Chapter 3 - Waves & optics

The propagation of light and geometric optics

Ques. - Define Fermat's principle and use it to deduce the laws of reflection and refraction.

Ans. - Fermat's Principle (1658) - The French mathematician Fermat gave the principle of least time in the form, " A ray of light in passing from one point to another through a set of media by any number of reflections or refractions choses a path along which the time taken is the least or minimum."

However, it has been found that there are a number of cases where the optical path (Vel. of light x time) is maximum or else neither a maximum nor a minimum but stationary. Hence, in general form Fermat's principle may be stated as-

" A ray of light in passing from one point to another through a set of media by any number of reflections or refractions choses a path along which the optical path is either a minimum or a maximum or a stationary."

Soon, it was realised that time is decisive factor rather path. Hence, the Fermat's principle may be stated as-

Between any two points the time taken by the light along actual path is extremum: it may be a minimum or a maximum.

Explanation of Fermat's Principle → suppose, A and B are the terminals of any optical path (fig. 1). Path ACB is actual path and path ADB is any neighbouring path. Time difference between two paths is δt is very small in comparison of displacement of path. δt vanishes along actual path ACB. Therefore, according to condition of maxima and minima t is extremum (maximum or minimum) i.e. time taken by light along the actual path is extremum.

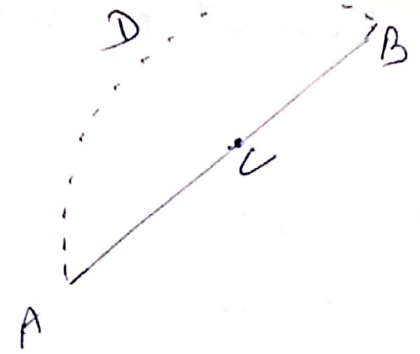

Fig. 1.

$$\int_A^B \frac{ds}{v} = \text{max. or min. or stationary} \quad \text{——} (1) \qquad \left(\text{time} = \frac{\text{distance}}{\text{Speed}}\right) \quad \delta t = \frac{ds}{v}$$

or $$\int_A^B \frac{\mu\, ds}{c} = \text{max. or min. or stationary} \quad \text{——} (2) \qquad \left(\because \mu = \frac{c}{v}\right)$$

We know that speed of light in vacuum is constant.

$\therefore$ Eqn (2) can be written as in the form or Fermat's principle of extremum path as.

$$\int_A^B \mu\, ds = \text{max. or min. or stationary}$$

i.e. $$\boxed{\delta \int_A^B \mu\, ds = 0} \quad \text{——} (3)$$

With the help of Fermat's principle we can drive following laws of geometrical optics.

(i) Rectilinear propagation of light

(ii) Laws of Reflection

(iii) Laws of Refraction.

Deduction of Laws of Reflection -

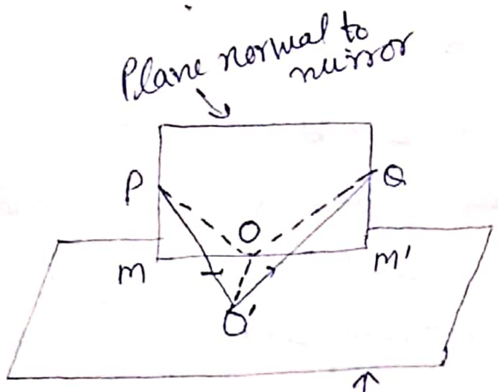

Fig. 2.

(i) First Law - Let a light ray from P, after reflection at point O' of the plane mirror passes through point Q. Draw a plane through P and Q normal to the plane mirror and draw OO' perpendicular on this plane from the point O'.

Clearly, $PO' > PO$ and $QO' > QO$

Consequently the time taken by the light ray along path $PO'Q$ will be greater than that along the path POQ which is contrary to Fermat's principle. Hence, the points O and O' must coincide. Thus, the incident ray, the reflected ray and normal to mirror at the point of incidence lie in one plane. This is first law.

(ii) Second Law - Let us consider,

$MM' = d$, A plane mirror

PO = incident ray

i = Angle of incidence

OQ = reflected ray

r = Angle of reflection

$PM = a$, ⊥ar from P

$QM' = b$ ⊥ar from Q

$MO = x$

$M'O = d - x$

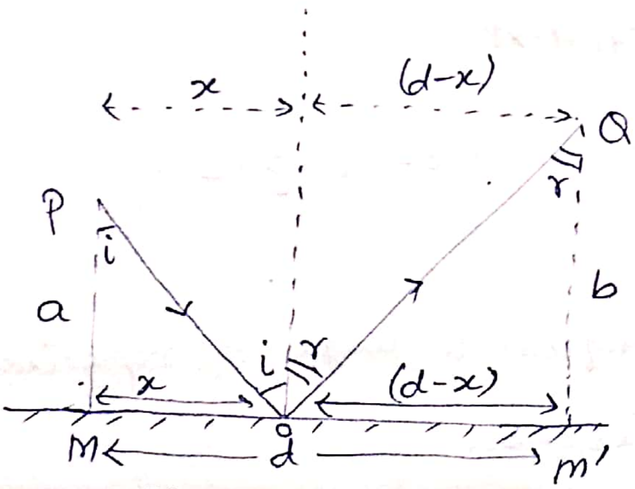

Fig. - 3

As the light traverses the entire path with the air the length of optical path between P and Q

$$l = POQ = PO + OQ$$

$$l = \sqrt{a^2+x^2} + \sqrt{b^2+(d-x)^2} \quad \text{—— (1)}$$

For different paths between P and Q, the position of O and hence x will change. But according to Fermat's principle the length of optical path is minimum or maximum or stationary.

Therefore we may write.

$$\frac{dl}{dx} = 0 \quad \text{—— (2)}$$

differentiating eqn. (1) with respect to x we get

$$\frac{dl}{dx} = \frac{1}{2}(a^2+x^2)^{-\frac{1}{2}}(2x) + \frac{1}{2}[b^2+(d-x)^2]^{-\frac{1}{2}}\{-2(d-x)\}$$

$$\frac{dl}{dx} = \frac{x}{\sqrt{a^2+x^2}} - \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

But, according to Fermat's principle

$$\frac{dl}{dx} = 0$$

$$\therefore \frac{x}{\sqrt{a^2+x^2}} - \frac{(d-x)}{\sqrt{b^2+(d-x)^2}} = 0$$

$$\text{or,} \quad \frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{b^2+(d-x)^2}} \quad \text{—— (3)}$$

$$\text{or} \quad \sin i = \sin r \quad \text{(from fig. 3)}$$

$$\text{or} \quad \boxed{\angle i = \angle r}$$

i.e., angle of incidence is equal to angle of reflection. This is second law of reflection.

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Deduction of Laws of Refraction (Snell's Law) -

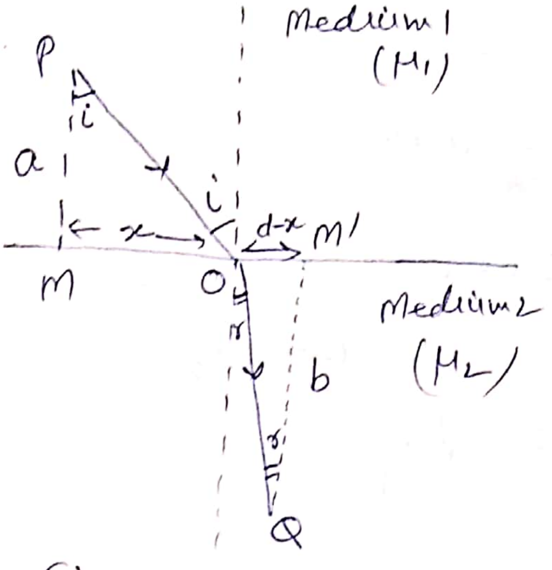

Fig. 4

Let us consider, MM' to be a refracting surface. O is point of incidence.

$MM' = d$,

PO = incident ray

OQ = refracted ray

i = Angle of incidence

r = Angle of refraction

$PM = a$, $\perp$ar from P to MM'

$QM' = b$, $\perp$ar from Q to MM'

μ_1 = refractive index of medium 1

μ_2 = refractive index of medium 2

Incident ray, refracted ray and normal at point of refraction are in same plane. This is first law.

Now:- optical path between P and Q

$$l = \mu_1 PO + \mu_2 OQ$$

$$l = \mu_1\left(\sqrt{a^2+x^2}\right) + \mu_2\left(\sqrt{b^2+(d-x)^2}\right) \quad \text{—— (1)}$$

Now, for different paths between P and Q, the position of O and hence x will change. But, according to Fermat's principle the length of optical path is minimum or maximum or stationary. Therefore we may write

$$\frac{dl}{dx} = 0 \quad \text{—— (2)}$$

differentiating eqn. (1) with respect to x we get -

$$\frac{dl}{dx} = \mu_1 \cdot \frac{1}{2} \cdot 2x \cdot (a^2+x^2)^{-\frac{1}{2}} + \mu_2 \frac{1}{2}\left[b^2+(d-x)^2\right]^{-\frac{1}{2}} \cdot \{-2(d-x)\}$$

$$\frac{dl}{dx} = \frac{\mu_1 x}{\sqrt{a^2+x^2}} - \frac{\mu_2 (d-x)}{\sqrt{b^2+(d-x)^2}}$$

But, according to Fermat's principle

$$\frac{dl}{dx} = 0$$

$$\therefore \mu_1 \frac{x}{\sqrt{a^2+x^2}} = \frac{\mu_2 (d-x)}{\sqrt{b^2+(d-x)^2}}$$

or. $\mu_1 \sin i = \mu_2 \sin r$ ——— (3)

(from Fig. 4)

or, $\boxed{\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2}$

i.e. the ratio of ~~sine~~ angle of sin of incidence and angle of sin of refraction is a constant and is equal to refractive index of second medium with respect to first. This is Snell's law of refraction.

Ques.- Give the electromagnetic theory of light.

Ans. **Electromagnetic Theory of Light →** Electromagnetic theory of light is an extension of wave theory of light. According to Faraday's law of electromagnetic induction a time varying magnetic field behaves as a source of electric field and according to Maxwell's modification of Ampere's law a changing electric field gives rise to a magnetic field. Hence, due to change in electric field, magnetic field is generated and due to change in magnetic field, electric field is generated. Therefore due to change in electric field and magnetic field an electromagnetic disturbance of varying electric field and magnetic field is generated in a medium. Thus electromagnetic waves have electric field & magnetic field varying with time. The electric field and magnetic field are perpendicular to each other and both are perpendicular to the direction of propagation of wave. These wave travel with the speed of 3×10^8 m/s in free space. X-rays, γ-rays, visible light, infrared waves, ultra violet rays, short waves, radio waves etc behave as electromagnetic wave.

Maxwell developed four field equations of electrostatics and magnetostatics. These equations form the basis of electromagnetic theory of light.

In ~~free space~~ general Maxwell's eqns. are written as-

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \text{or} \quad \text{div}\,\vec{E} = \frac{\rho}{\epsilon_0} \qquad \text{——} (1)$$ (Gauss' law in electrostatics)

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{or} \quad \text{div}\,\vec{B} = 0 \qquad \text{——} (2)$$ (no name)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{or} \quad \text{curl}\,\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{——} (3)$$ (Faraday's Law of EM induction)

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) \quad \text{or} \quad \text{curl}\,\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) \qquad \text{——} (4)$$ (Ampere's circuital law)

where,
- E = Electric field intensity
- ρ = charge density
- B = magnetic induction
- J = current Density
- D = Electric displacement vector
- ϵ_0 = Permittivity of free space
- μ_0 = Permeability of free space

These equations can't be verified directly, although their application to any situation can be verified.

Ques – Derive equation for plane electromagnetic wave in free space and show that light is also an electromagnetic wave.

Sol. - Maxwell's Electromagnetic equations are

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \text{---} \ (1)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{---} \ (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---} \ (3)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \quad \text{---} \ (4)$$

In free space, there is no charge

$\therefore q = 0$

Hence $\rho = 0$, $\vec{J} = 0$

Therefore maxwell's eqns. in free space can be written as :-

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{---} \ (5)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{---} \ (6)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---} \ (7)$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{---} \ (8) \qquad \vec{D} = \epsilon_0\vec{E}$$

Taking curl of eqn. (7) we get

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\frac{\partial \vec{B}}{\partial t} \qquad (\because \text{curl curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2\vec{A})$$

$$\text{grad div } \vec{E} - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$0 - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial E}{\partial t}\right) \qquad [\text{Using eqn (5) \& (8)}]$$

$$\text{or. } \nabla^2 E = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\text{or } \boxed{\nabla^2 E - \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2} = 0} \quad \text{---} \ (9)$$

Now, taking curl of eqn (8), we get,

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\text{grad div }\vec{B} - \nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times E)$$

$$0 - \nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right) \qquad \text{[Using eqn (6) \& (7)]}$$

$$-\nabla^2\vec{B} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

or, $$\boxed{\nabla^2\vec{B} - \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0} \qquad (10)$$

eqns (9) & (10) are plane wave equations governing electromagnetic fields $\vec{E}$ & $\vec{H}$ in free space.

Eqns (9) and (10) are of the form of general wave equation which is given as

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

or, $$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \qquad (11)$$

where, v is the velocity of wave.

$\therefore$ Comparing eqns. (9), (10) & (11) we see that field vectors $\vec{E}$ and $\vec{H}$ propagate in free space with a velocity

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}}} \qquad (12)$$

Now, $$\mu_0 = 4\pi\times10^{-7}\ N/A^2$$

$$\epsilon_0 = \frac{1}{4\pi}\times\frac{1}{9\times10^9}\ F/m$$

$$\therefore v = \frac{1}{\sqrt{4\pi\times10^{-7}\times\frac{1}{4\pi}\times\frac{1}{9\times10^9}}} = \frac{3}{\sqrt{10^{-16}}} = \frac{3}{10^{-8}}$$

$$\boxed{v = 3\times10^8\ m/s} \qquad (13)$$

Hence, electromagnetic waves in free space propagate with a speed of 3×10^8 m/s. Also, from experiments we know that speed of light in free space is 3×10^8 m/s. Hence, light is also electromagnetic wave.

Ques. - Write Boundary conditions at the surface of discontinuity.

Sol. - To study reflection and refraction of electro-magnetic waves at the boundary of two media we should know certain boundary conditions. These boundary conditions should be satisfied by definite relationship between $\vec{D}$, $\vec{B}$, $\vec{E}$ and $\vec{H}$.

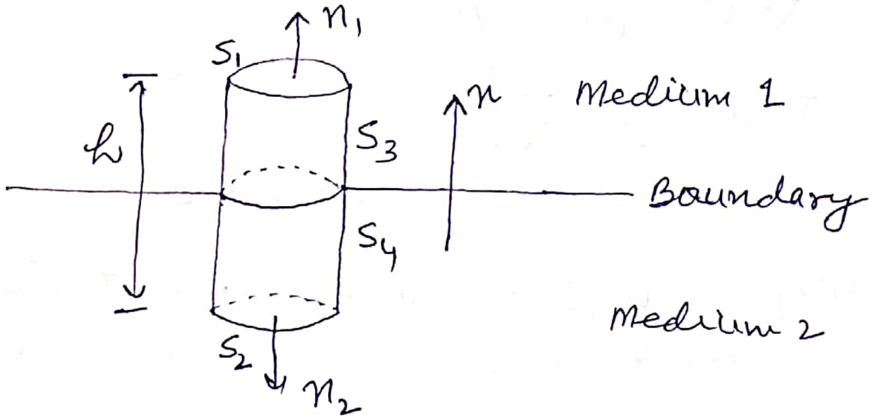

Fig - 1

① Boundary condition for electric displacement vector $\vec{D}$ → Let's construct a pillbox, like surface S composed of S_1, S_2, S_3 & S_4 as shown in fig 1. A is area of cross section. n_1, n_2 & n represent unit vectors normal to concerned surface. At the boundary we get for electric displacement vector,

$$\vec{D}_1 \cdot \hat{n} - \vec{D}_2 \cdot \hat{n} = \sigma$$

i.e, $\boxed{D_{1n} - D_{2n} = \sigma}$ ——— (1)

The boundary condition is, the normal component of electric displacement is not continuous at the ~~surface~~ interface but changes by an amount equal to the free surface charge density of charge at the interface.

(2) Boundary condition for magnetic Induction $\vec{B}$ —

At the bounary for magnetic induction $\vec{B}$, we have,

$$\vec{B}_1 \cdot \hat{n} - \vec{B}_2 \cdot \hat{n} = 0$$

i.e., $\boxed{B_{1n} = B_{2n}}$

the boundary condition is, the normal component of magnetic induction ($\vec{B}$) is continuous across the interface.

(3) Boundary condition for electric intensity vector $\vec{E}$ →

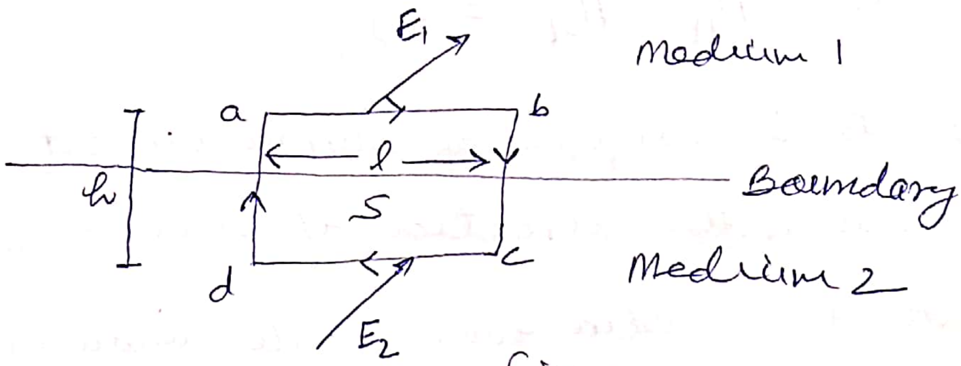

Fig-2

at the interface between two media, we construct a rectangular loop abcd bounding a surface S (fig.2).

At the boundary for electric intensity vector $\vec{E}$ we have,

$$E_{1t}\, l - E_{2t}\, l = 0$$

i.e. $\boxed{E_{1t} = E_{2t}}$

where, E_{1t} and E_{2t} are the tangential components of the electric field in the two media.

Boundary condition is, tangential component of electric field vector $\vec{E}$ must be continuous across the interface.

(ii) <u>Boundary condition for magnetic field intensity $\vec{H}$</u>

Boundary condition is

(a) If either of the two media has infinite conductivity, the tangential component of magnetic field intensities is not continuous at the surface, but changes by an amount equal to the components of surface current density perpendicular to tangential component of $\vec{H}$.

i.e. $$H_{1t}\, l - H_{2t}\, l = J_{s\perp}\, l$$

or $$\boxed{H_{1t} - H_{2t} = J_{s\perp}}$$

where,

$J_{s\perp}$ represents the component of surface current density perpendicular to the direction of $\vec{H}$ component.

(b) If both the media have finite conductivity the tangential component of magnetic field intensity is continuous across the interface.

i.e. $$H_{1t} - H_{2t} = 0$$

or $$\boxed{H_{1t} = H_{2t}}$$

Reflection and Refraction of Electromagnetic waves at the interface of Non-conducting media →

The reflection and refraction of light at plane surface between two media of different dielectric properties are familiar phenomena and may be divided in two categories –

① **Kinematic properties** –

ⓐ **Law of Reflection** –

$$\theta_i = \theta_r'$$

angle of incidence is equal to angle of reflection.

θ_i = angle of incidence

θ_r' = angle of reflection

ⓑ **Snell's Law of refraction** –

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_2}{n_1} = {}_1\mu_2$$

θ_i = angle of incidence

θ_r = angle of refraction

ⓒ **Law of Frequency** – The incident, reflected and refracted waves all have same frequency.

ⓓ The incident, reflected and refracted waves all lie in same plane but normal to boundary surface.

② **Dynamic Properties** → These properties are concerned with –

(a) intensity of reflected wave & refracted wave.

ⓑ phase changes and polarization.

The kinematic properties do not depend upon the nature of waves (transverse or longitudinal) or boundary condition. But, the dynamic properties depend entirely on the specific nature of electromagnetic field and their boundary conditions.

Fresnel's equations deal with the dynamic properties of Reflection and Refraction.

<u>Ques</u>→ Establish Fresnel's equations of Reflection and Refraction.

OR

Discuss dynamic properties of Reflection & Refraction.

<u>Sol.</u> — The equations relating the amplitudes of the reflected and transmitted waves with that of incident wave are known as Fresnel's equations or Fresnel's Formulae (1820)

We know that field vectors $\vec{E}$ and $\vec{B}$ in a plane electromagnetic wave are mutually perpendicular, also they are perpendicular to direction of propagation of wave.

Now, let's consider two extreme cases-

(i) in which incident wave is polarised such that the electric field vector $\vec{E}$ is normal to the plane of incidence.

(ii) in which incident wave is polarised such that the electric field vector $\vec{E}$ is parallel to the plane of incidence.

The general result may be obtained by a suitable linear combination of these two extreme results.

Case I - When $\vec{E}$-vector is perpendicular to the plane of incidence -

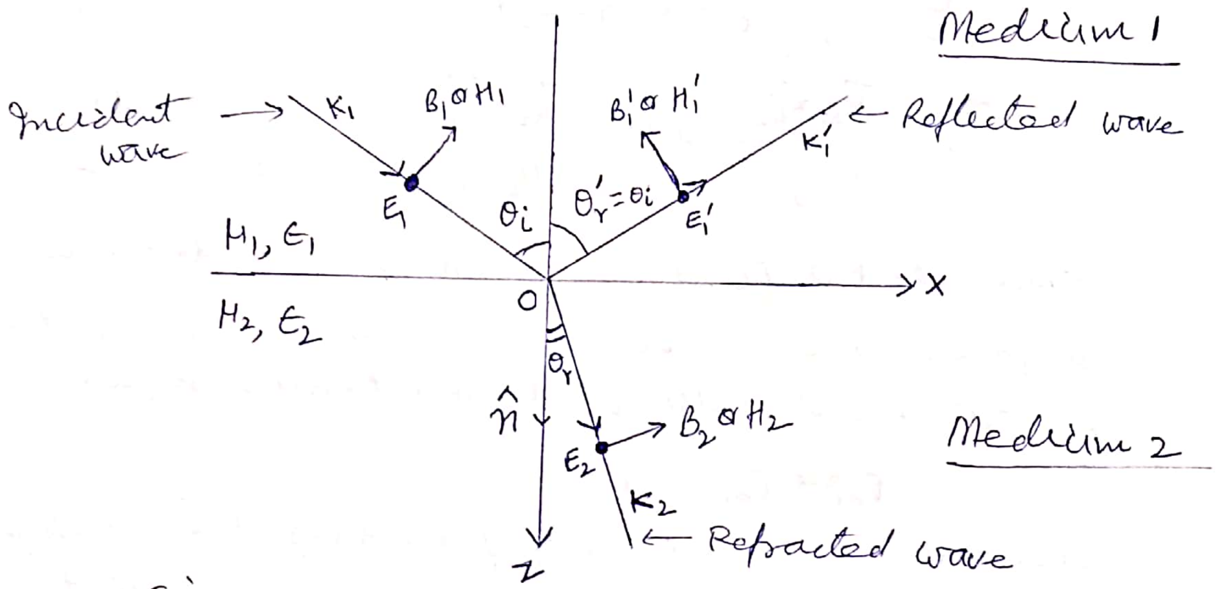

Fig.1 - Reflection and refraction with polarization perpendicular ~~parallel~~ to the plane of incidence.

The electric field vector $\vec{E}_1$ and magnetic field vector $\vec{B}$ of incident wave are perpendicular to direction of propagation $\vec{K}_1$, as shown in fig. 1. $\vec{E}_1$ is perpendicular to plane of incidence. Electric field vectors of reflected wave ($\vec{E}_1'$) and refracted wave ($\vec{E}_2$) are also perpendicular to plane of incidence. The field vectors of incident, reflected, and refracted waves ~~arg~~ are given as -

For incident wave -

$$\left.\begin{aligned} \vec{E}_1 &= \vec{E}_{01}\, e^{i(\vec{K}_1 \cdot \vec{r} - \omega_1 t)} \\ \vec{B}_1 &= \frac{\vec{k}_1 \times \vec{E}_1}{\omega_1} \text{ or } \vec{H}_1 = \frac{\vec{K}_1 \times \vec{E}_1}{\mu_1 \omega_1} \end{aligned}\right\} \quad \text{'1' stands for medium 1.} \qquad (1)$$

For reflected wave.-

$$\left.\begin{aligned}\vec{E}_1' &= \vec{E}_{01}'\, e^{i(\vec{k}_1'\cdot\vec{r}-\omega_1' t)}\\ \vec{B}_1' &= \frac{\vec{k}_1'\times\vec{E}_1'}{\omega_1'} \quad \text{or} \quad \vec{H}_1' = \frac{\vec{k}_1'\times\vec{E}_1'}{\mu_1\omega_1'}\end{aligned}\right\} \text{——— (2)}$$

For refracted wave -

$$\left.\begin{aligned}\vec{E}_2 &= \vec{E}_{02}\, e^{i(k_2\cdot r-\omega_2 t)}\\ \vec{B}_2 &= \frac{\vec{k}_2\times\vec{E}_2}{\omega_2} \quad \text{or} \quad \vec{H}_2 = \frac{\vec{k}_2\times\vec{E}_2}{\mu_2\omega_2}\end{aligned}\right\} \text{——— (3)}$$

'2' stands for medium 2

Since, electric field vectors are parallel to the boundary surface, the continuity of the tangential component of electric field vector $\vec{E}$ at the interface requires that-

$$\vec{E}_{01} + \vec{E}_{01}' = \vec{E}_{02} \quad \text{——— (4)}$$

Similarly, the continuity of the tangential component of the magnetic field at the interface requires that,

$$\vec{H}_{1t} + \vec{H}_{1t}' = \vec{H}_{2t}$$

i.e., $-H_{01}\cos\theta_i + H_{01}'\cos\theta_r' = -H_{02}\cos\theta_r$

$$\Rightarrow \quad (H_{01} - H_{01}')\cos\theta_i = H_{02}\cos\theta_r \quad \text{——— (5)} \quad (\because \theta_r' = \theta_i)$$

But, $\vec{B}_1 = \dfrac{\vec{k}_1\times\vec{E}_1}{\omega_1}$ $\quad\therefore \vec{H}_1 = \dfrac{\vec{k}_1\times\vec{E}_1}{\mu_1\omega_1}$ $\quad (\because B_1 = \mu_1 H_1)$

or, $\vec{H}_1 = \dfrac{k_1\,\hat{n}_1\times\vec{E}_1}{\mu_1\omega_1}$ where, $\hat{n}_1$ is unit vector along $\vec{k}_1$.

$$= \frac{\omega_1\sqrt{\mu_1\epsilon_1}\,\hat{n}_1\times\vec{E}_1}{\mu_1\omega_1} \quad \left(\text{Since, } k_1 = \frac{\omega_1}{v_1} = \omega_1\sqrt{\mu_1\epsilon_0}\right)$$

$$\vec{H}_1 = \sqrt{\frac{\epsilon_1}{\mu_1}}\,\hat{n}_1\times\vec{E}_1 \quad \text{——— (6)}$$

Eqn (6) gives -

$$\left.\begin{aligned} H_{01} &= \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{01} \\ H'_{01} &= \sqrt{\frac{\epsilon_1}{\mu_1}}\, E'_{01} \\ H_{02} &= \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{02} \end{aligned}\right\} \quad \text{——— (7)}$$

Using these results, from eqn (5) we get -

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\,(E_{01} - E'_{01}) \cos\theta_i = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{02} \cos\theta_r \quad \text{— (8)}$$

Elimination E_{02} from eqns. (4) and (8) we get

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\,(E_{01} - E'_{01}) \cos\theta_i = \sqrt{\frac{\epsilon_2}{\mu_2}}\,(E_{01} + E'_{01}) \cos\theta_r$$

On further simplification we get the ratio of amplitudes of electric field vector in reflected wave and incident wave i.e. coefficient of Reflectance. It is denoted by $R_\perp$.

$$\therefore \boxed{R_\perp = \left(\frac{E'_{01}}{E_{01}}\right)_\perp = \frac{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_r}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_r}} \quad \text{——— (9)}$$

where, symbol $\perp$ denotes that in the case under consideration electric field is perpendicular to plane of incidence.

Similarly, eliminating E'_{01} from eqns (4) and (8) we get,

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\,\{E_{01} - (E_{02} - E_{01})\} \cos\theta_i = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{02} \cos\theta_r$$

or, $$\sqrt{\frac{\epsilon_1}{\mu_1}}\,(2E_{01} - E_{02}) \cos\theta_i = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{02} \cos\theta_r$$

On further simplification we get the ratio of amplitudes of electric field vector in refracted wave

and incident wave i.e. coefficient of transmittance.
It is denoted by $T_\perp$.

$$\boxed{T_\perp = \left(\frac{E_{02}}{E_{01}}\right)_\perp = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_r}} \quad \text{—— (10)}$$

Eqns. (9) & (10) are known as Fresnel's equations.

For a non conducting medium we have,

$$\mu_1 = \mu_2 = \mu_0$$

so, that, $n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$ and $n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}$

$$\therefore \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad \text{—— (11)}$$

$n_1 \rightarrow$ refractive index of medium 1
$n_2 \rightarrow$ refractive index of medium 2

in this case, Reflectance is.

$$\boxed{R_\perp = \left(\frac{E'_{01}}{E_{01}}\right)_\perp = \frac{\cos\theta_i - \frac{n_2}{n_1}\cos\theta_r}{\cos\theta_i + \frac{n_2}{n_1}\cos\theta_r} = -\frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)}} \quad \text{—— (12)}$$

~~(for $n_2 < n_1$)~~ (for $n_2 > n_1$)

and transmittance is given as -

$$\boxed{T_\perp = \left(\frac{E_{02}}{E_{01}}\right)_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\frac{n_2}{n_1}}\cos\theta_r} = \frac{2\cos\theta_i \sin\theta_r}{\sin(\theta_i - \theta_r)}} \quad \text{—— (13)}$$

Eqns. (12) & (13) represent Fresnel's equations for non conducting media when electric field vector $\vec{E}$ is perpendicular to the plane of incidence.

Deduction from Fresnel's Equations $\rightarrow$

(a) when medium '1' is rarer and medium '2' is denser i.e, $n_1 < n_2$ then from snell's law we get $\theta_i > \theta_r$

i.e., refracted ray deviates towards the normal.
From eqn. (12) $(E'_{01}/E_{01})_{\perp}$ is negative hence, reflected ray and incident ray are in opposite phase i.e. on reflection from the denser medium at the interface the reflected ray suffers a phase change of π.

(b) when medium '1' is denser and medium '2' is rarer, i.e., $n_1 > n_2$, then from snell's law we get

$$\theta_i < \theta_r$$

i.e., refracted ray deviates away from the normal and in this case from eqn (12) $(E'_{01}/E_{01})_{\perp}$ is positive hence reflected ray and incident ray are in same phase.

(c) in both the cases, i.e. either $n_1 < n_2$ or $n_1 > n_2$ eqn. (13) is always positive, hence, incident ray and refracted ray do't suffer any phase change i.e. they are in same phase.

Case II - When $\vec{E}$ vector is parallel to plane of incidence -

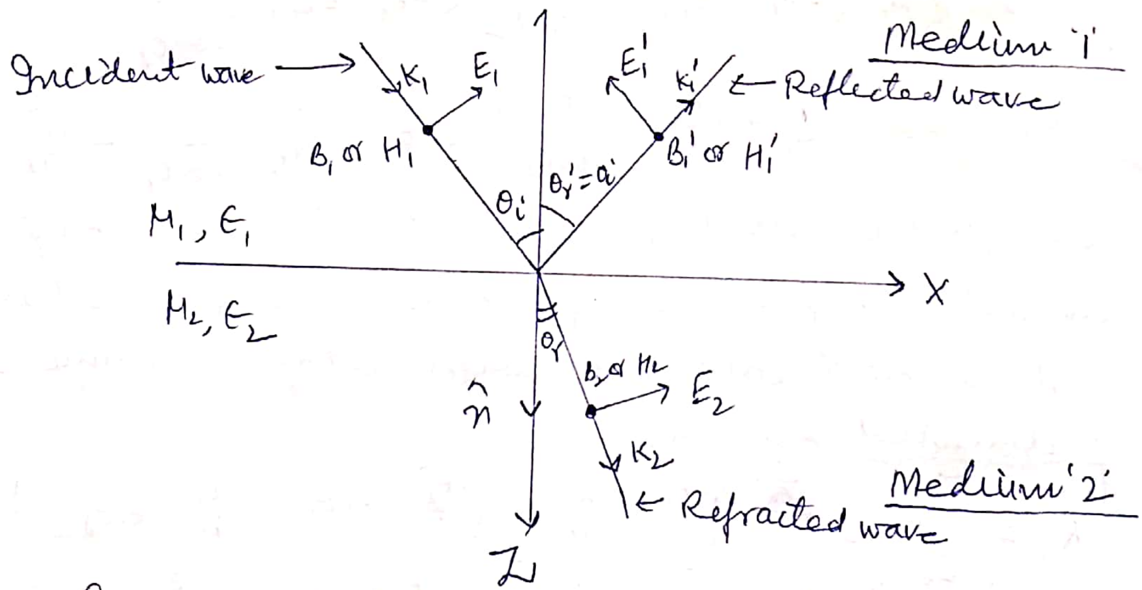

Fig 2: Reflection and refraction with polarization parallel to plane of incidence.

In this case electric field vectors of all the three waves are parallel to plane of incidence. Magnetic field vectors $\vec{H}$ are parallel to boundary surface, hence the continuity of tangential component of magnetic field vectors $\vec{H}$ at the interface requires that,

$$\vec{H}_{01} + \vec{H}'_{01} = \vec{H}_{02} \quad \text{——} (14)$$

Similarly, the continuity of tangential component of electric field vectors $\vec{E}$ require that.

$$\vec{E}_{01} \cos\theta_i - \vec{E}'_{01} \cos\theta'_r = \vec{E}_{02} \cos\theta_r$$

$$\text{or} \quad (\vec{E}_{01} - \vec{E}'_{01}) \cos\theta_i = \vec{E}_{02} \cos\theta_r \quad (\because \theta'_r = \theta_i) \quad \text{——} (15)$$

Now using eqn (7), eqn (14) gives.

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\, \vec{E}_{01} + \sqrt{\frac{\epsilon_1}{\mu_1}}\, \vec{E}'_{01} = \sqrt{\frac{\epsilon_2}{\mu_2}}\, \vec{E}_{02} \quad \text{——} (16)$$

substituting the value of $\vec{E}_{02}$ from eqn (15) in eqn (16) we get.

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\, \vec{E}_{01} + \sqrt{\frac{\epsilon_1}{\mu_1}}\, E'_{01} = \sqrt{\frac{\epsilon_2}{\mu_2}} \left(\frac{\vec{E}_{01} - \vec{E}'_{01}}{\vec{E}_{02} \cos\theta_r} \right) \cos\theta_i$$

$$\text{or,} \quad \sqrt{\frac{\epsilon_1}{\mu_1}} (\vec{E}_{01} + \vec{E}'_{01}) = \sqrt{\frac{\epsilon_2}{\mu_2}} (\vec{E}_{01} - \vec{E}'_{01}) \frac{\cos\theta_i}{\cos\theta_r}$$

$$\text{or,} \quad \vec{E}'_{01} \left(\sqrt{\frac{\epsilon_1}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\mu_2}} \frac{\cos\theta_i}{\cos\theta_r} \right) = \vec{E}_{01} \left[\sqrt{\frac{\epsilon_2}{\mu_2}} \frac{\cos\theta_i}{\cos\theta_r} - \sqrt{\frac{\epsilon_1}{\mu_1}} \right)$$

on further simplification we get the ratio of amplitudes of electric field vector in reflected wave and incident wave i.e. coefficient of reflectance. It is denoted by $R_{\parallel}$.

$$\boxed{R_{\parallel} = \left(\frac{\vec{E}'_{01}}{\vec{E}_{01}} \right)_{\parallel} = \frac{\sqrt{\frac{\epsilon_2}{\mu_2}} \frac{\cos\theta_i}{\cos\theta_r} - \sqrt{\frac{\epsilon_1}{\mu_1}}}{\sqrt{\frac{\epsilon_1}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\mu_2}} \frac{\cos\theta_i}{\cos\theta_r}} = \frac{\sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_i - \sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta_r}{\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\theta_r + \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_i}} \quad \text{——} (17)$$

where, symbol '∥' (parallel) denotes that in the case under consideration the electric field vector is parallel to plane of incidence.

Now substituting the value of $\vec{E}'_{01}$ from eqn (15) in eqn (16) we get,

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\,\vec{E}_{01} + \sqrt{\frac{\epsilon_1}{\mu_1}}\left(\vec{E}_{01} - \vec{E}_{02}\,\frac{\cos\theta_r}{\cos\theta_i}\right) = \sqrt{\frac{\epsilon_2}{\mu_2}}\,\vec{E}_{02}$$

or, $$\vec{E}_{02}\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\,\frac{\cos\theta_r}{\cos\theta_i} + \sqrt{\frac{\epsilon_2}{\mu_2}}\right) = 2\vec{E}_{01}\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\right)$$

On further simplification we get the ratio of electric field vector in refracted wave and incident wave. i.e. coefficient of transmittance. It is denoted by $T_{\parallel}$.

$$\boxed{T_{\parallel} = \left(\frac{\vec{E}_{02}}{\vec{E}_{01}}\right)_{\parallel} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}}{\sqrt{\frac{\epsilon_1}{\mu_1}}\,\frac{\cos\theta_r}{\cos\theta_i} + \sqrt{\frac{\epsilon_2}{\mu_2}}} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_r + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_i}} \qquad (18)$$

Equations (17) and (18) represent Fresnel's equations for the case when electric field vector ($\vec{E}$) is parallel to plane of incidence.

For, non conducting media.

$$\mu_1 = \mu_2 = \mu_0 \text{ so}$$

$$n_1 = \sqrt{\frac{\mu_1\epsilon_1}{\mu_0\epsilon_0}} = \sqrt{\frac{\epsilon_1}{\epsilon_0}} \quad \text{and} \quad n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}$$

$$\therefore \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \qquad (19)$$

where,

n_1 = refractive index of medium 1

n_2 = refractive index of medium 2

In this case eqn (17) may be written as.

$$R_{\|} = \left(\frac{\vec{E}'_{01}}{\vec{E}_{01}}\right)_{\|} = \frac{\sqrt{\frac{\epsilon_2}{\mu_0}}\cos\theta_i - \sqrt{\frac{\epsilon_1}{\mu_0}}\cos\theta_r}{\sqrt{\frac{\epsilon_1}{\mu_0}}\cos\theta_r + \sqrt{\frac{\epsilon_2}{\mu_0}}\cos\theta_i} \qquad (\because \mu_1 = \mu_2 = \mu_0)$$

$$R_{\|} = \frac{\vec{E}'_{01}}{\vec{E}_{01}} = \frac{\sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_i - \cos\theta_r}{\sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_i + \cos\theta_r} \qquad \left(\text{multiplying \& dividing by } \sqrt{\frac{\mu_0}{\epsilon_1}}\right)$$

——— (20a)

Using Snell's law

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_2}{n_1} \quad \text{we get}$$

$$R_{\|} = \left(\frac{\vec{E}'_{01}}{\vec{E}_{01}}\right)_{\|} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)} \qquad \text{——— (20b)}$$

Similarly eqn. (18) may be written as.

$$T_{\|} = \left(\frac{\vec{E}_{02}}{\vec{E}_{01}}\right)_{\|} = \frac{2\cos\theta_i}{\cos\theta_r + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_i} \qquad \left(\because \mu_1 = \mu_2 = \mu_0 \text{ \& multiplying \& dividing by } \sqrt{\mu_0/\epsilon_1}\right)$$

$$T_{\|} = \frac{2\cos\theta_i}{\cos\theta_r + \frac{n_2}{n_1}\cos\theta_i} = \frac{2\cos\theta_i}{\cos\theta_r + \frac{\sin\theta_i}{\sin\theta_r}\cdot\cos\theta_i} \qquad \text{——— (21a)}$$

$$\therefore T_{\|} = \left(\frac{\vec{E}_{02}}{\vec{E}_{01}}\right)_{\|} = \frac{2\cos\theta_i \sin\theta_r}{\frac{1}{2}(\sin 2\theta_r + \sin 2\theta_i)}$$

$$T_{\|} = \left(\frac{\vec{E}_{02}}{\vec{E}_{01}}\right)_{\|} = \frac{2\cos\theta_i \sin\theta_r}{\sin(\theta_i + \theta_r)\cos(\theta_i - \theta_r)} \qquad \text{——— (21b)}$$

Eqns. (20a), (20b), (21a), (21b) represents Fresnel's equations for non conducting media when electric field vector ($\vec{E}$) is parallel to plane of incidence.

It is clear from eqns (21a) & (21b) that $T_{\parallel}$ i.e. $(\vec{E}_{02}/\vec{E}_{01})_{\parallel}$ is always positive hence, the refracted ray and incident ray are always in ~~samp~~ same phase at the interface.

<u>Brewster Angle</u> → The angle of incidence at which incident radiation becomes polarised after reflection is known as Brewster angle or polarising angle and is denoted by θ_p.

From Fresnel's eqn (20b) we have.

$$R_{\parallel} = \left(\frac{E'_{01}}{E_{01}}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$$

if $\theta_i + \theta_r = 90^\circ$, $\tan(\theta_i + \theta_r) = \tan 90^\circ = \infty$
or, $(\theta_i = 90 - \theta_r)$,

so coefficient of Reflection, $\boxed{R_{\parallel} = 0}$

Thus if an unpolarized radiation consisting of a mixture of both type of radiation falls on interface at Brewster angle, only radiations with its electric vector at right angles to the plane of incidence will be reflected and the reflected radiation will be polarised.

Further from eqn (20b), $R_{\parallel}$ may be positive or negative.

<u>Case a</u> - if $\theta_i > \theta_r$ and $\theta_i + \theta_r < \frac{\pi}{2}$
or $\theta_i < \theta_r$ and $\theta_i + \theta_r > \frac{\pi}{2}$

$R_{\parallel}$ will be positive i.e. reflected waves and incidents waves will be in same phase at the boundary or interface.

<u>Case b</u> - in all other cases except case a, $\frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$ i.e. $R_{\parallel}$ will be negative i.e., the reflected wave will suffer a phase change of π radian at the boundary.

Thus, in this case the phase of reflected wave does not depend the ratio (n_2/n_1) but on relative values of θ_i and θ_r and their sum.

Fig. 3. shows the amplitudes of reflected and refracted waves as a function of incident amplitude for $\frac{n_2}{n_1} = 1.5$ and corresponds to light incident from air to glass.

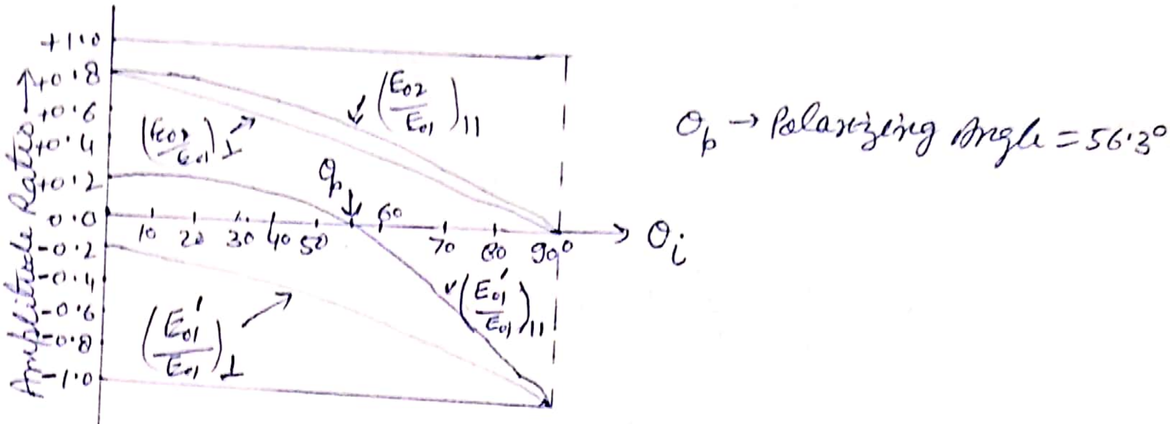

Fig. 3 - Reflected and refracted amplitudes for glass.

For normal incidence, $\theta_i = 0°$, $\theta_r = 0°$.

Fresnel's equation for non conducting media for the two cases (Reflection and Refraction ~~perpendicular and parallel~~) become identical and may be written as.

$$\frac{E'_{01}}{E_{01}} = \frac{n_2 - n_1}{n_2 + n_1} \qquad (22)$$

and

$$\frac{E_{02}}{E_{01}} = \frac{2n_1}{n_2 + n_1} \qquad (23)$$

Discussion →

① The two components $E_\perp$ and $E_\parallel$ are not transmitted and reflected equally.

② In case $E_\perp$ to plane of incidence there is reflection unless the two media have the same optical properties.

③ In case $E_{\parallel}$ to plane of incidence, the reflected amplitude vanishes if $\theta_i + \theta_r = 90^\circ$. The angle of incidence θ_i satisfying this condition is called Brewster's angle θ_p.

④ In case $E_{\perp}$ to plane of incidence the reflected ray suffers a phase change of π on reflection from a denser surface.

⑤ In case $E_{\parallel}$ to plane of incidence the reflected ray suffers a phase change of π only when angle of incidence is greater than Brewster's angle. i.e. $\theta_i > \theta_p$.

⑥ The refracted ray never suffers a phase change at the interface.

⑦ At grazing incidence $\theta_i = 90^\circ$, the reflection coefficient is equal to unity.

The Electromagnetic theory explains very well, reflection, refraction, flow of energy in free space, polarization, transverse character of light, etc. But, this theory could't explain photo electric effect, compton effect, emission and absorption of radiation Atomic spectra etc. which were later explained by quantum theory of light.

Ques. - Define Total Internal Reflection (TIR) and Evanescent Wave.

Sol. - Total Internal Reflection (TIR) - from snell's law,

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_2}{n_1}$$

when medium 1 is denser and medium 2 is rarer

i.e., $n_1 > n_2$

$\theta_i < \theta_r$

i.e, reflected ray deviates away from the normal. In such a case the velocity of light in medium 2 is greater than medium 1. Above a critical angle of incidence there is no refracted wave corresponding to incident wave and so no refraction is possible.

$$\theta_{crit} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This phenomenon is known as Total Internal reflection.

Evanescent Wave – Evanescent waves do't transfer energy. These waves are used to satisfy boundary conditions. Mathematically Evanescent waves can be characterized by a wave vector where one or more of the vector components has an imaginary value. If the angle of incidence exceeds the critical angle (TIR), then the wave vector of transmitted wave has the form.