Graph Coloring problem Using Backtracking

- Presented by:-

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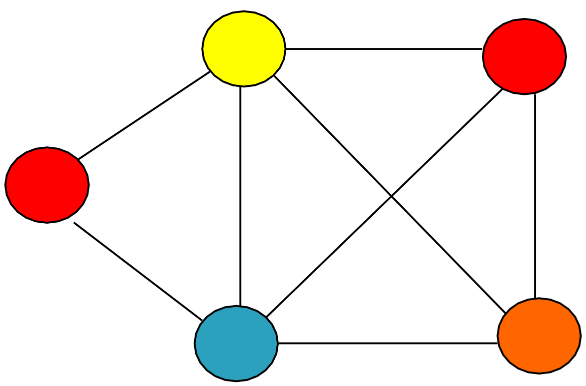
What is Coloring?

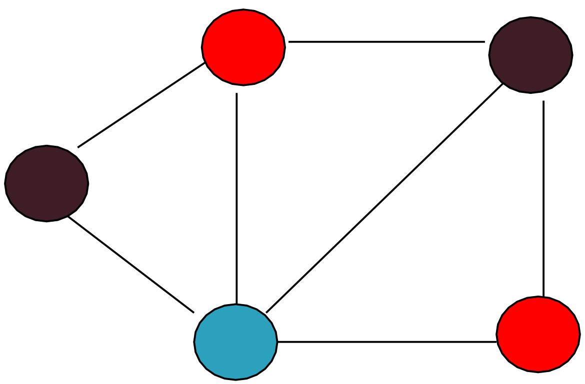
- Graph Coloring is an assignment of colors (or any distinct marks) to the vertices of a graph. Strictly speaking, a coloring is a proper coloring if no two adjacent vertices have the same color.

$$f : V(G) \rightarrow S$$



Coloring Planar graphs

- **Definition**: A graph is planar if it can be drawn in a plane without edge-crossings.



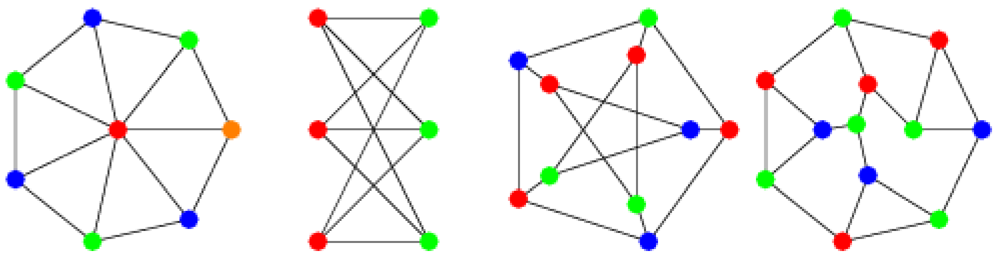
- **The four color theorem:** For every planar graph, the chromatic number is ≤ 4.

The Four-Color Theorem

- The four color theorem states that any planar map can be colored with at most four colors.
- In graph terminology, this means that using at most four colors, any planar graph can have its nodes colored such that no two adjacent nodes have the same color.
- Four-color conjecture – Francis Guthrie, 1852 (F.G.)
- Many incomplete proofs (Kempe).
- 5-color theorem proved in 1890 (Heawood)
- 4-color theorem finally proved in 1977 (Appel, Haken)
  - First major computer-based proof

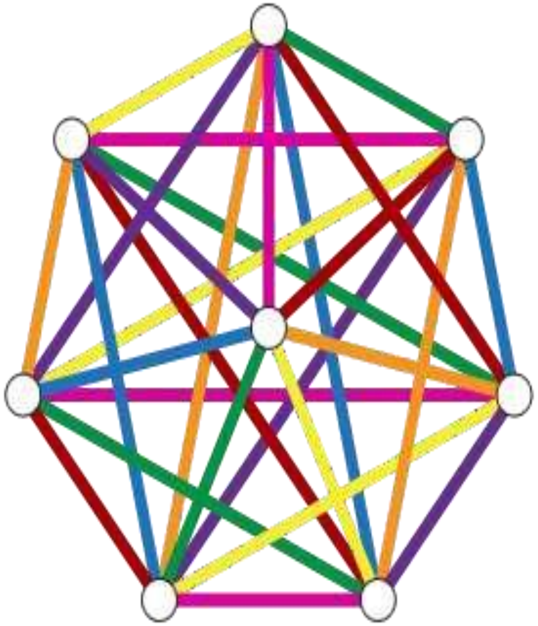
Vertex Coloring

- A **vertex coloring** is an assignment of labels or **colors** to each **vertex** of a **graph** such that no edge connects two identically colored **vertices**



Edge Coloring

- Similar to vertex coloring, except edges are color.
- Adjacent edges have different colors.



Edge Coloring

- Every edge-coloring problem can be transformed into a vertex-coloring problem
- Coloring the edges of graph G is the same as coloring the vertices in L(G)
- Not every vertex-coloring problem can be transformed to an edge-coloring problem
- Every graph has a line graph, but not every graph is a line graph of some other graph

Properties

- K-Coloring
  - A k-coloring of a graph G is a mapping of V(G) onto the integers 1..k such that adjacent vertices map into different integers.
  - A k-coloring partitions V(G) into k disjoint subsets such that vertices from different subsets have different colors.

Terminology

- K-colorable
  - A graph G is k-colorable if it has a k-coloring.

- Chromatic Number
  - The smallest integer k for which G is k-colorable is called the chromatic number of G, is denoted by the $\chi(G)$.

Backtracking Algorithm

- The idea is to assign colors one by one to different vertices, starting from the vertex 0. Before assigning a color, we check for safety by considering already assigned colors to the adjacent vertices.

- If we find a color assignment which is safe, we mark the color assignment as part of solution.

- If we do not a find color due to clashes then we backtrack and return false.

NP hard problem

NP: the class of decision problems that are solvable in polynomial time on a *nondeterministic* machine (or with a nondeterministic algorithm)

- (A *deterministic* computer is what we know)
- A *nondeterministic* computer is one that can "guess" the right answer or solution think of a nondeterministic computer as a parallel machine that can freely spawn ***an infinite number*** of processes
- Thus *NP* can also be thought of as the class of problems whose solutions can be verified in polynomial time
- Note that *NP* stands for "Nondeterministic Polynomial-time"

Examples Of NP hard problem

- Fractional Knapsack
- Sorting
- Others?
  - Graph Coloring
  - Satisfiability (SAT)
    - the problem of deciding whether a given Boolean formula is satisfiable.

Applications of Graph Coloring

- Many problems can be formulated as a graph coloring problem including Time Tabling, Scheduling, Register Allocation, Channel Assignment.
- A lot of research has been done in this area so much is already known about the problem space.

Thank You