16

Turning Moment Diagrams and Flywheel

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16.1. Introduction

The turning moment diagram (also known as ***crank-effort diagram***) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. 16.1. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

We have discussed in Chapter 15 (Art. 15.10.) that the turning moment on the crankshaft,

$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

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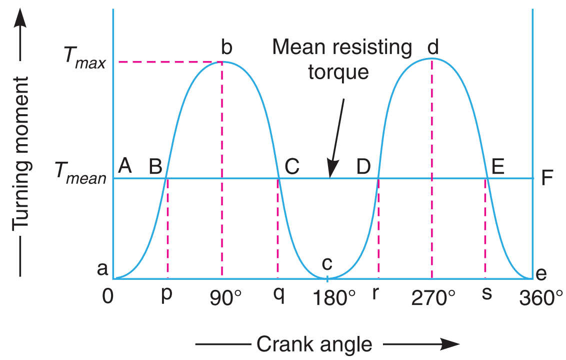

Fig. 16.1. Turning moment diagram for a single cylinder, double acting steam engine.

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.

This is shown by the curve *abc* in Fig. 16.1 and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *a A* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

For flywheel, have a look at your tailor's manual sewing machine.

Notes: 1. When the turning moment is positive (*i.e.* when the engine torque is more than the mean resisting torque) as shown between points *B* and *C* (or *D* and *E*) in Fig. 16.1, the crankshaft accelerates and the work is done by the steam.

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2. When the turning moment is negative (*i.e.* when the engine torque is less than the mean resisting torque) as shown between points *C* and *D* in Fig. 16.1, the crankshaft retards and the work is done on the steam.

3. If T = Torque on the crankshaft at any instant, and

T_{mean} = Mean resisting torque.

Then accelerating torque on the rotating parts of the engine

$$= T - T_{mean}$$

4. If $(T - T_{mean})$ is positive, the flywheel accelerates and if $(T - T_{mean})$ is negative, then the flywheel retards.

16.3. Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. 16.2. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

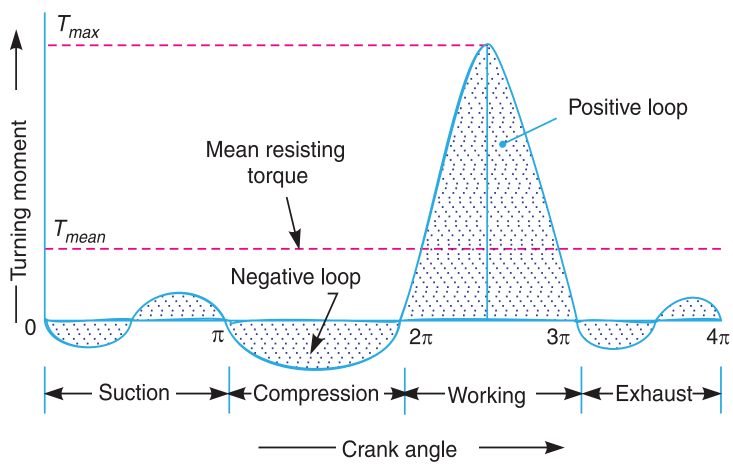

Fig. 16.2. Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig. 16.2.

16.4. Turning Moment Diagram for a Multi-cylinder Engine

A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. 16.3. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.

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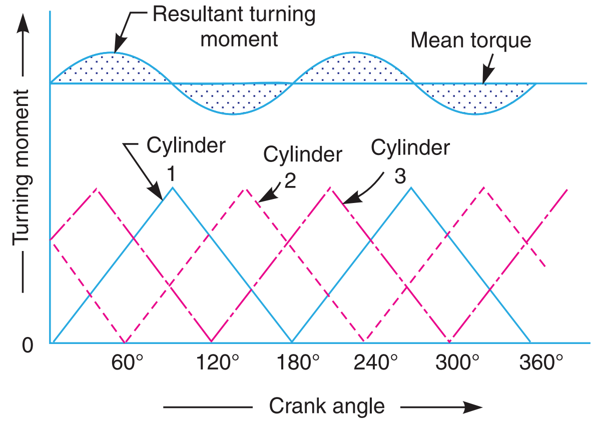

Fig. 16.3. Turning moment diagram for a multi-cylinder engine.

16.5. Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. 16.1. We see that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from *a* to *p*, the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *a AB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore, the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

Similarly, when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *C c D*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *D d E* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***. The areas *BbC*, *CcD*, *DdE*, etc. represent fluctuations of energy.

A little consideration will show that the engine has a maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crank moves from *p* to *q* and from *r* to *s*. On the other hand, the engine has a minimum speed either at *p* or at *r*. The reason is that the flywheel gives out some of its energy when the crank moves from *a* to *p* and *q* to *r*. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

16.6. Determination of Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 16.4. The horizontal line *AG* represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

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Let the energy in the flywheel at $A = E$, then from Fig. 16.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (*i.e.* cycle repeats after G)

A flywheel stores energy when the supply is in excess and releases energy when energy is in deficit.

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in flywheel

$$= E + a_1$$

Minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

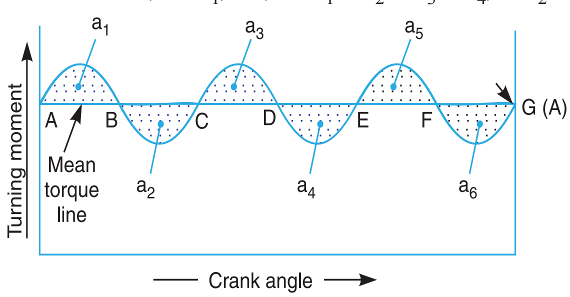

Fig. 16.4. Determination of maximum fluctuation of energy.

16.7. Coefficient of Fluctuation of Energy

It may be defined as the **ratio of the maximum fluctuation of energy to the work done per cycle.** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned (in radians), in one revolution.

= 2π, in case of steam engine and two stroke internal combustion engines

= 4π, in case of four stroke internal combustion engines.

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The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s $= 2\pi N/60$

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Table 16.1. Coefficient of fluctuation of energy (C_E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy (C_E)*
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinders, single acting, four stroke gas engine	0.066
5.	Six cylinders, single acting, four stroke gas engine	0.031

16.8. Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, ***a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.***

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In machines where the operation is intermittent like *punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Note: The function of a **governor in an engine is entirely different from that of a flywheel. It regulates the mean speed of an engine when there are variations in the load, *e.g.*, when the load on the engine increases, it becomes necessary to increase the supply of working fluid. On the other hand, when the load decreases, less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed of the engine within certain limits.

As discussed above, the flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by the varying load.

16.9. Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed***.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

Note. The reciprocal of the coefficient of fluctuation of speed is known as ***coefficient of steadiness*** and is denoted by *m*.

$$\therefore \quad m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

16.10. Energy Stored in a Flywheel

A flywheel is shown in Fig. 16.5. We have discussed in Art. 16.5 that when a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

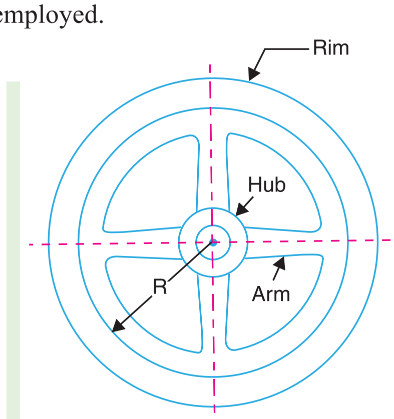

Fig. 16.5. Flywheel.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

* See Art. 16.12.

** See Chapter 18 on Governors.

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I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \qquad \text{(in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \qquad \text{...}(i)$$

$$\text{...}\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \text{... (Multiplying and dividing by } \omega)$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \text{... } (\because I = m.k^2) \quad \text{...}(ii)$$

$$= 2.E.C_S \text{ (in N–m or joules)} \qquad \text{...}\left(\because E = \frac{1}{2} \times I.\omega^2\right) \quad \text{...}(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting $k = R$, in equation ***(ii)***, we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

where v = Mean linear velocity (*i.e.* at the mean radius) in m/s = $\omega.R$

Notes. 1. Since $\omega = 2\,\pi\,N/60$, therefore equation ***(i)*** may be written as

$$\Delta E = I \times \frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right) = \frac{4\pi^2}{3600} \times I \times N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m.k^2.N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m.k^2.N^2.C_S \qquad \text{...}\left(\because C_S = \frac{N_1 - N_2}{N}\right)$$

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2. In the above expressions, only the mass moment of inertia of the flywheel rim (I) is considered and the mass moment of inertia of the hub and arms is neglected. This is due to the fact that the major portion of the mass of the flywheel is in the rim and a small portion is in the hub and arms. Also the hub and arms are nearer to the axis of rotation, therefore the mass moment of inertia of the hub and arms is small.

Example 16.1. *The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds.*

Solution. Given : m = 6.5 t = 6500 kg ; k = 1.8 m ; ΔE = 56 kN-m = 56×10^3 N-m ; N = 120 r.p.m.

Let N_1 and N_2 = Maximum and minimum speeds respectively.

We know that fluctuation of energy (ΔE),

$$56 \times 10^3 = \frac{\pi^2}{900} \times m.k^2 \, . \, N (N_1 - N_2) = \frac{\pi^2}{900} \times 6500 \, (1.8)^2 \, 120 \, (N_1 - N_2)$$

$$= 27\,715 \, (N_1 - N_2)$$

$$\therefore \quad N_1 - N_2 = 56 \times 10^3 / 27\,715 = 2 \text{ r.p.m.} \qquad \text{...(i)}$$

We also know that mean speed (N),

$$120 = \frac{N_1 + N_2}{2} \quad \text{or} \quad N_1 + N_2 = 120 \times 2 = 240 \text{ r.p.m.} \qquad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$N_1 = 121 \text{ r.p.m., and } N_2 = 119 \text{ r.p.m.}$$ **Ans.**

Example 16.2. *The flywheel of a steam engine has a radius of gyration of 1 m and mass 2500 kg. The starting torque of the steam engine is 1500 N-m and may be assumed constant. Determine:* ***1.*** *the angular acceleration of the flywheel, and* ***2.*** *the kinetic energy of the flywheel after 10 seconds from the start.*

Solution. Given : k = 1 m ; m = 2500 kg ; T = 1500 N-m

1. Angular acceleration of the flywheel

Let α = Angular acceleration of the flywheel.

We know that mass moment of inertia of the flywheel,

$$I = m.k^2 = 2500 \times 1^2 = 2500 \text{ kg-m}^2$$

$\therefore$ Starting torque of the engine (T),

$$1500 = I.\alpha = 2500 \times \alpha \quad \text{or} \quad \alpha = 1500 / 2500 = 0.6 \text{ rad/s}^2$$ **Ans.**

2. Kinetic energy of the flywheel

First of all, let us find out the angular speed of the flywheel after 10 seconds from the start (*i.e.* from rest), assuming uniform acceleration.

Let ω_1 = Angular speed at rest = 0

ω_2 = Angular speed after 10 seconds, and

t = Time in seconds.

We know that $\omega_2 = \omega_1 + \alpha\, t = 0 + 0.6 \times 10 = 6$ rad/s

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∴ Kinetic energy of the flywheel

$$= \frac{1}{2} \times I(\omega_2)^2 = \frac{1}{2} \times 2500 \times 6^2 = 45\ 000 \text{ N-m} = 45 \text{ kN-m} \quad \textbf{Ans.}$$

Example 16.3. *A horizontal cross compound steam engine develops 300 kW at 90 r.p.m. The coefficient of fluctuation of energy as found from the turning moment diagram is to be 0.1 and the fluctuation of speed is to be kept within ± 0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2 metres.*

Solution. Given : $P = 300$ kW $= 300 \times 10^3$ W; $N = 90$ r.p.m.; $C_E = 0.1$; $k = 2$ m

We know that the mean angular speed,

$$\omega = 2\pi N/60 = 2\pi \times 90/60 = 9.426 \text{ rad/s}$$

Let ω_1 and ω_2 = Maximum and minimum speeds respectively.

Since the fluctuation of speed is ± 0.5% of mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 1\%\ \omega = 0.01\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.01$$

We know that work done per cycle

$$= P \times 60 / N = 300 \times 10^3 \times 60 / 90 = 200 \times 10^3 \text{ N-m}$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Work done per cycle} \times C_E = 200 \times 10^3 \times 0.1 = 20 \times 10^3 \text{ N-m}$$

Let m = Mass of the flywheel.

We know that maximum fluctuation of energy (ΔE),

$$20 \times 10^3 = m.k^2.\omega^2.C_S = m \times 2^2 \times (9.426)^2 \times 0.01 = 3.554\ m$$

∴ $m = 20 \times 10^3/3.554 = 5630$ kg **Ans.**

Example 16.4. *The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 5 N-m ; crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 r.p.m.*

Solution. Given : $m = 36$ kg ; $k = 150$ mm $= 0.15$ m ; $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad /s

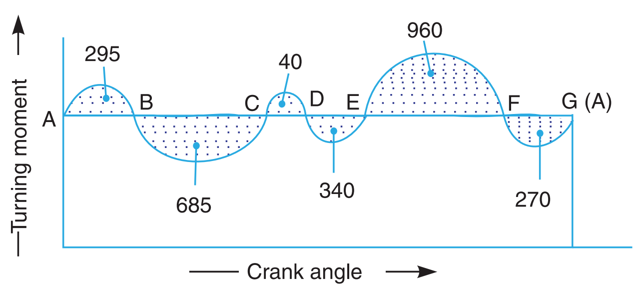

Fig. 16.6

The turning moment diagram is shown in Fig. 16.6.

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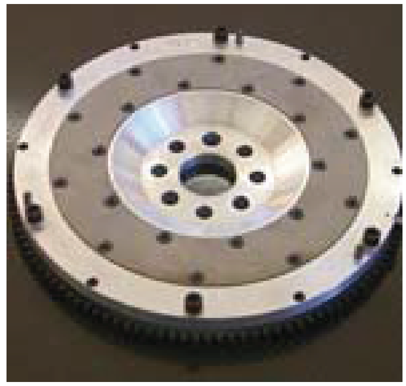
Since the turning moment scale is 1 mm = 5 N-m and crank angle scale is 1 mm = 1° = π /180 rad, therefore,

1 mm^2 on turning moment diagram

$$= 5 \times \frac{\pi}{180} = \frac{\pi}{36} \text{ N-m}$$



Flywheel of an electric motor.

Let the total energy at $A = E$, then referring to Fig. 16.6,

Energy at $B = E + 295$ ... **(Maximum energy)**

Energy at $C = E + 295 - 685 = E - 390$

Energy at $D = E - 390 + 40 = E - 350$

Energy at $E = E - 350 - 340 = E - 690$ ...**(Minimum energy)**

Energy at $F = E - 690 + 960 = E + 270$

Energy at $G = E + 270 - 270 = E$ = Energy at A

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 295) - (E - 690) = 985 \text{ mm}^2$$
$$= 985 \times \frac{\pi}{36} = 86 \text{ N-m} = 86 \text{ J}$$

Let C_S = Coefficient of fluctuation of speed.

We know that maximum fluctuation of energy (ΔE),

$$86 = m.k^2\,\omega^2.C_S = 36 \times (0.15)^2 \times (188.52)^2\, C_S = 28\,787\, C_S$$

$\therefore$ $C_S = 86 / 28\,787 = 0.003$ or 0.3% **Ans.**

Example 16.5. *The turning moment diagram for a multicylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows :*

+ 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm², when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.

Solution. Given : N = 600 r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; R = 0.5 m

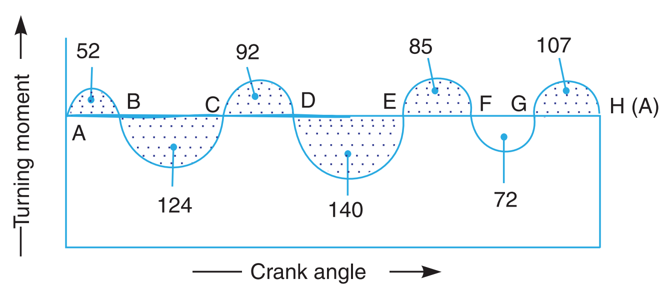


Fig. 16.7

Since the total fluctuation of speed is not to exceed ± 1.5% of the mean speed, therefore

$$\omega_1 - \omega_2 = 3\%\ \omega = 0.03\ \omega$$

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and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

The turning moment diagram is shown in Fig. 16.7.

Since the turning moment scale is 1 mm = 600 N-m and crank angle scale is 1 mm = 3° = 3° × π/180 = π / 60 rad, therefore

1 mm^2 on turning moment diagram

$$= 600 \times \pi/60 = 31.42 \text{ N-m}$$

Let the total energy at $A = E$, then referring to Fig. 16.7,

Energy at $B = E + 52$ ...**(Maximum energy)**

Energy at $C = E + 52 - 124 = E - 72$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$ ...**(Minimum energy)**

Energy at $F = E - 120 + 85 = E - 35$

Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E$ = Energy at A

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 52) - (E - 120) = 172 = 172 \times 31.42 = 5404 \text{ N-m}$$

Let m = Mass of the flywheel in kg.

We know that maximum fluctuation of energy (ΔE),

$$5404 = m.R^2.\omega^2.C_S = m \times (0.5)^2 \times (62.84)^2 \times 0.03 = 29.6\, m$$

$\therefore$ $m = 5404 / 29.6 = 183$ kg **Ans.**

Example 16.6. *A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter.*

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Solution. Given : $N = 250$ r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s ; $m = 500$ kg ; $k = 600$ mm = 0.6 m

The turning moment diagram for the complete cycle is shown in Fig. 16.8.

We know that the torque required for one complete cycle

$$= \text{Area of figure } OABCDEF$$

$$= \text{Area } OAEF + \text{Area } ABG + \text{Area } BCHG + \text{Area } CDH$$

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$

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$$= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \frac{1}{2} \times \pi(3000 - 750)$$

$$= 11\,250\,\pi \text{ N-m} \quad \text{...}(i)$$

If T_{mean} is the mean torque in N-m, then torque required for one complete cycle

$$= T_{mean} \times 6\pi \text{ N-m} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$T_{mean} = 11\,250\,\pi / 6\,\pi = 1875 \text{ N-m}$$

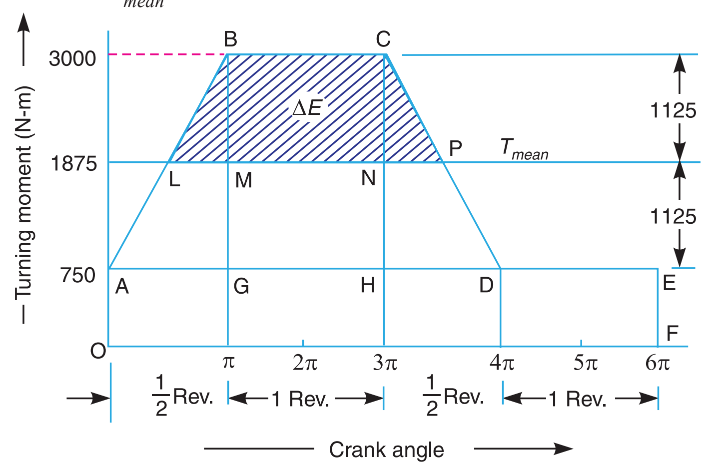

Fig. 16.8

Power required to drive the machine

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 1875 \times 26.2 = 49\,125 \text{ W} = 49.125 \text{ kW} \quad \textbf{Ans.}$$

Coefficient of fluctuation of speed

Let C_S = Coefficient of fluctuation of speed.

First of all, let us find the values of LM and NP. From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5\,\pi$$

Now, from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5\,\pi$$

From Fig. 16.8, we find that

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore, maximum fluctuation of energy,

$$\Delta E = \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

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$$= \frac{1}{2} \times 0.5\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\pi \times 1125$$

$$= 8837 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.C_S = 500 \times (0.6)^2 \times (26.2)^2 \times C_S = 123\ 559\ C_S$$

$$C_S = \frac{8837}{123559} = 0.071 \textbf{ Ans.}$$

Flywheel of a pump run by a diesel engine.

Example 16.7. *During forward stroke of the piston of the double acting steam engine, the turning moment has the maximum value of 2000 N-m when the crank makes an angle of 80° with the inner dead centre. During the backward stroke, the maximum turning moment is 1500 N-m when the crank makes an angle of 80° with the outer dead centre. The turning moment diagram for the engine may be assumed for simplicity to be represented by two triangles.*

If the crank makes 100 r.p.m. and the radius of gyration of the flywheel is 1.75 m, find the coefficient of fluctuation of energy and the mass of the flywheel to keep the speed within ± 0.75% of the mean speed. Also determine the crank angle at which the speed has its minimum and maximum values.

Solution. Given : $N = 100$ r.p.m. or $\omega = 2\pi \times 100/60 = 10.47$ rad /s; $k = 1.75$ m

Since the fluctuation of speed is ± 0.75% of mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 1.5\%\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 1.5\% = 0.015$$

Coefficient of fluctuation of energy

The turning moment diagram for the engine during forward and backward strokes is shown in Fig. 16.9. The point O represents the inner dead centre (I.D.C.) and point G represents the outer dead centre (O.D.C). We know that maximum turning moment when crank makes an angle of 80° (or $80 \times \pi / 180 = 4\pi/9$ rad) with I.D.C.,

$\therefore \quad AB = 2000$ N-m

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and maximum turning moment when crank makes an angle of 80° with outer dead centre (O.D.C.) or $180° + 80° = 260° = 260 \times \pi / 180 = 13\pi / 9$ rad with I.D.C.,

$$LM = 1500 \text{ N-m}$$

Let $T_{mean} = EB = QM$ = Mean resisting torque.

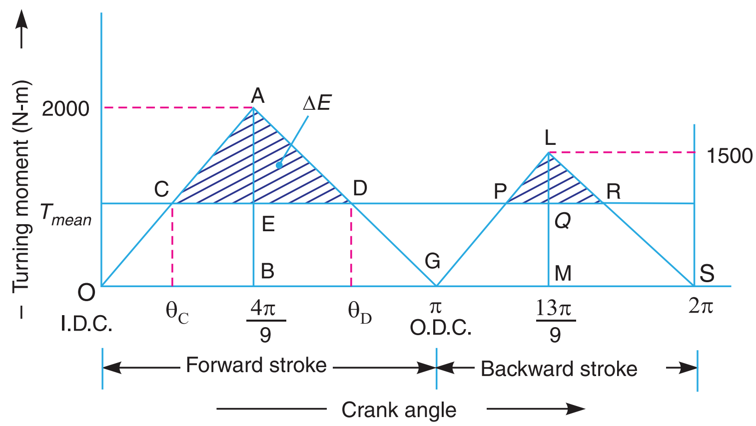

Fig. 16.9

We know that work done per cycle

$$= \text{Area of triangle } OAG + \text{Area of triangle } GLS$$

$$= \frac{1}{2} \times OG \times AB + \frac{1}{2} \times GS \times LM$$

$$= \frac{1}{2} \times \pi \times 2000 + \frac{1}{2} \times \pi \times 1500 = 1750\,\pi \text{ N-m} \quad \text{...}(i)$$

We also know that work done per cycle

$$= T_{mean} \times 2\,\pi \text{ N-m} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$T_{mean} = 1750\,\pi / 2\,\pi = 875 \text{ N-m}$$

From similar triangles ACD and AOG,

$$\frac{CD}{AE} = \frac{OG}{AB}$$

or

$$CD = \frac{OG}{AB} \times AE = \frac{OG}{AB}(AB - EB) = \frac{\pi}{2000}(2000 - 875) = 1.764 \text{ rad}$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Area of triangle } ACD = \frac{1}{2} \times CD \times AE$$

$$= \frac{1}{2} \times CD(AB - EB) = \frac{1}{2} \times 1.764(2000 - 875) = 992 \text{ N-m}$$

We know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Max. fluctuation of energy}}{\text{Work done per cycle}} = \frac{992}{1750\,\pi} = 0.18 \text{ or } 18\%$$ **Ans.**

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Mass of the flywheel

Let m = Mass of the flywheel.

We know that maximum fluctuation of energy (ΔE),

$$992 = m.k^2.\omega^2.C_S = m \times (1.75)^2 \times (10.47)^2 \times 0.015 = 5.03\ m$$

$$\therefore \quad m = 992 / 5.03 = 197.2 \text{ kg} \quad \textbf{Ans.}$$

Crank angles for the minimum and maximum speeds

Flywheel of small steam engine.

We know that the speed of the flywheel is minimum at point C and maximum at point D (See Art. 16.5).

Let θ_C and θ_D = Crank angles from I.D.C., for the minimum and maximum speeds.

From similar triangles ACE and AOB,

$$\frac{CE}{OB} = \frac{AE}{AB}$$

or
$$CE = \frac{AE}{AB} \times OB = \frac{AB - EB}{AB} \times OB = \frac{2000 - 875}{2000} \times \frac{4\pi}{9} = \frac{\pi}{4} \text{ rad}$$

$$\therefore \quad \theta_C = \frac{4\pi}{9} - \frac{\pi}{4} = \frac{7\pi}{36} \text{ rad} = \frac{7\pi}{36} \times \frac{180}{\pi} = 35° \quad \textbf{Ans.}$$

Again from similar triangles AED and ABG,

$$\frac{ED}{BG} = \frac{AE}{AB}$$

or
$$ED = \frac{AE}{AB} \times BG = \frac{AB - EB}{AB}(OG - OB)$$

$$= \frac{2000 - 875}{2000}\left(\pi - \frac{4\pi}{9}\right) = \frac{2.8\pi}{9} \text{ rad}$$

$$\therefore \quad \theta_D = \frac{4\pi}{9} + \frac{2.8\pi}{9} = \frac{6.8\pi}{9} \text{ rad} = \frac{6.8\pi}{9} \times \frac{180}{\pi} = 136° \quad \textbf{Ans.}$$

Example 16.8. *A three cylinder single acting engine has its cranks set equally at 120° and it runs at 600 r.p.m. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque of 90 N-m at 60° from dead centre of corresponding crank. The torque on the return stroke is sensibly zero. Determine :* **1.** *power developed.* **2.** *coefficient of fluctuation of speed, if the mass of the flywheel is 12 kg and has a radius of gyration of 80 mm,* **3.** *coefficient of fluctuation of energy, and 4. maximum angular acceleration of the flywheel.*

Solution. Given : N = 600 r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad /s; T_{max} = 90 N-m; m = 12 kg; k = 80 mm = 0.08 m

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The torque-crank angle diagram for the individual cylinders is shown in Fig. 16.10 (*a*), and the resultant torque-crank angle diagram for the three cylinders is shown in Fig. 16.10 (*b*).

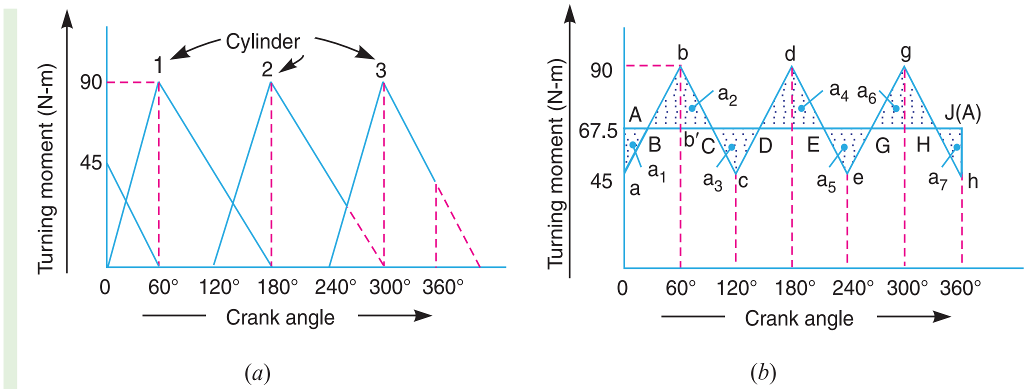

Fig. 16.10

1. ***Power developed***

We know that work done/cycle

$$= \text{Area of three triangles} = 3 \times \frac{1}{2} \times \pi \times 90 = 424 \text{ N-m}$$

and mean torque, $$T_{mean} = \frac{\text{Work done / cycle}}{\text{Crank angle / cycle}} = \frac{424}{2\pi} = 67.5 \text{ N-m}$$

$\therefore$ Power developed $= T_{mean} \times \omega = 67.5 \times 62.84 = 4240 \text{ W} = 4.24 \text{ kW}$ **Ans.**

2. ***Coefficient of fluctuation of speed***

Let C_S = Coefficient of fluctuation of speed.

First of all, let us find the maximum fluctuation of energy (ΔE).

From Fig. 16.10 (*b*), we find that

$$a_1 = \text{Area of triangle } AaB = \frac{1}{2} \times AB \times Aa$$

$$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45) = 5.89 \text{ N-m} = a_7 \quad \ldots(\because AB = 30° = \pi/6 \text{ rad})$$

$$a_2 = \text{Area of triangle } BbC = \frac{1}{2} \times BC \times bb'$$

$$= \frac{1}{2} \times \frac{\pi}{3}(90 - 67.5) = 11.78 \text{ N-m} \quad \ldots(\because BC = 60° = \pi/3 \text{ rad})$$

$$= a_3 = a_4 = a_5 = a_6$$

Now, let the total energy at $A = E$, then referring to Fig. 16.10 (*b*),

Energy at $B = E - 5.89$

Energy at $C = E - 5.89 + 11.78 = E + 5.89$

Energy at $D = E + 5.89 - 11.78 = E - 5.89$

Energy at $E = E - 5.89 + 11.78 = E + 5.89$

Energy at $G = E + 5.89 - 11.78 = E - 5.89$

Energy at $H = E - 5.89 + 11.78 = E + 5.89$

Energy at $J = E + 5.89 - 5.89 = E$ = Energy at A

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From above we see that maximum energy

$$= E + 5.89$$

and minimum energy $= E - 5.89$

∴ * Maximum fluctuation of energy,

$$\Delta E = (E + 5.89) - (E - 5.89) = 11.78 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$11.78 = m.k^2.\omega^2.C_S = 12 \times (0.08)^2 \times (62.84)^2 \times C_S = 303.3\ C_S$$

$$\therefore \quad C_S = 11.78 / 303.3 = 0.04 \text{ or } 4\% \quad \textbf{Ans.}$$

3. *Coefficient of fluctuation of energy*

We know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Max. fluctuation of energy}}{\text{Work done/cycle}} = \frac{11.78}{424} = 0.0278 = 2.78\% \quad \textbf{Ans.}$$

4. *Maximum angular acceleration of the flywheel*

Let α = Maximum angular acceleration of the flywheel.

We know that,

$$T_{max} - T_{mean} = I.\alpha = m.k^2.\alpha$$

$$90 - 67.5 = 12 \times (0.08)^2 \times \alpha = 0.077\ \alpha$$

$$\therefore \quad \alpha = \frac{90 - 67.5}{0.077} = 292 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Example 16.9. *A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2 per cent of the mean speed and the turning moment diagram during compression and expansion is assumed to be triangular in shape, find the moment of inertia of the flywheel.*

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

Since the total fluctuation of speed ($\omega_1 - \omega_2$) is not to exceed ± 2 per cent of the mean speed (ω), therefore

$$\omega_1 - \omega_2 = 4\%\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 4\% = 0.04$$

The turning moment-crank angle diagram for a four stroke engine is shown in Fig. 16.11. It is assumed to be triangular during compression and expansion strokes, neglecting the suction and exhaust strokes.

* Since the area above the mean torque line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,

$$\Delta E = \text{Area } Bbc = \text{Area } DdE = \text{Area } Ggh$$

$$= \frac{1}{2} \times \frac{\pi}{3}(90 - 67.5) = 11.78 \text{ N-m}$$

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We know that for a four stroke engine, number of working strokes per cycle,

$$n = N/2 = 300 / 2 = 150$$

∴ Work done/cycle $= P \times 60/n = 20 \times 10^3 \times 60/150 = 8000$ N-m ...(*i*)

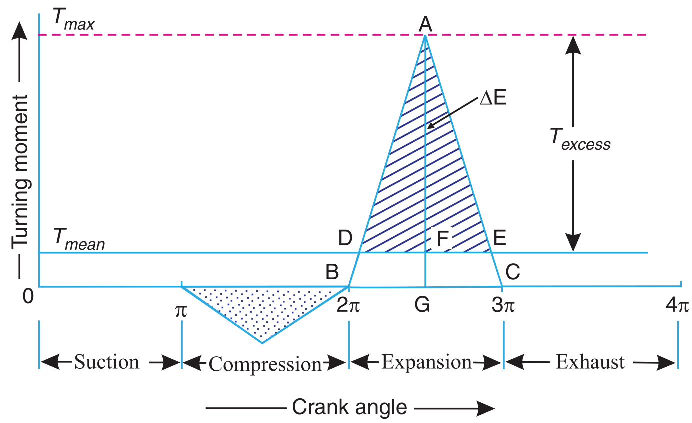

Fig. 16.11

Since the work done during suction and exhaust strokes is negligible, therefore net work done per cycle (during compression and expansion strokes)

$$= W_E - W_C = W_E - \frac{W_E}{3} = \frac{2}{3} W_E \qquad \ldots (\because W_E = 3W_C) \ldots(ii)$$

Equating equations (*i*) and (*ii*), work done during expansion stroke,

$$W_E = 8000 \times 3/2 = 12\,000 \text{ N-m}$$

We know that work done during expansion stroke (W_E),

$$12\,000 = \text{Area of triangle } ABC = \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG$$

$$\therefore \quad AG = T_{max} = 12\,000 \times 2/\pi = 7638 \text{ N-m}$$

and mean turning moment,

$$^* T_{mean} = FG = \frac{\text{Work done/cycle}}{\text{Crank angle/cycle}} = \frac{8000}{4\pi} = 637 \text{ N-m}$$

∴ Excess turning moment,

$$T_{excess} = AF = AG - FG = 7638 - 637 = 7001 \text{ N-m}$$

Now, from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{7001}{7638} \times \pi = 2.88 \text{ rad}$$

Since the area above the mean turning moment line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta\, ADE = \frac{1}{2} \times DE \times AF = \frac{1}{2} \times 2.88 \times 7001 = 10\,081 \text{ N-m}$$

* The mean turning moment (T_{mean}) may also be obtained by using the following relation :

$$P = T_{mean} \times \omega \text{ or } T_{mean} = P/\omega = 20 \times 10^3/31.42 = 637 \text{ N-m}$$

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Let I = Moment of inertia of the flywheel in kg-m^2 .

We know that maximum fluctuation of energy (ΔE),

$$10\ 081 = I.\omega^2.C_S = I \times (31.42)^2 \times 0.04 = 39.5\ I$$

$\therefore$ $I = 10081/ 39.5 = 255.2$ kg-m^2 **Ans.**

Example 16.10. *The turning moment diagram for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows :*

Suction stroke = 0.45 × 10^{-3} m^2; Compression stroke = 1.7 × 10^{-3} m^2; Expansion stroke = 6.8 × 10^{-3} m^2; Exhaust stroke = 0.65 × 10^{-3} m^2. Each m^2 of area represents 3 MN-m of energy.

Assuming the resisting torque to be uniform, find the mass of the rim of a flywheel required to keep the speed between 202 and 198 r.p.m. The mean radius of the rim is 1.2 m.

Solution. Given : $a_1 = 0.45 \times 10^{-3}$ m^2 ; $a_2 = 1.7 \times 10^{-3}$ m^2 ; $a_3 = 6.8 \times 10^{-3}$ m^2; $a_4 = 0.65 \times 10^{-3}$ m^2; $N_1 = 202$ r.p.m; $N_2 = 198$ r.p.m.; $R = 1.2$ m

The turning moment crank angle diagram for a four stroke engine is shown in Fig. 16.12. The areas below the zero line of pressure are taken as negative while the areas above the zero line of pressure are taken as positive.

$\therefore$ Net area $= a_3 - (a_1 + a_2 + a_4)$

$$= 6.8 \times 10^{-3} - (0.45 \times 10^{-3} + 1.7 \times 10^{-3} + 0.65 \times 10^{-3}) = 4 \times 10^{-3} \text{ m}^2$$

Since the energy scale is 1 m^2 = 3 MN-m = 3 × 10^6 N-m, therefore,

Net work done per cycle $= 4 \times 10^{-3} \times 3 \times 10^6 = 12 \times 10^3$ N-m . . . *(i)*

We also know that work done per cycle,

$$= T_{mean} \times 4\pi \text{ N-m} \quad \ldots (ii)$$

From equations *(i)* and *(ii)*,

$$T_{mean} = FG = 12 \times 10^3/4\pi = 955 \text{ N-m}$$

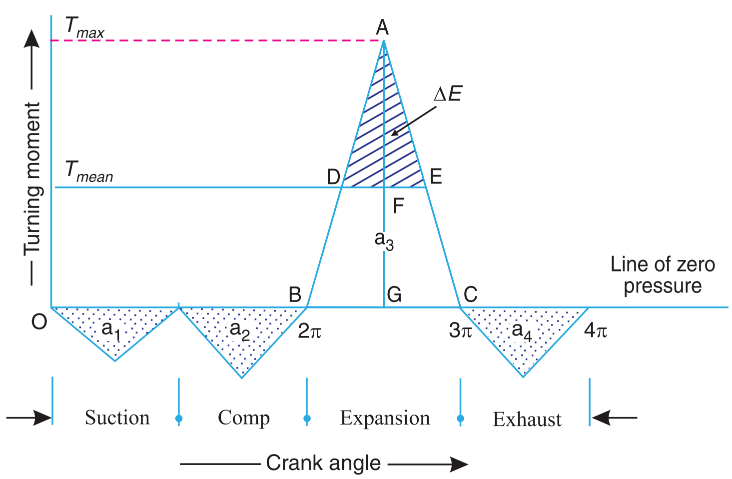

Fig. 16.12

Work done during expansion stroke

$$= a_3 \times \text{Energy scale} = 6.8 \times 10^{-3} \times 3 \times 10^6 = 20.4 \times 10^3 \text{ N-m} \quad \ldots (iii)$$

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Also, work done during expansion stroke

$$= \text{Area of triangle } ABC$$

$$= \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG = 1.571 \times AG \qquad \ldots (iv)$$

From equations ***(iii)*** and ***(iv)***,

$$AG = 20.4 \times 10^3 / 1.571 = 12\ 985 \text{ N-m}$$

∴ Excess torque,

$$T_{excess} = AF = AG - FG = 12\ 985 - 955 = 12\ 030 \text{ N-m}$$

Now from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{12\ 030}{12\ 985} \times \pi = 2.9 \text{ rad}$$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta ADE = \frac{1}{2} \times DE \times AF = \frac{1}{2} \times 2.9 \times 12\,030 \text{ N-m}$$

$$= 17\ 444 \text{ N-m}$$

Mass of the rim of a flywheel

Let m = Mass of the rim of a flywheel in kg, and

N = Mean speed of the flywheel

$$= \frac{N_1 + N_2}{2} = \frac{202 + 198}{2} = 200 \text{ r.p.m.}$$

We know that the maximum fluctuation of energy (ΔE),

$$17\ 444 = \frac{\pi^2}{900} \times m.R^2.N\ (N_1 - N_2) = \frac{\pi^2}{900} \times (1.2)^2\ 200 \times (202 - 198)$$

$$= 12.63\ m$$

∴ $m = 17\ 444 / 12.36 = 1381$ kg **Ans.**

Example 16.11. *The turning moment curve for an engine is represented by the equation, T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:*

1. *Power developed by the engine ;* ***2.*** *Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not exceed 1% of mean speed which is 180 r.p.m; and* ***3.*** *Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.*

Solution. Given : T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m ; N = 180 r.p.m. or ω = 2π × 180/60 = 18.85 rad/s

Since the total fluctuation of speed ($\omega_1 - \omega_2$) is 1% of mean speed (ω), therefore coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. ***Power developed by the engine***

We know that work done per revolution

$$= \int_0^{2\pi} T\,d\theta = \int_0^{2\pi} (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\, d\theta$$

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$$= \left[20\,000\,\theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20\,000 \times 2\pi = 40\,000\,\pi \text{ N-m}$$

and mean resisting torque of the engine,

$$T_{mean} = \frac{\text{Work done per revolution}}{2\pi} = \frac{40\,000}{2\pi} = 20000 \text{ N-m}$$

We know that power developed by the engine

$$= T_{mean} \cdot \omega = 20\,000 \times 18.85 = 377\,000 \text{ W} = 377 \text{ kW}$$ **Ans.**

2. *Moment of inertia of the flywheel*

Let I = Moment of inertia of the flywheel in kg-m^2.

The turning moment diagram for one stroke (*i.e.* half revolution of the crankshaft) is shown in Fig. 16.13. Since at points B and D, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\,000$$

or $\quad 9500 \sin 2\theta = 5700 \cos 2\theta$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$\therefore \quad 2\theta = 31° \text{ or } \theta = 15.5°$

$\therefore \quad \theta_B = 15.5° \text{ and } \theta_D = 90° + 15.5° = 105.5°$

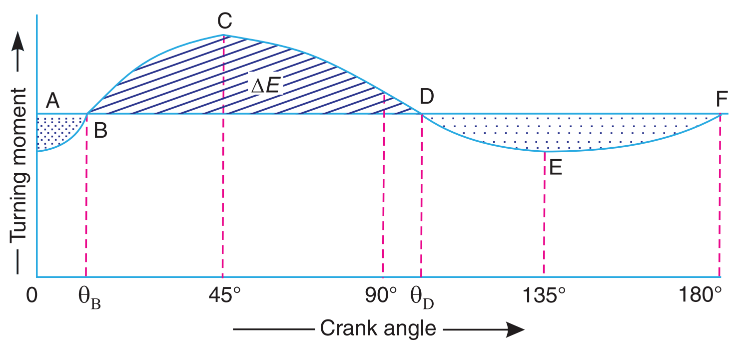

Fig. 16.13

Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean})\, d\theta$$

$$= \int_{15.5°}^{105.5°} (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000)\, d\theta$$

$$= \left[-\frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_{15.5°}^{105.5°} = 11\,078 \text{ N-m}$$

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We know that maximum fluctuation of energy (ΔE),

$$11\,078 = I.\omega^2.C_S = I \times (18.85)^2 \times 0.01 = 3.55\, I$$

$\therefore \quad I = 11078/3.55 = 3121$ kg-m^2 **Ans.**

Nowadays steam turbines like this can be produced entirely by computer-controlled machine tools, directly from the engineer's computer.

Note : This picture is given as additional information and is not a direct example of the current chapter.

3. *Angular acceleration of the flywheel*

Let α = Angular acceleration of the flywheel, and

θ = Angle turned by the crank from inner dead centre = 45° . . . (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. We know that excess torque at any instant,

$$T_{excess} = T - T_{mean}$$
$$= 20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20000$$
$$= 9500 \sin 2\theta - 5700 \cos 2\theta$$

$\therefore$ Excess torque at 45°

$$= 9500 \sin 90° - 5700 \cos 90° = 9500 \text{ N-m} \quad \ldots (i)$$

We also know that excess torque

$$= I.\alpha = 3121 \times \alpha \quad \ldots (ii)$$

From equations (*i*) and (*ii*),

$\alpha = 9500/3121 = 3.044$ rad /s^2 **Ans.**

Example 16.12. *A certain machine requires a torque of (5000 + 500 sin θ) N-m to drive it, where θ is the angle of rotation of shaft measured from certain datum. The machine is directly coupled to an engine which produces a torque of (5000 + 600 sin 2θ) N-m. The flywheel and the other rotating parts attached to the engine has a mass of 500 kg at a radius of gyration of 0.4 m. If the mean speed is 150 r.p.m., find :* **1.** *the fluctuation of energy,* **2.** *the total percentage fluctuation of speed, and* **3.** *the maximum and minimum angular acceleration of the flywheel and the corresponding shaft position.*

Solution. Given : T_1 = (5000 + 500 sin θ) N-m ; T_2 = (5000 + 600 sin 2θ) N-m ; m = 500 kg; k = 0.4 m ; N = 150 r.p.m. or $\omega = 2\pi \times 150/60 = 15.71$ rad/s

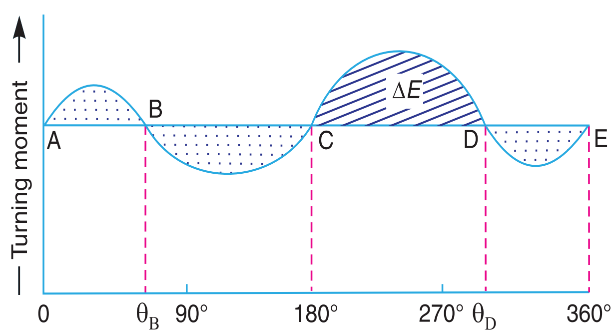

Fig. 16.14

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1. *Fluctuation of energy*

We know that change in torque

$$= T_2 - T_1 = (5000 + 600 \sin 2\theta) - (5000 + 500 \sin \theta)$$
$$= 600 \sin 2\theta - 500 \sin \theta$$

This change is zero when

$$600 \sin 2\theta = 500 \sin \theta \quad \text{or} \quad 1.2 \sin 2\theta = \sin \theta$$

$$1.2 \times 2 \sin \theta \cos \theta = \sin \theta \quad \text{or} \quad 2.4 \sin \theta \cos \theta = \sin \theta \quad \ldots (\because \sin 2\theta = 2 \sin \theta \cos \theta)$$

$\therefore$ Either $\sin \theta = 0$ or $\cos \theta = 1/2.4 = 0.4167$

when $\sin \theta = 0, \theta = 0°, 180°$ and $360°$

i.e. $\theta_A = 0°, \theta_C = 180°$ and $\theta_E = 360°$

when $\cos \theta = 0.4167, \theta = 65.4°$ and $294.6°$

i.e. $\theta_B = 65.4°$ and $\theta_D = 294.6°$

The turning moment diagram is shown in Fig. 16.14. The maximum fluctuation of energy lies between *C* and *D* (*i.e.* between 180° and 294.6°), as shown shaded in Fig. 16.14.

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \int_{180°}^{294.6°} (T_2 - T_1)\, d\theta$$

$$= \int_{180°}^{294.6°} \left[(5000 + 600 \sin 2\theta) - (5000 + 500 \sin \theta)\right] d\theta$$

$$= \left[-\frac{600 \cos 2\theta}{2} + 500 \cos \theta \right]_{180°}^{294.6°} = 1204 \text{ N-m} \quad \textbf{Ans.}$$

2. *Total percentage fluctuation of speed*

Let C_S = Total percentage fluctuation of speed.

We know that maximum fluctuation of energy (ΔE),

$$1204 = m.k^2.\omega^2.C_S = 500 \times (0.4)^2 \times (15.71)^2 \times C_S = 19\,744\, C_S$$

$\therefore$ $C_S = 1204 / 19\,744 = 0.061$ or 6.1% **Ans.**

3. *Maximum and minimum angular acceleration of the flywheel and the corresponding shaft positions*

The change in torque must be maximum or minimum when acceleration is maximum or minimum. We know that

Change in torque, $T = T_2 - T_1 = (5000 + 600 \sin 2\theta) - (5000 + 500 \sin \theta)$

$= 600 \sin 2\theta - 500 \sin \theta$ ...(*i*)

Differentiating this expression with respect to θ and equating to zero for maximum or minimum values.

$$\therefore \quad \frac{d}{d\theta}(600 \sin 2\theta - 500 \sin \theta) = 0 \quad \text{or} \quad 1200 \cos 2\theta - 500 \cos \theta = 0$$

or $12 \cos 2\theta - 5 \cos \theta = 0$

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$12\,(2\cos^2\theta - 1) - 5\cos\theta = 0$ . . . ($\because \cos 2\theta = 2\cos^2\theta - 1$)

$24\cos^2\theta - 5\cos\theta - 12 = 0$

$$\therefore \quad \cos\theta = \frac{5 \pm \sqrt{25 + 4 \times 12 \times 24}}{2 \times 24} = \frac{5 \pm 34.3}{48}$$

$$= 0.8187 \quad \text{or} \quad -0.6104$$

$$\therefore \quad \theta = 35° \quad \text{or} \quad 127.6°$$ **Ans.**

Substituting $\theta = 35°$ in equation **(*i*)**, we have maximum torque,

$$T_{max} = 600 \sin 70° - 500 \sin 35° = 277 \text{ N-m}$$

Substituting $\theta = 127.6°$ in equation **(*i*)**, we have minimum torque,

$$T_{min} = 600 \sin 255.2° - 500 \sin 127.6° = -976 \text{ N-m}$$

We know that maximum acceleration,

$$\alpha_{max} = \frac{T_{max}}{I} = \frac{277}{500 \times (0.4)^2} = 3.46 \text{ rad/s}^2$$ **Ans.** . . . ($\because I = m.k^2$)

and minimum acceleration (or maximum retardation),

$$\alpha_{min} = \frac{T_{min}}{I} = \frac{976}{500 \times (0.4)^2} = 12.2 \text{ rad/s}^2$$ **Ans.**

Example 16.13. *The equation of the turning moment curve of a three crank engine is (5000 + 1500 sin 3 θ) N-m, where θ is the crank angle in radians. The moment of inertia of the flywheel is 1000 kg-m² and the mean speed is 300 r.p.m. Calculate :* ***1.*** *power of the engine, and* ***2.*** *the maximum fluctuation of the speed of the flywheel in percentage when* **(*i*)** *the resisting torque is constant, and* **(*ii*)** *the resisting torque is (5000 + 600 sin θ) N-m.*

Solution. Given : $T = (5000 + 1500 \sin 3\theta)$ N-m ; $I = 1000$ kg-m² ; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

1. ***Power of the engine***

We know that work done per revolution

$$= \int_0^{\pi} (5000 + 1500 \sin 3\theta)\, d\theta = \left[5000\,\theta - \frac{1500 \cos 3\theta}{3}\right]_0^{2\pi}$$

$$= 10\,000\,\pi \text{ N-m}$$

$\therefore$ Mean resisting torque,

$$T_{mean} = \frac{\text{Work done/rev}}{2\pi} = \frac{10\,000\,\pi}{2\pi} = 5000 \text{ N-m}$$

We know that power of the engine,

$$P = T_{mean} \,.\, \omega = 5000 \times 31.42 = 157\,100 \text{ W} = 157.1 \text{ kW}$$ **Ans.**

2. ***Maximum fluctuation of the speed of the flywheel***

Let C_S = Maximum or total fluctuation of speed of the flywheel.

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(i) When resisting torque is constant

The turning moment diagram is shown in Fig. 16.15. Since the resisting torque is constant, therefore the torque exerted on the shaft is equal to the mean resisting torque on the flywheel.

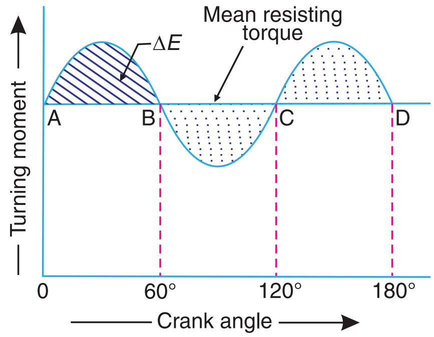

Fig. 16.15

$$\therefore \quad T = T_{mean}$$

$$5000 + 1500 \sin 3\theta = 5000$$

$$1500 \sin 3\theta = 0 \quad \text{or} \quad \sin 3\theta = 0$$

$$\therefore \quad 3\theta = 0° \quad \text{or} \quad 180°$$

$$\theta = 0° \quad \text{or} \quad 60°$$

∴ Maximum fluctuation of energy,

$$\Delta E = \int_0^{60°} (T - T_{mean})\, d\theta = \int_0^{60°} (5000 + 1500 \sin 3\theta - 5000)\, d\theta$$

$$= \int_0^{60°} 1500 \sin 3\theta\, d\theta = \left[-\frac{1500 \cos 3\theta}{3} \right]_0^{60°} = 1000 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$1000 = I.\omega^2.C_S = 1000 \times (31.42)^2 \times C_S = 987\ 216\ C_S$$

$$\therefore \quad C_S = 1000 / 987\ 216 = 0.001 \text{ or } 0.1\% \quad \textbf{Ans.}$$

(ii) When resisting torque is (5000 + 600 sin θ) N-m

The turning moment diagram is shown in Fig. 16.16. Since at points *B* and *C*, the torque exerted on the shaft is equal to the mean resisting torque on the flywheel, therefore

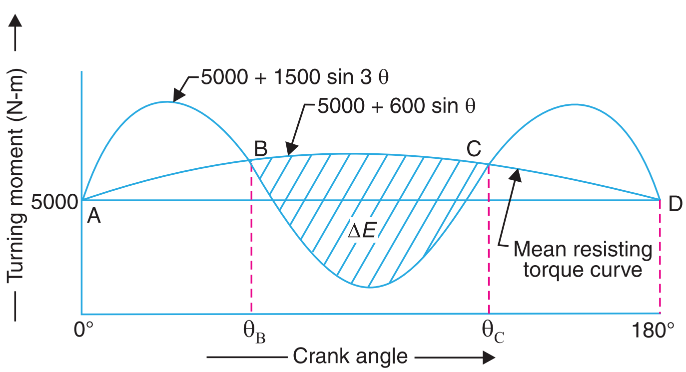

Fig. 16.16

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$5000 + 1500 \sin 3\theta = 5000 + 600 \sin \theta$ or $2.5 \sin 3\theta = \sin \theta$

$2.5 (3 \sin \theta - 4 \sin^3 \theta) = \sin \theta$ ...($\because \sin 3\theta = 3 \sin \theta - 4 \sin^3\theta$)

$3 - 4 \sin^2\theta = 0.4$ ...(Dividing by 2.5 sin θ)

$$\sin^2 \theta = \frac{3 - 0.4}{4} = 0.65 \quad \text{or} \quad \sin \theta = 0.8062$$

$\therefore$ $\theta = 53.7°$ or $126.3°$ *i.e.* $\theta_B = 53.7°$, and $\theta_C = 126.3°$

$\therefore$ Maximum fluctuation of energy,

$$^*\Delta E = \int_{53.7°}^{126.3°} \left[(5000 + 1500 \sin 3\theta) - (5000 + 600 \sin \theta)\right] d\theta$$

$$= \int_{53.7°}^{126.3°} (1500 \sin 3\theta - 600 \sin \theta) d\theta = \left[-\frac{1500 \cos 3\theta}{3} + 600 \cos \theta\right]_{53.7°}^{126.3°}$$

$= -1656$ N-m

We know that maximum fluctuation of energy (ΔE),

$1656 = I.\omega^2.C_S = 1000 \times (31.42)^2 \times C_S = 987\ 216\ C_S$

$\therefore$ $C_S = 1656 / 987\ 216 = 0.00\ 168$ or 0.168% **Ans.**

16.11. Dimensions of the Flywheel Rim

Consider a rim of the flywheel as shown in Fig. 16.17.

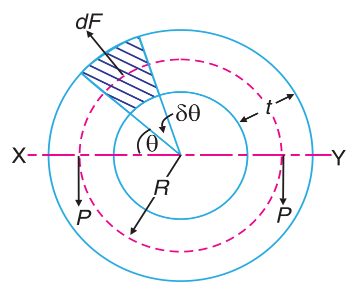

Fig. 16.17. Rim of a flywheel.

Let D = Mean diameter of rim in metres,

R = Mean radius of rim in metres,

A = Cross-sectional area of rim in m^2,

ρ = Density of rim material in kg/m^3,

N = Speed of the flywheel in r.p.m.,

ω = Angular velocity of the flywheel in rad/s,

v = Linear velocity at the mean radius in m/s $= \omega . R = \pi D.N/60$, and

σ = Tensile stress or hoop stress in N/m^2 due to the centrifugal force.

Consider a small element of the rim as shown shaded in Fig. 16.17. Let it subtends an angle $\delta\theta$ at the centre of the flywheel.

Volume of the small element

$$= A \times R.\delta\theta$$

$\therefore$ Mass of the small element

$$dm = \text{Density} \times \text{volume} = \rho.A.R.\delta\theta$$

and centrifugal force on the element, acting radially outwards,

$$dF = dm.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$$

* Since the fluctuation of energy is negative, therefore it is shown below the mean resisting torque curve, in Fig. 16.16.

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Vertical component of dF

$$= dF.\sin\theta = \rho.A.R^2.\omega^2.\delta\theta.\sin\theta$$

∴ Total vertical upward force tending to burst the rim across the diameter $X\ Y$.

$$= \rho.A.R^2.\omega^2 \int_0^{\pi} \sin\theta.d\theta = \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^{\pi}$$

$$= 2\rho.A.R^2.\omega^2 \qquad \text{. . . (i)}$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by $2P$, such that

$$2P = 2\sigma.A \qquad \text{. . . (ii)}$$

Equating equations (*i*) and (*ii*),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

or $$\sigma = \rho.R^2.\omega^2 = \rho.v^2 \qquad \text{....}(\because v = \omega.R)$$

∴ $$v = \sqrt{\frac{\sigma}{\rho}} \qquad \text{...(iii)}$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi D.A.\rho$$

∴ $$A = \frac{m}{\pi.D.\rho} \qquad \text{...(iv)}$$

From equations ***(iii)*** and ***(iv)***, we may find the value of the mean radius and cross-sectional area of the rim.

Note: If the cross-section of the rim is a rectangular, then

$$A = b \times t$$

where b = Width of the rim, and

t = Thickness of the rim.

Example 16.14. *The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N-m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in sq. mm are*

– 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 sq. mm, when the engine is running at 800 r.p.m.

The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m³. The width of the rim is to be 5 times the thickness.

Solution. Given : N = 800 r.p.m. or $\omega = 2\pi \times 800 / 60 = 83.8$ rad/s; *Stroke = 300 mm ; $\sigma = 7$ MPa $= 7 \times 10^6$ N/m² ; $\rho = 7200$ kg/m³

Since the fluctuation of speed is ± 2% of mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 4\%\ \omega = 0.04\ \omega$$

* Superfluous data.

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and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

Diameter of the flywheel rim

Let D = Diameter of the flywheel rim in metres, and

v = Peripheral velocity of the flywheel rim in m/s.

We know that centrifugal stress (σ),

$$7 \times 10^6 = \rho.v^2 = 7200\, v^2 \quad \text{or} \quad v^2 = 7 \times 10^6/7200 = 972.2$$

$\therefore$ $v = 31.2$ m/s

We know that $v = \pi\, D.N/60$

$\therefore$ $D = v \times 60 / \pi\, N = 31.2 \times 60/\pi \times 800 = 0.745$ m **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $5\,t$ ...(Given)

$\therefore$ Cross-sectional area of flywheel rim,

$$A = b.t = 5\,t \times t = 5\,t^2$$

First of all, let us find the mass (m) of the flywheel rim. The turning moment diagram is shown in Fig 16.18.

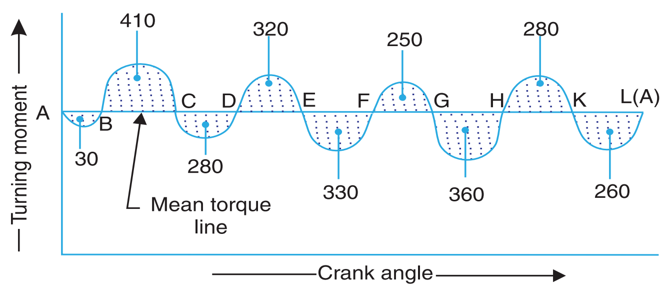

Fig. 16.18

Since the turning moment scale is 1 mm = 500 N-m and crank angle scale is 1 mm = 6° = π /30 rad, therefore

1 mm^2 on the turning moment diagram

$$= 500 \times \pi / 30 = 52.37 \text{ N-m}$$

Let the energy at $A = E$, then referring to Fig. 16.18,

Energy at $B = E - 30$ . . . **(Minimum energy)**

Energy at $C = E - 30 + 410 = E + 380$

Energy at $D = E + 380 - 280 = E + 100$

Energy at $E = E + 100 + 320 = E + 420$ . . . **(Maximum energy)**

Energy at $F = E + 420 - 330 = E + 90$

Energy at $G = E + 90 + 250 = E + 340$

Energy at $H = E + 340 - 360 = E - 20$

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Energy at $K = E - 20 + 280 = E + 260$

Energy at $L = E + 260 - 260 = E$ = Energy at A

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 420) - (E - 30) = 450 \text{ mm}^2$$

$$= 450 \times 52.37 = 23\ 566 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$23\ 566 = m.v^2.C_S = m \times (31.2)^2 \times 0.04 = 39\ m$$

$$\therefore \quad m = 23566 / 39 = 604 \text{ kg}$$

We know that mass of the flywheel rim (m),

$$604 = \text{Volume} \times \text{density} = \pi\, D.A.\rho$$

$$= \pi \times 0.745 \times 5t^2 \times 7200 = 84\ 268\ t^2$$

$$\therefore \quad t^2 = 604 / 84\ 268 = 0.007\ 17 \text{ m}^2 \quad \text{or} \quad t = 0.085 \text{ m} = 85 \text{ mm}$$ **Ans.**

and $b = 5t = 5 \times 85 = 425$ mm **Ans.**

Example 16.15. *A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean speed. If the mean diameter of the flywheel rim is 2 metre and the hub and spokes provide 5% of the rotational inertia of the flywheel, find the mass and cross-sectional area of the flywheel rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg/m³.*

Solution. Given : $P = 150$ kW $= 150 \times 10^3$ W; $N = 80$ r.p.m. or $\omega = 2\,\pi \times 80/60 = 8.4$ rad/s; $C_E = 0.1$; $D = 2$ m or $R = 1$ m ; $\rho = 7200$ kg/m^3

Since the fluctuation of speed is ± 2% of mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 4\%\ \omega = 0.04\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg, and

I = Mass moment of inertia of the flywheel in kg-m^2.

We know that work done per cycle

$$= P \times 60/N = 150 \times 10^3 \times 60 / 80 = 112.5 \times 10^3 \text{ N-m}$$

and maximum fluctuation of energy,

$$\Delta E = \text{Work done /cycle} \times C_E = 112.5 \times 10^3 \times 0.1 = 11\ 250 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$11\ 250 = I.\omega^2.C_S = I \times (8.4)^2 \times 0.04 = 2.8224\ I$$

$$\therefore \quad I = 11\ 250 / 2.8224 = 3986 \text{ kg-m}^2$$

Since the hub and spokes provide 5% of the rotational inertia of the flywheel, therefore, mass moment of inertia of the flywheel rim (I_{rim}) will be 95% of the flywheel, *i.e.*

$$I_{rim} = 0.95\ I = 0.95 \times 3986 = 3787 \text{ kg-m}^2$$

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and $I_{rim} = m.k^2$ or $^* m = \dfrac{I_{rim}}{k^2} = \dfrac{3787}{1^2} = 3787$ kg **Ans.** ... ($\because k = R$)

Cross-sectional area of the flywheel rim

Let A = Cross-sectional area of flywheel rim in m^2.

We know that the mass of the flywheel (m),

$$3787 = 2\,\pi\,R \times A \times \rho = 2\,\pi \times 1 \times A \times 7200 = 45\,245\,A$$

$\therefore$ $A = 3787/45\,245 = 0.084$ m^2 **Ans.**

Example 16.16. *A multi-cylinder engine is to run at a speed of 600 r.p.m. On drawing the turning moment diagram to a scale of 1 mm = 250 N-m and 1 mm = 3°, the areas above and below the mean torque line in mm² are : + 160, – 172, + 168, – 191, + 197, – 162*

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel. Determine the suitable dimensions of a rectangular flywheel rim if the breadth is twice its thickness. The density of the cast iron is 7250 kg/m³ and its hoop stress is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

Solution. Given : N = 600 r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad /s; $\rho = 7250$ kg/m^3; $\sigma = 6$ MPa $= 6 \times 10^6$ N/m^2

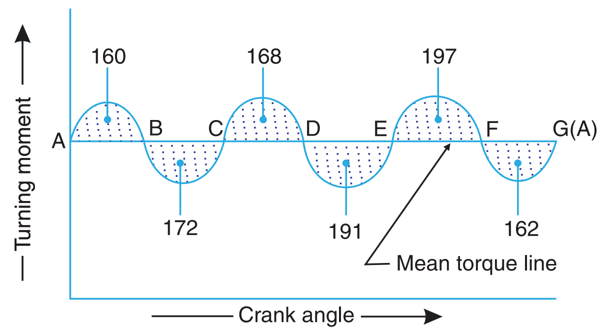

Fig. 16.19

Since the fluctuation of speed is ± 1% of mean speed, therefore, total fluctuation of speed,

$$\omega_1 - \omega_2 = 2\%\ \omega = 0.02\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

Moment of inertia of the flywheel

Let I = Moment of inertia of the flywheel in kg-m^2.

The turning moment diagram is shown in Fig. 16.19. The turning moment scale is 1 mm = 250 N-m and crank angle scale is 1 mm = 3° = π /60 rad, therefore,

1 mm^2 of turning moment diagram

$$= 250 \times \pi/60 = 13.1 \text{ N-m}$$

---

* The mass of the flywheel rim (m) may also be obtained by using the following relation:

$\Delta E_{rim} = 0.95\,(\Delta E) = 0.95 \times 11\,250 = 10\,687.5$ N-m

and $\Delta E_{rim} = m.k^2.\omega^2.C_S = m\,(1)^2 \times (8.4)^2 \times 0.04 = 2.8224\,m$

$\therefore$ $m = (\Delta E)_{rim} / 2.8224 = 10\,687.5 / 2.8224 = 3787$ kg

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Let the total energy at $A = E$. Therefore from Fig. 16.19, we find that

Energy at $B = E + 160$

Energy at $C = E + 160 - 172 = E - 12$

Energy at $D = E - 12 + 168 = E + 156$

Energy at $E = E + 156 - 191 = E - 35$ . . . **(Minimum energy)**

Energy at $F = E - 35 + 197 = E + 162$ . . . **(Maximum energy)**

Energy at $G = E + 162 - 162 = E =$ Energy at A

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 162) - (E - 35) = 197 \text{ mm}^2$$
$$= 197 \times 13.1 = 2581 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I \times (62.84)^2 \times 0.02 = 79\, I$$

$\therefore$ $I = 2581/79 = 32.7$ kg-m^2 **Ans.**

Dimensions of the flywheel rim

Let t = Thickness of the flywheel rim in metres,

b = Breadth of the flywheel rim in metres = $2\,t$ . . . (Given)

D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that hoop stress (σ),

$$6 \times 10^6 = \rho.v^2 = 7250\, v^2 \quad \text{or} \quad v^2 = 6 \times 10^6/7250 = 827.6$$

$\therefore$ $v = 28.8$ m/s

We know that $v = \pi\, DN/60$, or $D = v \times 60 / \pi\, N = 28.8 \times 60/\pi \times 600 = 0.92$ m

Now, let us find the mass (m) of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore maximum fluctuation of energy of rim,

$$\Delta E_{rim} = 0.92 \times \Delta E = 0.92 \times 2581 = 2375 \text{ N-m}$$

We know that maximum fluctuation of energy of rim (ΔE_{rim}),

$$2375 = m.v^2.C_S = m \times (28.8)^2 \times 0.02 = 16.6\, m$$

$\therefore$ $m = 2375/16.6 = 143$ kg

Also $m = \text{Volume} \times \text{density} = \pi\, D.A.\rho = \pi\, D.b.t.\rho$

$\therefore$ $143 = \pi \times 0.92 \times 2\,t \times t \times 7250 = 41\,914\, t^2$

$t^2 = 143 / 41\,914 = 0.0034$ m^2

or $t = 0.0584$ m $= 58.4$ mm **Ans.**

and $b = 2\,t = 116.8$ mm **Ans.**

Example 16.17. *The turning moment diagram of a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The areas of these triangles are as follows:*

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Suction stroke = 5×10^{-5} m^2; Compression stroke = 21×10^{-5} m^2; Expansion stroke = 85×10^{-5} m^2; Exhaust stroke = 8×10^{-5} m^2.

All the areas excepting expression stroke are negative. Each m^2 of area represents 14 MN-m of work.

Assuming the resisting torque to be constant, determine the moment of inertia of the flywheel to keep the speed between 98 r.p.m. and 102 r.p.m. Also find the size of a rim-type flywheel based on the minimum material criterion, given that density of flywheel material is 8150 kg/m^3 ; the allowable tensile stress of the flywheel material is 7.5 MPa. The rim cross-section is rectangular, one side being four times the length of the other.

Solution. Given: $a_1 = 5 \times 10^{-5}$ m^2; $a_2 = 21 \times 10^{-5}$ m^2; $a_3 = 85 \times 10^{-5}$ m^2; $a_4 = 8 \times 10^{-5}$ m^2; $N_2 = 98$ r.p.m.; $N_1 = 102$ r.p.m.; $\rho = 8150$ kg/m^3; $\sigma = 7.5$ MPa $= 7.5 \times 10^6$ N/m^2

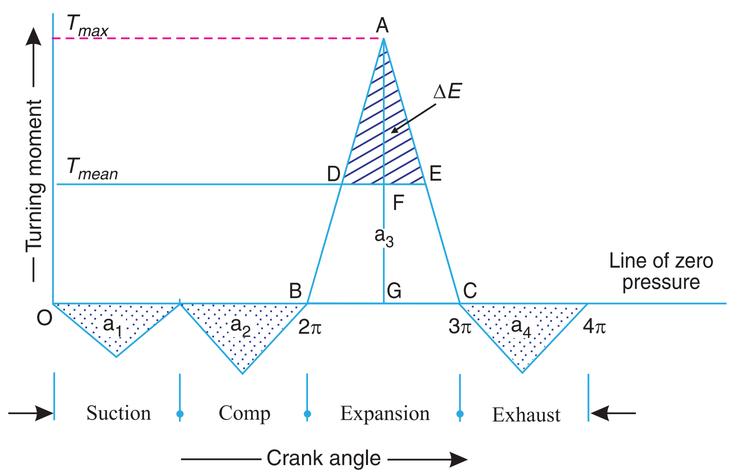

Fig. 16.20

The turning moment-crank angle diagram for a four stroke engine is shown in Fig. 16.20. The areas below the zero line of pressure are taken as negative while the areas above the zero line of pressure are taken as positive.

$\therefore$ Net area $= a_3 - (a_1 + a_2 + a_4)$

$= 85 \times 10^5 - (5 \times 10^5 + 21 \times 10^{-5} + 8 \times 10^{-5}) = 51 \times 10^{-5}$ m^2

Since $1m^2 = 14$ MN-m $= 14 \times 10^6$ N-m of work, therefore

Net work done per cycle

$$= 51 \times 10^{-5} \times 14 \times 10^6 = 7140 \text{ N-m} \quad \ldots(i)$$

We also know that work done per cycle

$$= T_{mean} \times 4\pi \text{ N-m} \quad \ldots(ii)$$

From equation (*i*) and (*ii*),

$$T_{mean} = FG = 7140 / 4\pi = 568 \text{ N-m}$$

Work done during expansion stroke

$$= a_3 \times \text{Work scale} = 85 \times 10^{-5} \times 14 \times 10^6 = 11\,900 \text{ N-m} \quad \ldots(iii)$$

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Also, work done during expansion stroke

$$= \frac{1}{2} \times BC \times AG = = \frac{1}{2} \times \pi \times AG = 1.571\, AG \qquad ...(iv)$$

From equations **(*iii*)** and **(*iv*),**

$AG = 11\ 900/1.571 = 7575$ N-m

∴ Excess torque $= AF = AG - FG = 7575 - 568 = 7007$ N-m

Now from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{7007}{7575} \times \pi = 2.9 \text{ rad}$$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta\, ADE = \frac{1}{2} \times DE \times AF$$

$$= \frac{1}{2} \times 2.9 \times 7007 = 10\ 160 \text{ N-m}$$

Moment of Inertia of the flywheel

Let I = Moment of inertia of the flywheel in kg-m^2.

We know that mean speed during the cycle

$$N = \frac{N_1 + N_2}{2} = \frac{102 + 98}{2} = 100 \text{ r.p.m.}$$

∴ Corresponding angular mean speed,

$$\omega = 2\pi N / 60 = 2\pi \times 100/60 = 10.47 \text{ rad/s}$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{102 - 98}{100} = 0.04$$

We know that maximum fluctuation of energy (ΔE),

$$10\ 160 = I.\omega^2.C_S = I\,(10.47)^2 \times 0.04 = 4.385\, I$$

∴ $I = 10160 / 4.385 = 2317$ kg-m^2 **Ans.**

Size of flywheel

Let t = Thickness of the flywheel rim in metres,

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that hoop stress (σ),

$$7.5 \times 10^6 = \rho\ .\ v^2 = 8150\, v^2$$

∴ $$v^2 = \frac{7.5 \times 10^6}{8150} = 920 \quad \text{or} \quad v = 30.3 \text{ m/s}$$

and $v = \pi DN/60$ or $D = v \times 60/\pi N = 30.3 \times 60/\pi \times 100 = 5.786$ m

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Now let us find the mass (m) of the flywheel rim. We know that maximum fluctuation of energy (ΔE),

$$10\ 160 = m.v^2\ C_S = m \times (30.3)^2 \times 0.04 = 36.72\ m$$

∴ $$m = 10\ 160/36.72 = 276.7 \text{ kg}$$

Also $$m = \text{Volume} \times \text{density} = \pi D \times A \times \frac{\rho}{g} = \pi D \times b \times t \times \frac{\rho}{g}$$

$$276.7 = \pi \times 5.786 \times 4t \times t \times \frac{8 \times 10^4}{9.81} = 59.3 \times 10^4\ t^2$$

∴ $$t^2 = 276.7/59.3 \times 10^4 = 0.0216 \text{ m or } 21.6 \text{ mm}$$ **Ans.**

and $$b = 4t = 4 \times 21.6 = 86.4 \text{ mm}$$ **Ans.**

Example 16.18. *An otto cycle engine develops 50 kW at 150 r.p.m. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed 0.5% of mean on either side. Find the mean diameter of the flywheel and a suitable rim cross-section having width four times the depth so that the hoop stress does not exceed 4 MPa. Assume that the flywheel stores 16/15 times the energy stored by the rim and the work done during power stroke is 1.40 times the work done during the cycle. Density of rim material is 7200 kg/m³.*

Solution. Given : $P = 50$ kW $= 50 \times 10^3$ W; $N = 150$ r.p.m. or $\omega = 2\pi \times 150/60 = 15.71$ rad/s; $n = 75$; $\sigma = 4$ MPa $= 4 \times 10^6$ N/m²; $r = 7200$ kg/m³

First of all, let us find the mean torque (T_{mean}) transmitted by the engine or flywheel. We know that the power transmitted (P),

$$50 \times 10^3 = T_{mean} \times \omega = T_{mean} \times 15.71$$

∴ $$T_{mean} = 50 \times 10^3/15.71 = 3182.7 \text{ N-m}$$

Since the explosions per minute are equal to $N/2$, therefore, the engine is a four stroke cycle engine. The turning moment diagram of a four stroke engine is shown in Fig. 16.21.

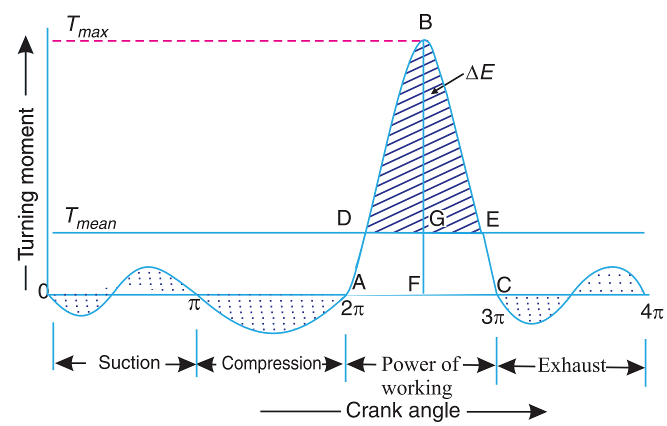

Fig. 16.21

We know that *work done per cycle

$$= T_{mean} \times \theta = 3182.7 \times 4\pi = 40\ 000 \text{ N-m}$$

* The work done per cycle for a four stroke engine is also given by

$$\text{Work done per cycle} = \frac{P \times 60}{\text{Number of explosions/min}} = \frac{P \times 60}{n} = \frac{50 \times 10^3 \times 60}{75} = 40000 \text{ N-m}$$

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∴ Workdone during power or working stroke

$= 1.4 \times \text{work done per cycle}$ ....(Given)

$= 1.4 \times 40\ 000 = 56\ 000$ N-m ...(*i*)

The workdone during power stroke is shown by a triangle ABC in Fig. 16.20, in which base $AC = \pi$ radians and height $BF = T_{max}$.

∴ Work done during working stroke

$$= \frac{1}{2} \times \pi \times T_{max} = 1.571\, T_{max} \quad \ldots (ii)$$

From equations (*i*) and (*ii*), we have

$$T_{max} = 56\ 000/1.571 = 35\ 646 \text{ N-m}$$

We know that the excess torque,

$$T_{excess} = BG = BF - FG = T_{max} - T_{mean} = 35\ 646 - 3182.7 = 32\ 463.3 \text{ N-m}$$

Now, from similar triangles BDE and ABC,

$$\frac{DE}{AC} = \frac{BG}{BF} \quad \text{or} \quad DE = \frac{BG}{BF} \times AC = \frac{32\,463.3}{35\,646} \times \pi = 0.9107\pi$$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Area of triangle } BDE = \frac{1}{2} \times DE \times BG$$

$$= \frac{1}{2} \times 0.9107\pi \times 32\,463.3 = 46\ 445 \text{ N-m}$$

Mean diameter of the flywheel

Let D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that hoop stress (σ),

$$4 \times 10^6 = \rho.v^2 = 7200\, v^2 \quad \text{or} \quad v^2 = 4 \times 10^6/7200 = 556$$

∴ $v = 23.58$ m/s

We know that $v = \pi DN/60$ or $D = v \times 60/N = 23.58 \times 60/\pi \times 150 = 3$ m **Ans.**

Cross-sectional dimensions of the rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $4\,t$ ...(Given)

∴ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2$$

First of all, let us find the mass of the flywheel rim.

Let m = Mass of the flywheel rim in kg, and

E = Total energy of the flywheel in N-m.

Since the fluctuation of speed is 0.5% of the mean speed on either side, therefore total fluctuation of speed,

$$N_2 - N_1 = 1\% \text{ of mean speed} = 0.01\, N$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = 0.01$$

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We know that the maximum fluctuation of energy (ΔE),

$$46\ 445 = E \times 2C_S = E \times 2 \times 0.01 = 0.02\ E$$

$$\therefore \qquad E = 46\ 445 / 0.02 = 2322 \times 10^3 \text{ N-m}$$

Since the energy stored by the flywheel is $\frac{16}{15}$ times the energy stored by the rim, therefore, the energy of the rim,

$$E_{rim} = \frac{15}{16} E = \frac{15}{16} \times 232 \times 10^3 = 2177 \times 10^3 \text{ N-m}$$

We know that energy of the rim (E_{rim}),

$$2177 \times 10^3 = \frac{1}{2} \times m \times v^2 = m\,(23.58)^2 = 278\ m$$

$$\therefore \qquad m = 2177 \times 10^3 / 278 = 7831 \text{ kg}$$

We also knonw that mass of the flywheel rim (m),

$$7831 = \pi D \times A \times \rho = \pi \times 3 \times 4t^2 \times 7200 = 271\ 469 t^2$$

$$\therefore \qquad t^2 = 831 / 271\ 469 = 0.0288 \text{ or } t = 0.17 \text{ m} = 170 \text{ mm}$$ **Ans.**

and $\qquad b = 4\,t = 4 \times 170 = 680$ mm **Ans.**

16.12. Flywheel in Punching Press

We have discussed in Art. 16.8 that the function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle. The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in punching press or in a rivetting machine. A punching press is shown diagrammatically in Fig. 16.22. The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism. From Fig. 16.22, we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied. On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from

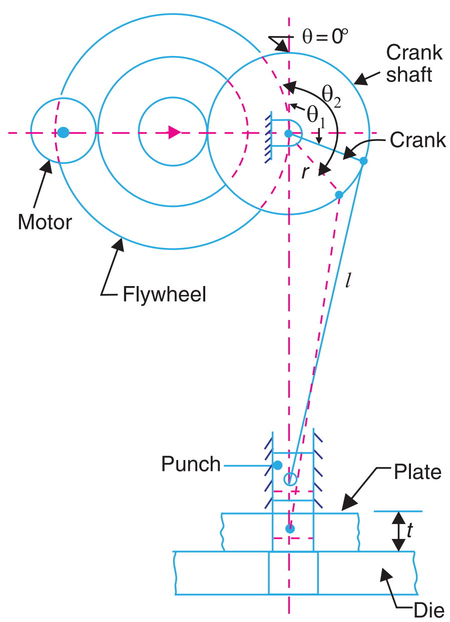

Fig. 16.22. Operation of flywheel in a punching press.

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$\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy supplied. Thus the flywheel has to absorb excess energy available at one stage and has to make up the deficient energy at the other stage to keep to fluctuations of speed within permissible limits. This is done by choosing the suitable moment of inertia of the flywheel.

Punching press and flywheel.

Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

Let d_1 = Diameter of the hole punched,

t_1 = Thickness of the plate, and

τ_u = Ultimate shear stress for the plate material.

∴ Maximum shear force required for punching,

$$F_S = \text{Area sheared} \times \text{Ultimate shear stress} = \pi d_1 . t_1 \tau_u$$

It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

∴ Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1. The energy supplied by the motor to the crankshaft during actual punching operation,

$$E_2 = E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right)$$

∴ Balance energy required for punching

$$= E_1 - E_2 = E_1 - E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right) = E_1 \left(1 - \frac{\theta_2 - \theta_1}{2\pi} \right)$$

This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = E_1 \left(1 - \frac{\theta_2 - \theta_1}{2\pi} \right)$$

The values of θ_1 and θ_2 may be determined only if the crank radius (r), length of connecting rod (l) and the relative position of the job with respect to the crankshaft axis are known. In the absence of relevant data, we assume that

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

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where t = Thickness of the material to be punched,

s = Stroke of the punch = $2 \times \text{Crank radius} = 2r$.

By using the suitable relation for the maximum fluctuation of energy (ΔE) as discussed in the previous articles, we can find the mass and size of the flywheel.

Example 16.19. *A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 r.p.m. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour; each punching operation takes 2 second and requires 15 kN-m of energy. Find the power of the motor and the minimum mass of the flywheel if speed of the same is not to fall below 200 r. p. m.*

Solution. Given $N_1 = 225$ r.p.m ; $k = 0.5$ m ; Hole punched = 720 per hr; $E_1 = 15$ kN-m $= 15 \times 10^3$ N-m ; $N_2 = 200$ r.p.m.

Power of the motor

We know that the total energy required per second

= Energy required / hole × No. of holes / s

$= 15 \times 10^3 \times 720/3600 = 3000$ N-m/s

∴ Power of the motor = 3000 W = 3 kW **Ans.** (∵ 1 N-m/s = 1 W)

Minimum mass of the flywheel

Let m = Minimum mass of the flywheel.

Since each punching operation takes 2 seconds, therefore energy supplied by the motor in 2 seconds,

$$E_2 = 3000 \times 2 = 6000 \text{ N-m}$$

∴ Energy to be supplied by the flywheel during punching or maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 15 \times 10^3 - 6000 = 9000 \text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2} = 212.5 \text{ r.p.m}$$

We know that maximum fluctuation of energy (ΔE),

$$9000 = \frac{\pi^2}{900} \times m.k^2.N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m \times (0.5)^2 \times 212.5 \times (225 - 200) = 14.565\, m$$

∴ m = 9000/14.565 = 618 kg **Ans.**

Example 16.20. *A machine punching 38 mm holes in 32 mm thick plate requires 7 N-m of energy per sq. mm of sheared area, and punches one hole in every 10 seconds. Calculate the power of the motor required. The mean speed of the flywheel is 25 metres per second. The punch has a stroke of 100 mm.*

Find the mass of the flywheel required, if the total fluctuation of speed is not to exceed 3% of the mean speed. Assume that the motor supplies energy to the machine at uniform rate.

Solution. Given : $d = 38$ mm ; $t = 32$ mm ; $E_1 = 7$ N-m/mm^2 of sheared area ; $v = 25$ m/s ; $s = 100$ mm ; $v_1 - v_2 = 3\%\, v = 0.03\, v$

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Power of the motor required

We know that sheared area,

$$A = \pi d.t = \pi \times 38 \times 32 = 3820 \text{ mm}^2$$

Since the energy required to punch a hole is 7 N-m/mm^2 of sheared area, therefore total energy required per hole,

$$E_1 = 7 \times 3820 = 26\ 740 \text{ N-m}$$

Also the time required to punch a hole is 10 second, therefore energy required for punching work per second

$$= 26\ 740/10 = 2674 \text{ N-m/s}$$

∴ Power of the motor required

$$= 2674 \text{ W} = 2.674 \text{ kW}$$ **Ans.**

Mass of the flywheel required

Let m = Mass of the flywheel in kg.

Since the stroke of the punch is 100 mm and it punches one hole in every 10 seconds, therefore the time required to punch a hole in a 32 mm thick plate

$$= \frac{10}{2 \times 100} \times 32 = 1.6 \text{ s}$$

∴ Energy supplied by the motor in 1.6 seconds,

$$E_2 = 2674 \times 1.6 = 4278 \text{ N-m}$$

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 26\ 740 - 4278 = 22\ 462 \text{ N-m}$$

Coefficient of fluctuation of speed,

$$C_S = \frac{v_1 - v_2}{v} = 0.03$$

We know that maximum fluctuation of energy (ΔE),

$$22\ 462 = m.v^2.C_S = m \times (25)^2 \times 0.03 = 18.75\ m$$

∴ $m = 22\ 462 / 18.75 = 1198$ kg **Ans.**

Note : The value of maximum fluctuation of energy (ΔE) may also be determined as discussed in Art. 16.12. We know that energy required for one punch,

$$E_1 = 26\ 740 \text{ N-m}$$

and

$$\Delta E = \left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right) = E_1\left(1 - \frac{t}{2s}\right) \quad \ldots\ldots\left(\because \frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s}\right)$$

$$= 26\ 740\left[1 - \frac{32}{2 \times 100}\right] = 22\ 462 \text{ N-m}$$

Example 16.21. *A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute?*

Solution. Given : P = 3 kW ; m = 150 kg ; k = 0.6 m ; N_1 = 300 r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$ rad/s

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