teeth.

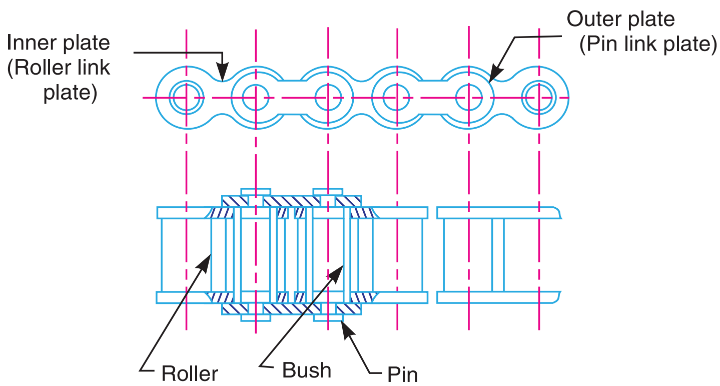


Fig. 11.32. Bush roller chain.

3. *Inverted tooth or silent chain*. An inverted tooth or silent chain is shown in Fig. 11.33. It is designed to eliminate the evil effects caused by stretching and to produce noiseless running. When the chain stretches and the pitch of the chain increases, the links ride on the teeth of the sprocket wheel at a slightly increased radius. This automatically corrects the small change in the pitch. There is no relative sliding between the teeth of the inverted tooth chain and the sprocket wheel teeth. When properly lubricated, this chain gives durable service and runs very smoothly and quietly.

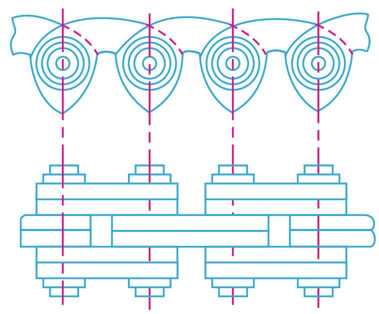


Fig. 11.33. Inverted tooth or silent chain.

11.39. Length of Chain

An open chain drive system connecting the two sprockets is shown in Fig. 11.34. We have already discussed in Art. 11.11 that the length of belt for an open belt drive connecting the two pulleys of radii r_1 and r_2 and a centre distance x, is

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{...(i)}$$

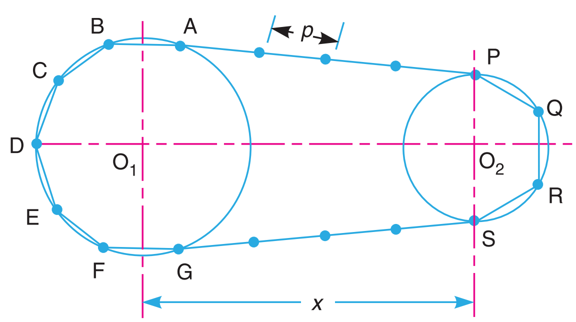


Fig. 11.34. Length of chain

If this expression is used for determining the length of chain, the result will be slightly greater than the required length. This is due to the fact that the pitch lines *A B C D E F G* and *P Q R S* of the sprockets are the parts of a polygon and not that of a circle. The exact length of the chain may be determined as discussed below :

Let T_1 = Number of teeth on the larger sprocket,

T_2 = Number of teeth on the smaller sprocket, and

p = Pitch of the chain.

We have discussed in Art. 11.32, that diameter of the pitch circle,

$$d = p \operatorname{cosec}\left(\frac{180°}{T}\right) \quad \text{or} \quad r = \frac{p}{2}\operatorname{cosec}\left(\frac{180°}{T}\right)$$

∴ For larger sprocket,

$$r_1 = \frac{p}{2}\operatorname{cosec}\left(\frac{180°}{T_1}\right)$$

and for smaller sprocket,
$$r_2 = \frac{p}{2}\operatorname{cosec}\left(\frac{180°}{T_2}\right)$$

Since the term $\pi (r_1 + r_2)$ is equal to half the sum of the circumferences of the pitch circles, therefore the length of chain corresponding to

$$\pi(r_1 + r_2) = \frac{p}{2}(T_1 + T_2)$$

Substituting the values of r_1, r_2 and $\pi (r_1 + r_2)$ in equation ***(i)***, the length of chain is given by

$$L = \frac{p}{2}(T_1 + T_2) + 2x + \frac{\left[\frac{p}{2}\operatorname{cosec}\left(\frac{180°}{T_1}\right) - \frac{p}{2}\operatorname{cosec}\left(\frac{180°}{T_2}\right)\right]^2}{x}$$

If $x = m.p$, then

$$L = p\left[\frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\operatorname{cosec}\left(\frac{180°}{T_1}\right) - \operatorname{cosec}\left(\frac{180°}{T_2}\right)\right]^2}{4\,m}\right] = p.K$$

where K = Multiplying factor

$$= \frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\operatorname{cosec}\left(\frac{180°}{T_1}\right) - \operatorname{cosec}\left(\frac{180°}{T_2}\right)\right]^2}{4\,m}$$

The value of multiplying factor (K) may not be a complete integer. But the length of the chain must be equal to an integer number of times the pitch of the chain. Thus, the value of K should be rounded off to the next higher integral number.

Example 11.23. *A chain drive is used for reduction of speed from 240 r.p.m. to 120 r.p.m. The number of teeth on the driving sprocket is 20. Find the number of teeth on the driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and centre to centre distance between the two sprockets is 800 mm, determine the pitch and length of the chain.*