Module 5 : Pulse propagation through third order nonlinear optical medium

Lecture 34 : Propagation in fibers

Objectives

In this lecture we will discuss

1. Dispersion of refractive index and its effects on the pulse propagation.
2. Wave equation to describe pulse propagation in linear regime and
3. Propagation of a pulse in dispersion(higher order) free medium.

Propagation in fibers

Propagation of optical radiation through any medium is governed by the refractive index of the medium. We have already seen that the refractive index depends on the extent of detuning of the frequency ω of the optical radiation and the internal resonance, of the medium. This is called chromatic dispersion. Far away from the internal resonance of the medium, the refractive index is approximated by Sellmeir equation

$$n^2(\omega) = 1 + \sum_{i=1}^{m} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \qquad (34.1)$$

where ω_j is the resonant frequency and B_j is the strength of the jth resonance.

Dispersion plays an important role in the propagation of optical pulses. It leads to the broadening of pulses and in combination with the nonlinearity it can lead to the soliton propagation as we will see in the lectures to follow. All these have important implications for fiber optic communication systems as the pulses travel long distances in optical fibers.

Effect of dispersion on the propagation of an optical pulse can be accounted by expanding the propagation constant $\beta(\omega) = n(\omega)\dfrac{\omega}{c}$ of the mode in a Taylor series around the central frequency ω_0 of the pulse.

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \frac{1}{6}\beta_3(\omega - \omega_0)^3 + \ldots \qquad (34.2)$$

Where

$$\beta_m = \left[\frac{d^m\beta}{d\omega^m}\right]_{\omega=\omega_0} \qquad (34.3)$$

and

$$m = 0,1,2,3 \ldots$$

The first term in the equation (34.2) leads to the addition of the phase shift upon propagation. The second term simply leads to the temporal delay of the pulse envelop as a whole upon propagation. The pulse envelope moves at the group velocity v_g defined as

$$v_g = \frac{1}{\beta_1} \qquad (34.4)$$

Where

$$\beta_1 = \frac{1}{c}\left[n + \omega\frac{dn}{d\omega}\right]$$
$$= \frac{n_g}{c} \qquad (34.5)$$

n_g is called the group index.

As a result of the chromatic dispersion, pulses of different wavelength will propagate with different speeds due to their group velocity mismatch when two or more pulses of different wavelengths propagate in a fibers, it results in-their temporal separation as shown in figure 34.1.
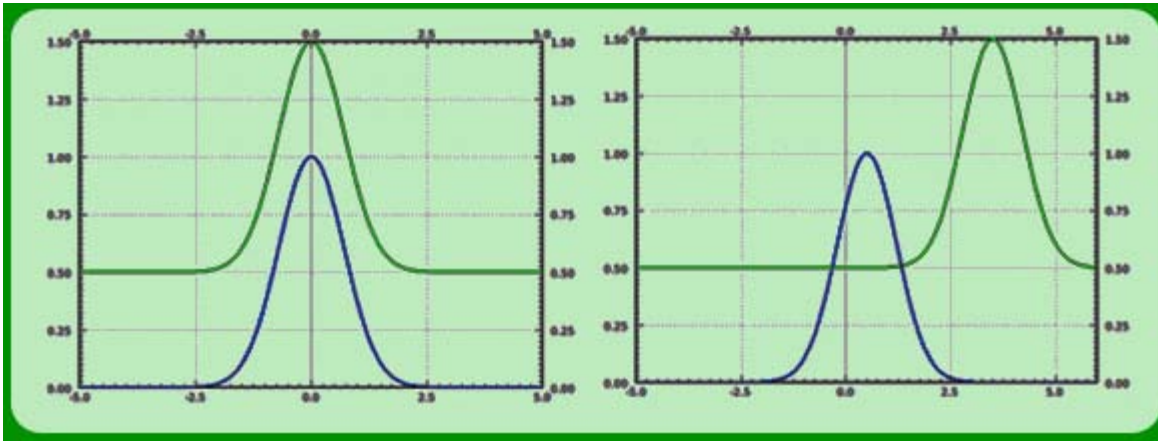
Figure 34.1 Two pulses of wavelengths ,and as at the input end of the fiber (time t=0) -left panel and temporally separated pulses after propagation for time t-right panel

The walk off parameter d is defined as

$$d = \beta_1(\lambda_1) - \beta_1(\lambda_2)$$

$$= \left[\frac{1}{v_g(\lambda_1)} - \frac{1}{v_g(\lambda_2)}\right] \tag{34.6}$$

For a given pulse duration the walk-off length L_w is given by

$$L_w = \frac{\tau}{d} \tag{34.7}$$

Example: Two pulses of 100 ps each having their central wavelength λ_1 = 1.06 μm and λ_2 =1.127 μm propagating in a fiber with walk-off parameter d = 1.67 ps/m will completely separate in time after traversing its walk of length L_w=60 m.

The parameter β_2 appearing in the third term of equation (34.2) is called the group velocity dispersion parameter

$$\beta_2 = \frac{d\beta_1}{d\omega} = \frac{d}{d\omega}\left[\frac{1}{v_g}\right]$$

$$= -\frac{1}{v_g^2}\frac{dv_g}{d\omega} \tag{34.8}$$

It leads to the pulse broadening.

$$\beta_2 = \frac{1}{c}\left[2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}\right]$$

$$\simeq \frac{\omega}{c}\frac{d^2n}{d\omega^2} \tag{34.9}$$

Alternatively, one can write

$$\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2} \tag{34.10}$$

If $\lambda < \lambda_D$ -zero dispersion wavelength then $\beta_2 > 0$. It is called the normally dispersive behavior.

In this case, since $v_{\lambda+\Delta\lambda} > v_\lambda$ the red wavelengths travel faster than blue ones.

On the other hand if $\lambda > \lambda_D$, $\beta_2 < 0$ and it is the negatively or anomalously dispersive regime. and $v_{\lambda+\Delta\lambda} < v_\lambda$ in this regime. Consequently, blue travels faster than the red wavelengths.

In fiber optics, one uses the dispersion parameter D instead of β_2. It is defined as

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \approx -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2} \tag{34.11}$$

and is commonly expressed in the units of ps/km.nm.

In addition to the wavelength dependence of β arising from the material chromatic dispersion, it also

depends upon the ratio of fiber core diameter and wavelength as well as on refractive index difference between core and cladding.

This dependence of mode propagation properties on fiber design parameters is termed as wave guide dispersion and must be added to the material chromatic dispersion to get the total chromatic dispersion. Its effect is small except near the zero dispersion wavelength and can be used to shift the dispersion wavelength of the fiber by appropriate choice of fiber design parameters. Wavelength dispersion of refractive index and group index is shown in figure 34.3a. Figure 34.3b shows the group velocity dispersion of the silica fiber.

Pulse propagation in optical fibers in linear regime:

Propagation in the fiber is governed by the wave equation

$$\nabla^2 E = \frac{n_0^2}{c^2}\frac{\partial^2 E}{\partial t^2}$$ (34.12)
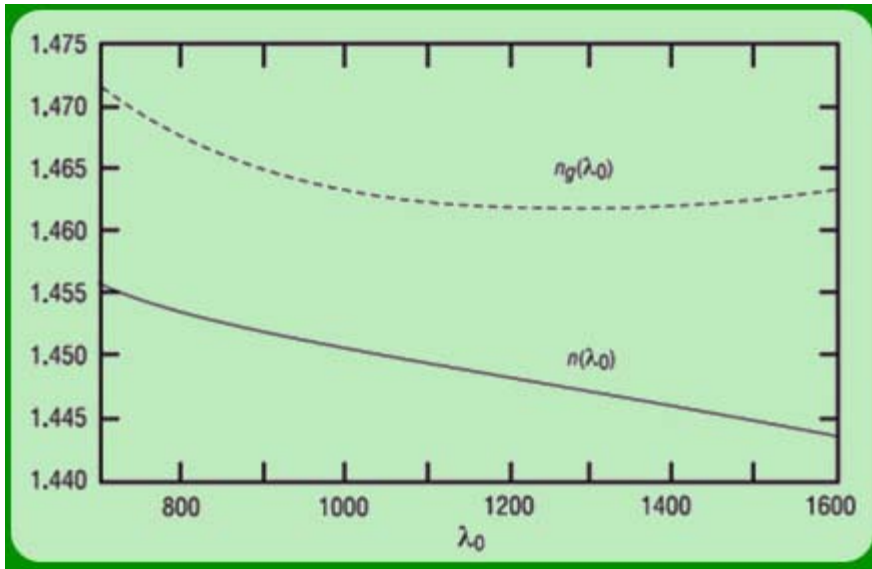

Figure 34.3a

For the monochromatic waves, it admits solutions of the form

$$E = F(x,y,\omega)\,e^{-i(\omega t - \beta z)}$$ (34.13)
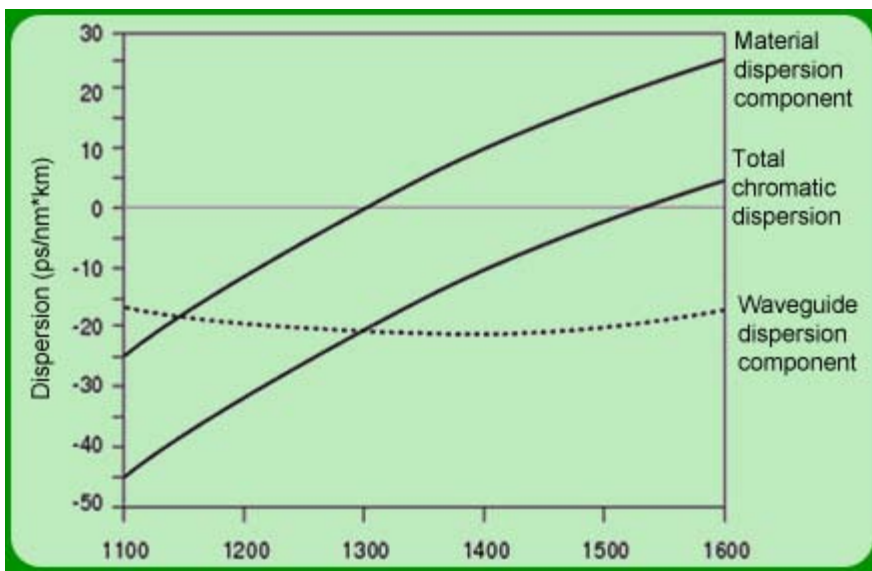

Figure 34.3b

Where $F(x,y)$, gives mode dependence on transverse dimensions. The mode distribution $F(x,y)$ is a solution of

$$E = F(x, y, \omega) e^{-i(\omega t - \beta z)} \qquad (34.14)$$

Where $\nabla_t^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is transverse Laplacian

A pulse is effectively a superposition of monochromatic waves of different frequencies Mathematically it can be described as

$$E = \int_{-\infty}^{\infty} \phi(\omega) F(x, y, \omega) e^{-i(\omega t - \beta z)} d\omega \qquad (34.15)$$

We will assume that the optical pulse has narrow spectral width compared to its central frequency, ω_0 i.e.

$$\frac{\Delta \omega}{\omega_0} \ll 1$$

so that we can ignore the weak frequency dependence of $F(x, y, \omega)$.

Thereby

$$E = F(x, y) \int_{-\infty}^{\infty} \Phi(\omega) e^{-i(\omega t - \beta z)} d\omega \qquad (34.16)$$

Alternatively, we can write the expression for the pulse as

$$E = F(x, y) u(z, t) \exp[-i(\omega_0 t - \beta_0 z)]$$

Where $\beta_0 = \beta(\omega_0)$ and

$$u(z, t) = \int_{-\infty}^{\infty} \phi(\omega) e^{-i((\omega - \omega_0)t - (\beta - \beta_0)z)} d\omega \qquad (34.17)$$

describes the pulse envelop.

Pulse envelop varies slowly in time as compared to the rapidly varying carrier wave as shown in figure 34.2. The former moves with group velocity v_g and the later with the phase velocity v_p.
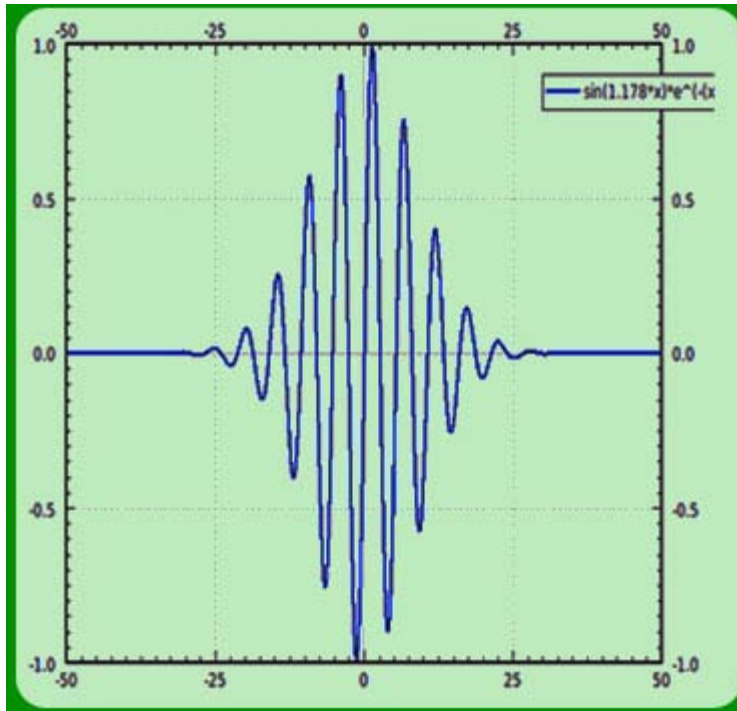

Figure 34.2

Propagation constant β is frequency dependent. Its frequency dependence leads to

- chromatic dispersion

- Wave guide dispersion

Frequency dependence of β near central frequency ω_0 can be expressed using Taylor series as

$$\beta = n(\omega)\frac{\omega}{c}$$

$$= \beta_0 + \dot{\beta}_0\Omega + \frac{1}{2}\ddot{\beta}_0\Omega^2 + \frac{1}{6}\dddot{\beta}_0\Omega^3 + \ldots \ldots \qquad (34.18)$$

Where

$$\Omega = \omega - \omega_0$$

$$\dot{\beta}_0 = \left(\frac{\partial\beta}{\partial\omega}\right)_{\omega=\omega_0} \quad \text{and} \quad \ddot{\beta}_0 = \left(\frac{\partial^2\beta}{\partial\omega^2}\right)_{\omega=\omega_0}$$

Hence

$$u(z,t) = \int_{-\infty}^{\infty}\phi(\Omega)e^{-i\Omega t}\exp[i(\dot{\beta}_0\Omega + \frac{1}{2}\ddot{\beta}_0\Omega)z]d\Omega \qquad (34.19)$$

Note: higher order dispersion terms have been neglected.

Recall that $\dot{\beta}_0 = \frac{1}{v_g}$

and $\ddot{\beta}_0$ is group velocity dispersion coefficient. By substitution it can be shown that the pulse envelop is solution of

$$\frac{\partial u}{\partial z} + \dot{\beta}_0\frac{\partial u}{\partial z} = \frac{i}{2}\ddot{\beta}_0\frac{\partial^2 u}{\partial t^2} \qquad (34.20)$$

This is the wave equation for the optical pulse envelop in a dispersion free medium. In the absence of dispersion.

$$\ddot{\beta}_0 = 0 \text{ and } \dddot{\beta}_0 = 0$$

The solution of the wave equation comprises of the general form

$$u = g\left(t - \dot{\beta}_0 z\right) \qquad (34.21)$$

Thus in the absence of dispersion, pulses of arbitrary shape travel without distortion through the fiber.

Proof:

Let us consider the optical pulse at the input end of medium

$$E(x,y,z=0,t) = \phi(x,y)f(t) \qquad (34.22)$$

$f(t)$ is related to its counterpart, F(w) in the frequency domain as

$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F(\omega)e^{-i\omega t}d\omega \qquad (34.23)$$

After traveling a distance z

$$E(x,y,z,t) = \phi(x,y)u(z,t)e^{-i(\omega_0 t - \beta_0 z)} \qquad (34.24)$$

Where

$$u(z,t) = \int_{0}^{\infty}\phi(\omega)e^{-i((\omega-\omega_0)t-(\beta-\beta_0)z)}d\omega \qquad (34.25)$$

Put $\Omega = \omega - \omega_0$ and $\beta = \beta_0 + \dot{\beta}_0\Omega$ --for a linear system

$$u(z,t) = \int_{-\infty}^{\infty}\phi(\omega)e^{-i[\Omega t - \dot{\beta}_0\Omega z]}d\omega$$

Or

$$u(z,t) = \int_{-\infty}^{\infty} \phi(\Omega) e^{-i[\Omega t - \dot{\beta_0}\Omega z]} d\Omega$$

$$\equiv g(t - \dot{\beta_0} z) \qquad (34.26)$$

Therefore

$$E(x,y,z,t) = \phi(x,y) g\left(t - \dot{\beta_0} z\right) e^{-i(\omega_0 t - \beta_0 z)} \qquad (34.27)$$

∴ (Hence proves)

Recap:

In this lecture we have discussed the following

1. Dispersion of refractive index and its effects on the pulse propagation.
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3. Propagation of a pulse in dispersion(higher order) free medium.