

Title

KINEMATICS OF MACHINES

Sub-title

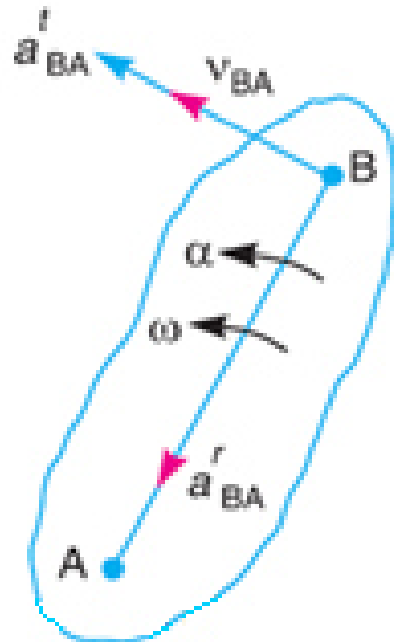
Velocity & Acceleration Analysis

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Acceleration in Mechanisms

Acceleration of a particle whose **velocity changes both in magnitude and direction** at any instant has the following two components :

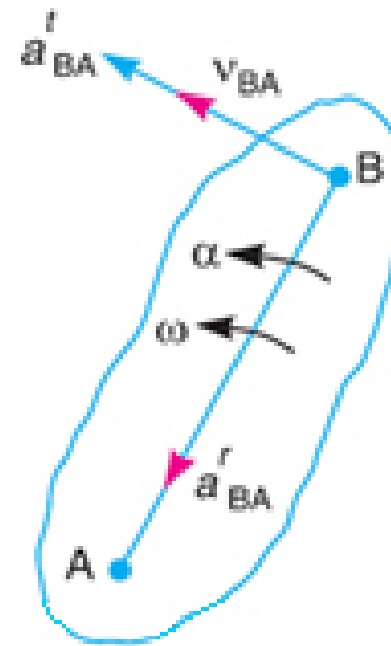
1. The *centripetal or radial component*, which is perpendicular to the velocity of the particle at the given instant.
2. The *tangential component*, which is parallel to the velocity of the particle at the given instant.



$$a_{BA}^r = \omega^2 \times \text{Length of link } AB$$

$$= \omega^2 \times AB = v_{BA}^2 / AB \quad \dots \left(\because \omega = \frac{v_{BA}}{AB} \right)$$

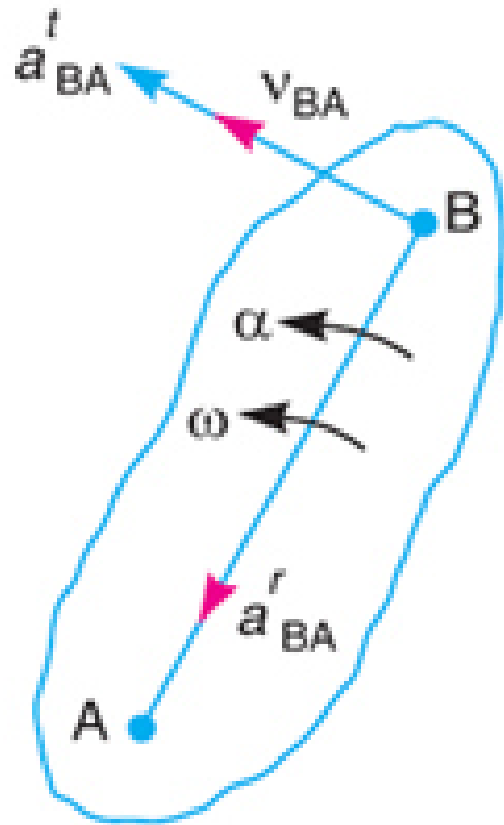
This radial component of acceleration acts perpendicular to the velocity v_{BA} . In other words, it acts **parallel to the link AB**.



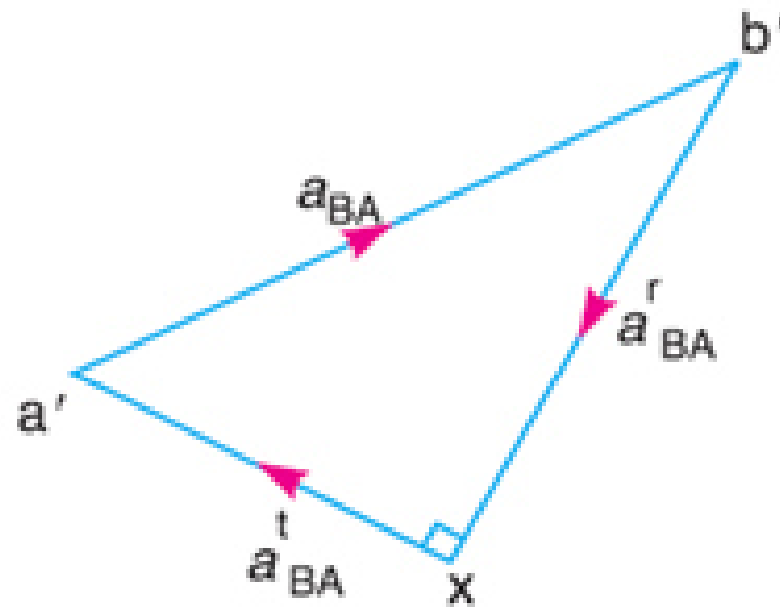
We know that tangential component of the acceleration of B with respect to A,

$$a_{BA}^t = \alpha \times \text{Length of the link } AB = \alpha \times AB = dv/dt$$

This tangential component of acceleration acts parallel to the velocity v_{BA} . In other words, it acts **perpendicular** to the link AB.

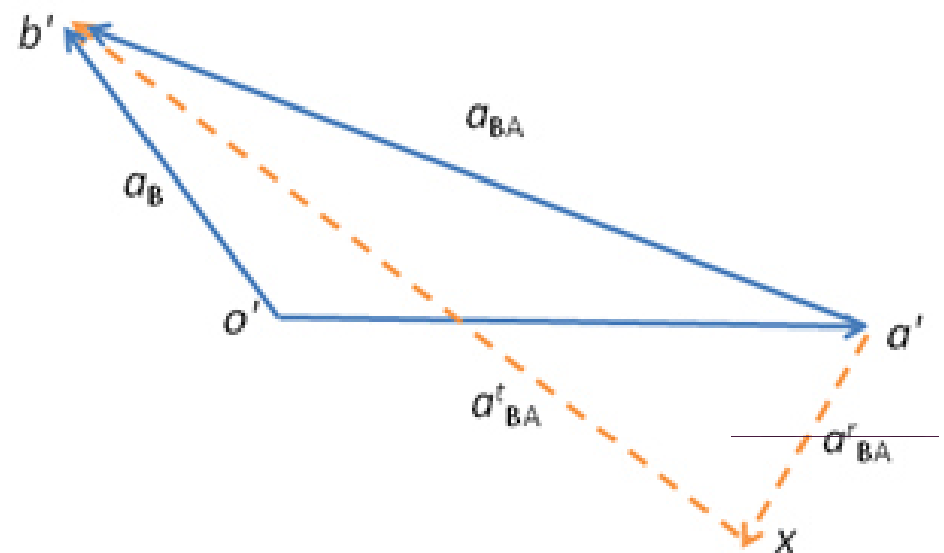
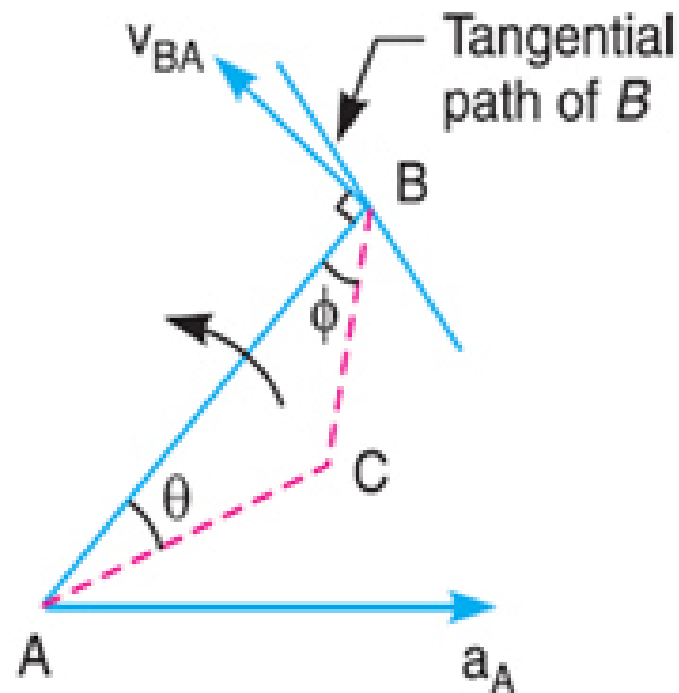


Link.



Acceleration diagram.

Acceleration of a Point on a Link

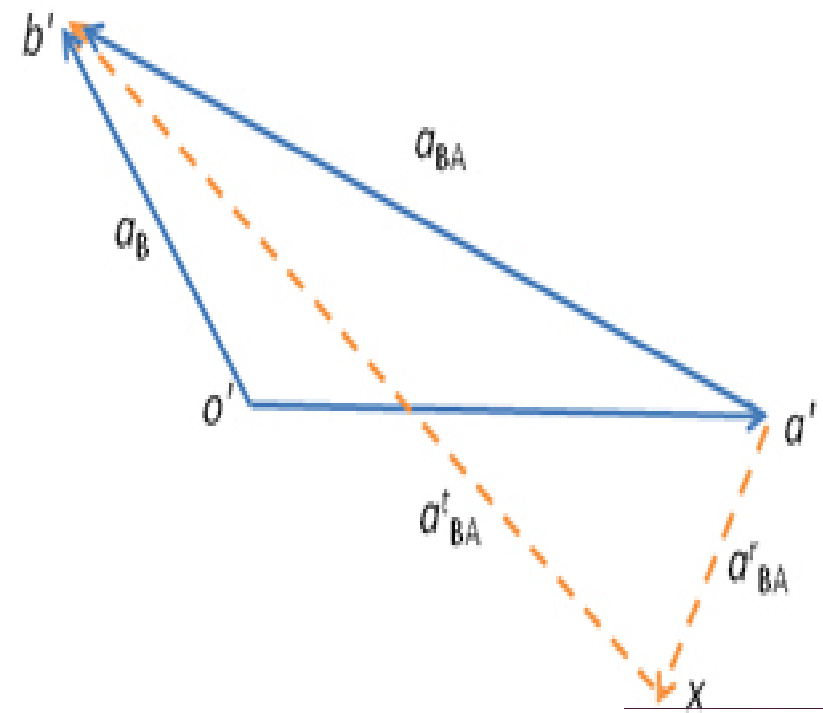
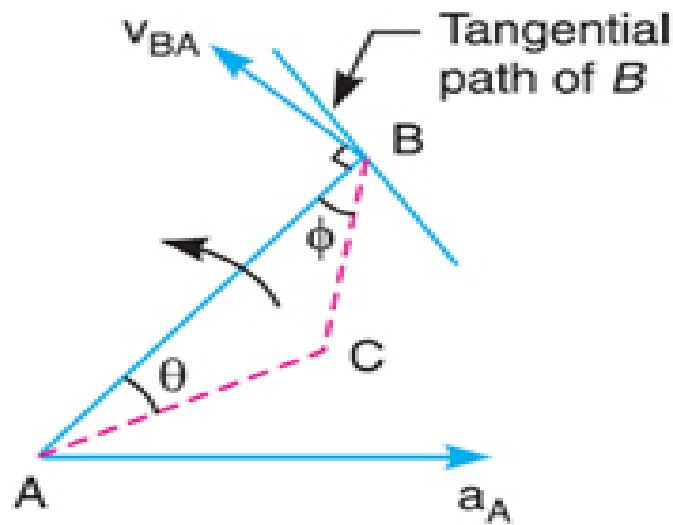


From any point o' , draw vector $o'a'$ parallel to the direction of absolute acceleration at point A i.e. a_A , to some suitable scale

Draw vector $a'x$ parallel to the link AB (because radial component of the acceleration of B with respect to A will pass through AB), such that

$$\text{vector } a'x = a_{BA}^r = v_{BA}^2 / AB$$

where v_{BA} = Velocity of B with respect to A .



From point x , draw vector xb' perpendicular to AB or vector $a'x$ i.e. a'_{BA}

through o' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B . The vectors xb' and $o'b'$ intersect at b' . Now the values of a_B and a'_{BA} may be measured, to the scale.

$$\alpha_{AB} = a'_{BA} / AB$$

By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA} . The vector $a'b'$ is known as acceleration image of the link AB .

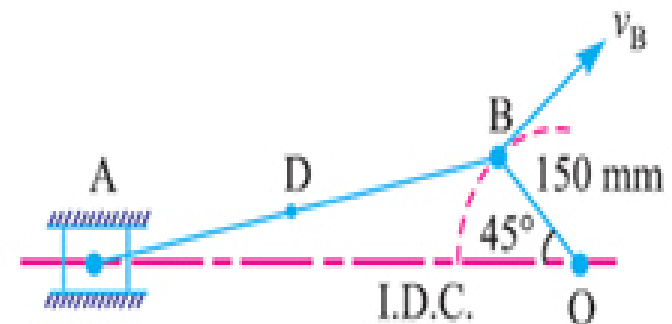
Example 8.1. The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : **1.** linear velocity and acceleration of the midpoint of the connecting rod, and **2.** angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Solution. Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; $OB = 150$ mm = 0.15 m ; $BA = 600$ mm = 0.6 m

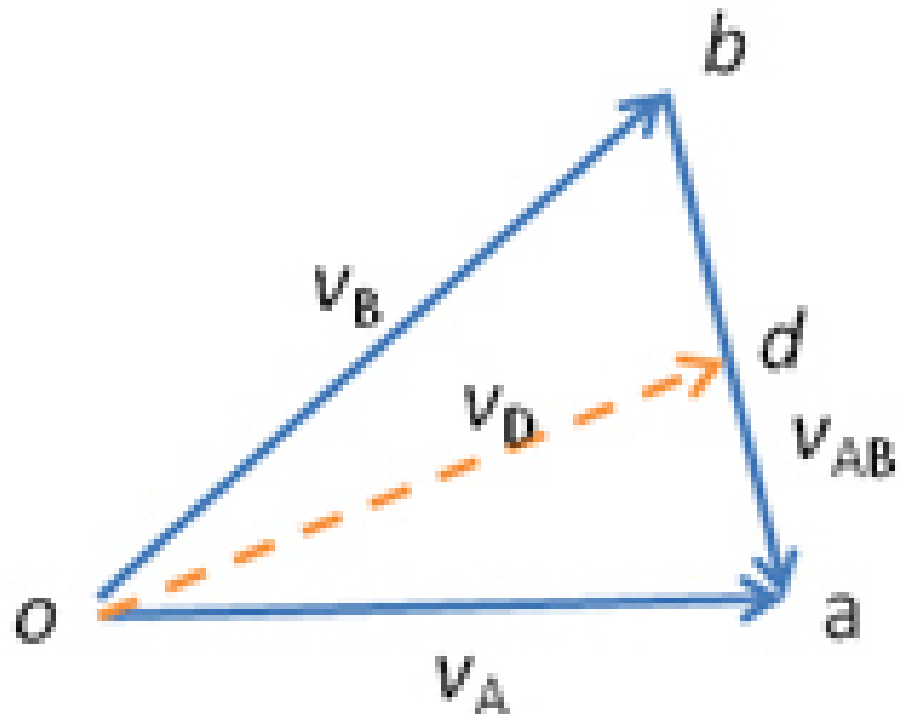
We know that linear velocity of B with respect to O or velocity of B ,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

1. First of all draw the space diagram, to some suitable scale; as shown in Fig. (a).



(a) Space diagram.



Acceleration of the midpoint of the connecting rod

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a_{BO}^r = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

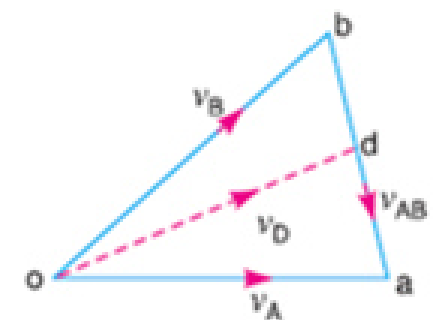
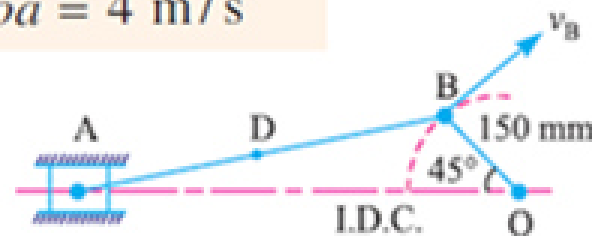
and the radial component of the acceleration of A with respect to B,

$$a_{AB}^r = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

- NOTE:1) A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.**
2) When a point moves along a straight line, it has no centripetal or radial component of the acceleration.

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A, v_A = \text{vector } oa = 4 \text{ m/s}$$

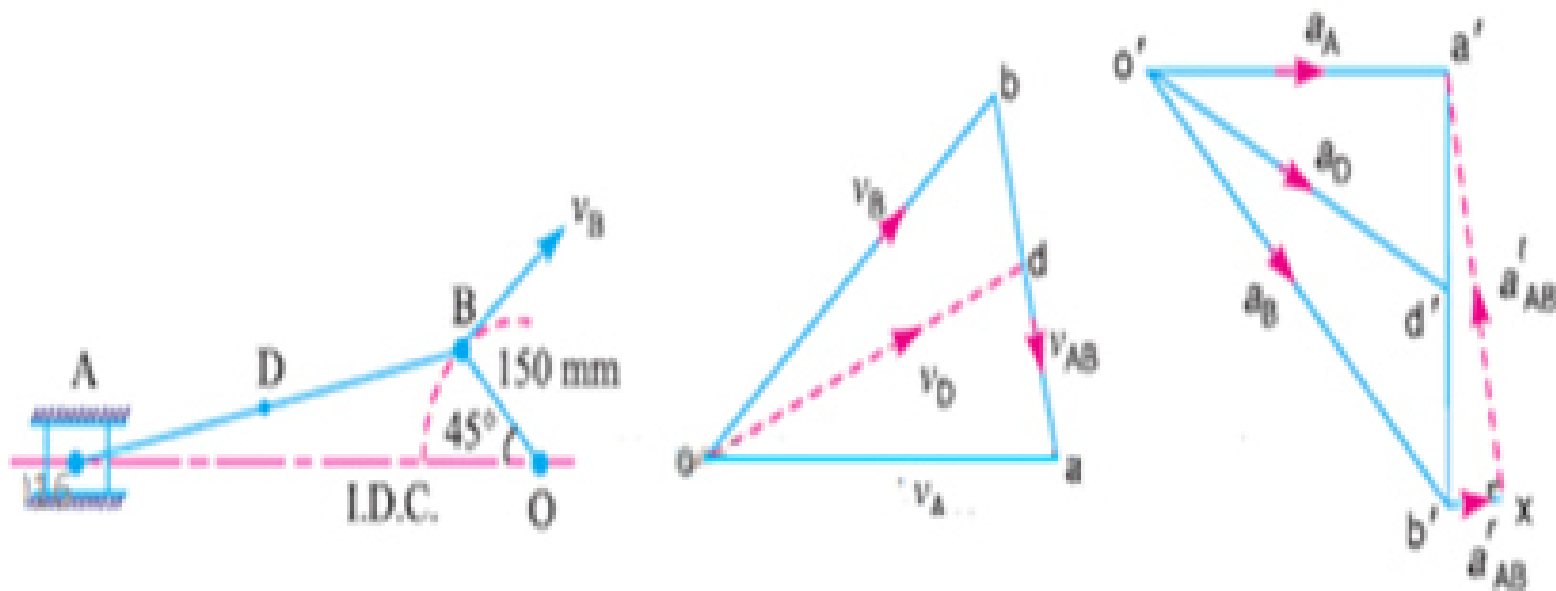


Acceleration of the midpoint of the connecting rod

Draw vector $o' b'$ parallel to BO , to some suitable scale,

$$\text{vector } o' b' = a'_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank OB rotates at a constant speed, therefore there will be no tangential component of the acceleration of B with respect to O .



from point b' , draw vector $b'x$ parallel to AB to represent $a'_{AB} = 19.3 \text{ m/s}^2$ and
 from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

Now from o' , draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A i.e. a_A . The vectors xa' and $o'a'$ intersect at a' . Join $a'b'$.

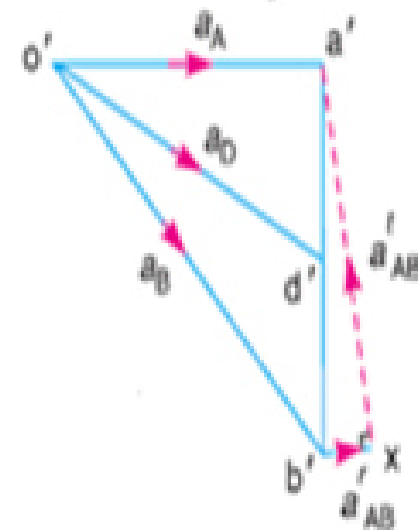
In order to find the acceleration of the midpoint D of the connecting rod AB , divide the vector $a'b'$ at d' in the same ratio as D divides AB . In other words

$$b'd' / b'a' = BD / BA$$

Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod i.e. a_D .

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \text{ Ans.}$$



Angular velocity of the connecting rod

We know that angular velocity of the connecting rod AB ,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}$$

Ans.

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

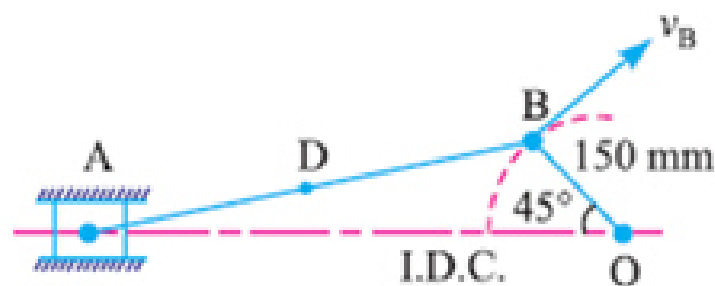
$$a'_{AB} = 103 \text{ m/s}^2$$

...(By measurement)

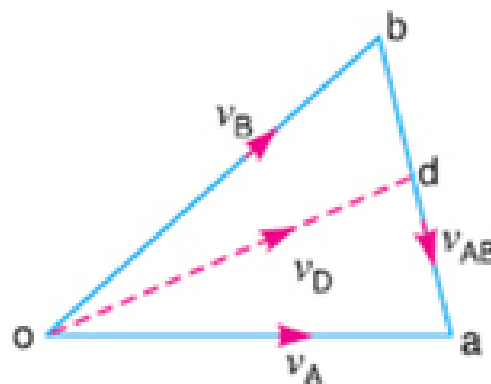
We know that angular acceleration of the connecting rod AB ,

$$\alpha_{AB} = \frac{a'_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2$$

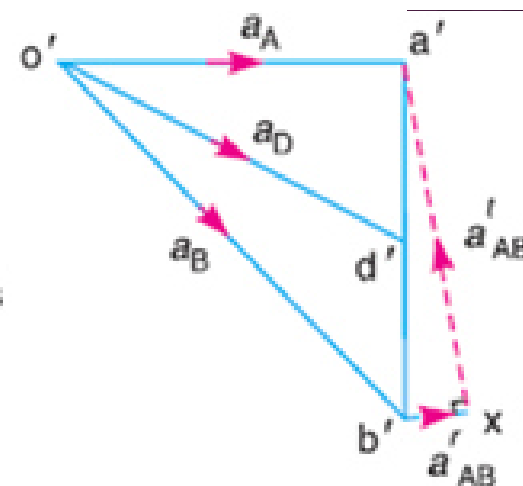
Ans.



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are $PQ = 62.5 \text{ mm}$; $QR = 175 \text{ mm}$; $RS = 112.5 \text{ mm}$; and $PS = 200 \text{ mm}$. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle $QPS = 60^\circ$ and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

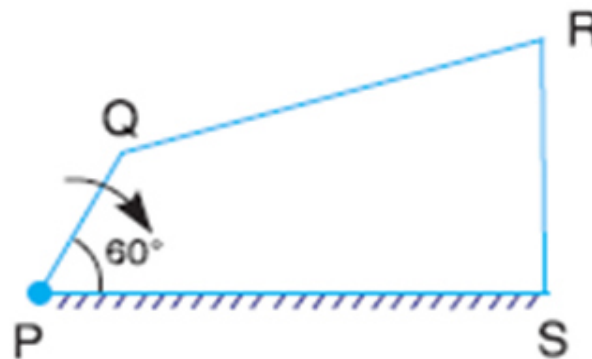
Solution. Given : $\omega_{QP} = 10 \text{ rad/s}$; $PQ = 62.5 \text{ mm} = 0.0625 \text{ m}$; $QR = 175 \text{ mm} = 0.175 \text{ m}$;
 $RS = 112.5 \text{ mm} = 0.1125 \text{ m}$; $PS = 200 \text{ mm} = 0.2 \text{ m}$

We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale _____



(a) Space diagram.

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ , to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q i.e. v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q , draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (i.e. v_{RQ}) and from point s , draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (i.e. v_{RS} or v_R). The vectors qr and sr intersect at r . By measurement, we find that

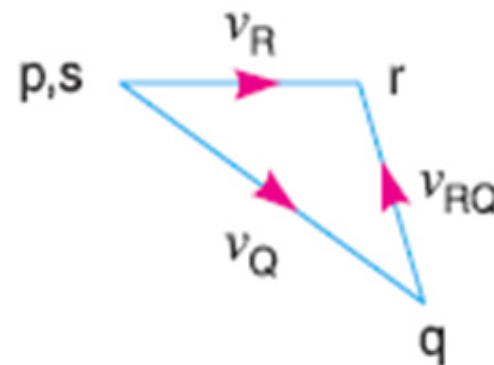
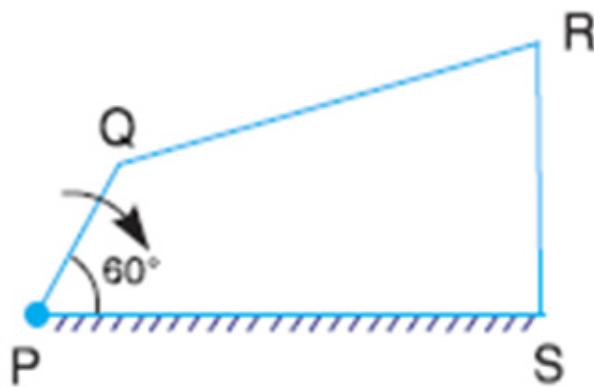
$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link QR ,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise) Ans.}$$

and angular velocity of link RS ,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise) Ans.}$$



Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P .

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

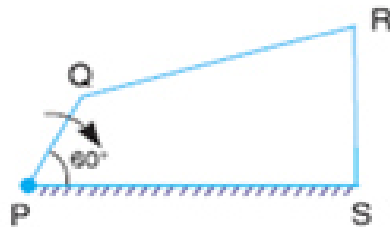
$$a_{QP}^r = a_{QP} = a_Q = \frac{v_{QP}^2}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q ,

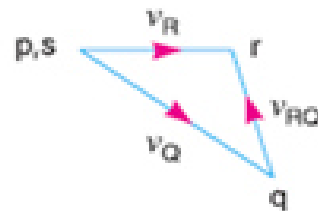
$$a_{RQ}^r = \frac{v_{RQ}^2}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

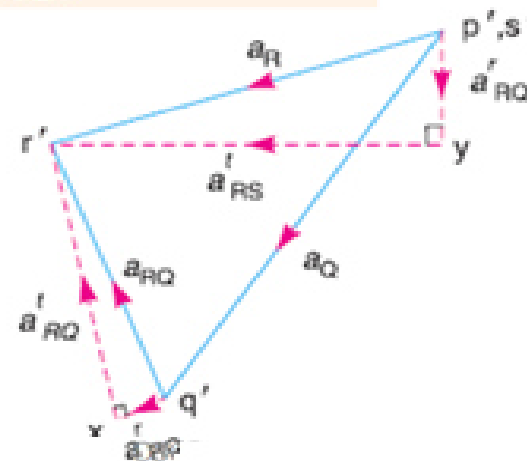
$$a_{RS}^r = a_{RS} = a_R = \frac{v_{RS}^2}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$



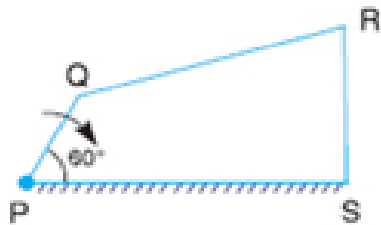
(a) Space diagram.



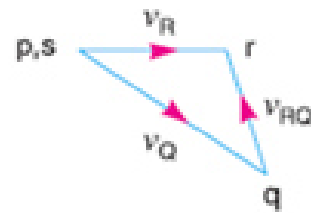
(b) Velocity diagram.



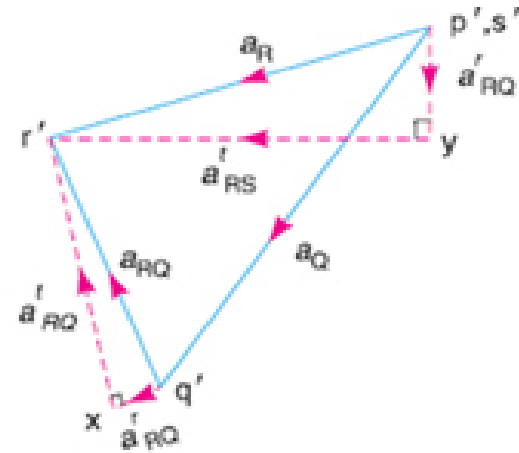
(c) Acceleration diagram.



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ , to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q i.e. a_{QP}^r or a_Q such that

$$\text{vector } p'q' = a_{QP}^r = a_Q = 6.25 \text{ m/s}^2$$

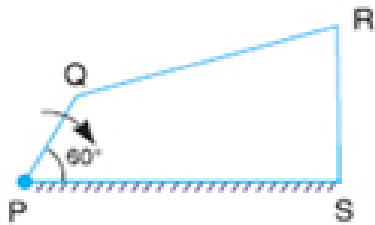
2. From point q' , draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q i.e. a_{RQ}^r such that

$$\text{vector } q'x = a_{RQ}^r = 0.634 \text{ m/s}^2$$

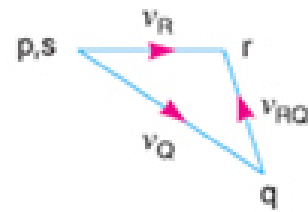
3. From point x , draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q i.e. a_{RQ}^t whose magnitude is not yet known.

4. Now from point s' , draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S i.e. a_{RS}^r such that

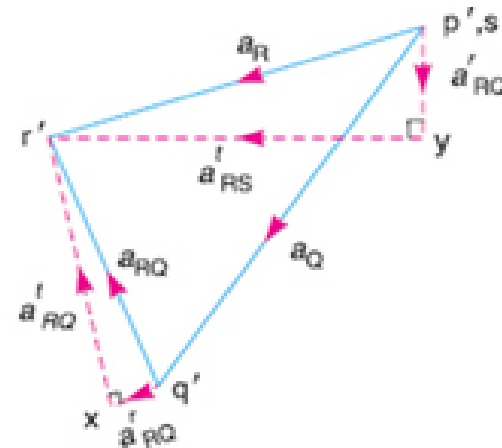
$$\text{vector } s'y = a_{RS}^r = 1.613 \text{ m/s}^2$$



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

5. From point y , draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S i.e. a'_{RS} .

6. The vectors xr' and yr' intersect at r' . Join $p'r$ and $q'r'$. By measurement, we find that

$$a'_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a'_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR ,

$$\alpha_{QR} = \frac{a'_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise) Ans.}$$

and angular acceleration of link RS ,

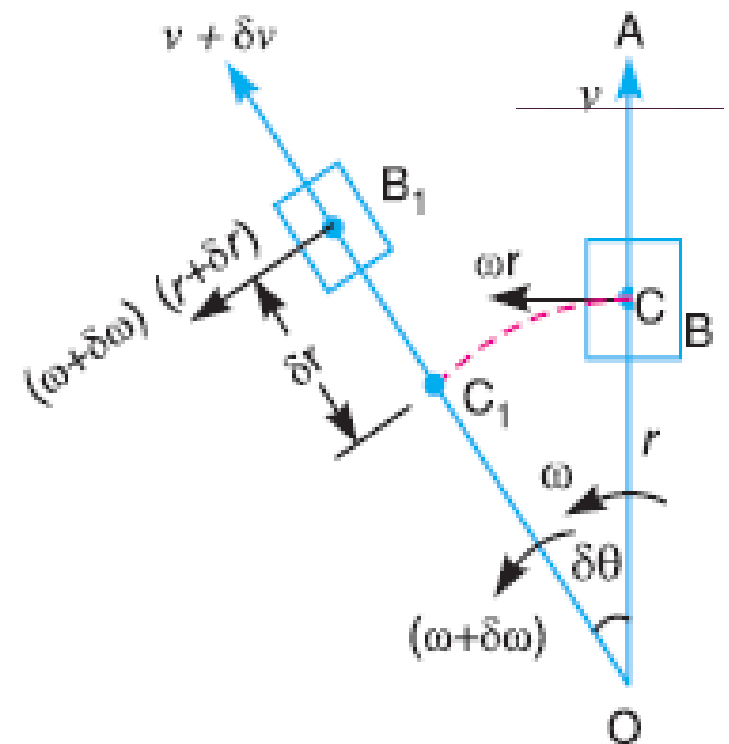
$$\alpha_{RS} = \frac{a'_{RS}}{SR} = \frac{5.3}{0.1125} \text{ Anticlockwise Ans.}$$

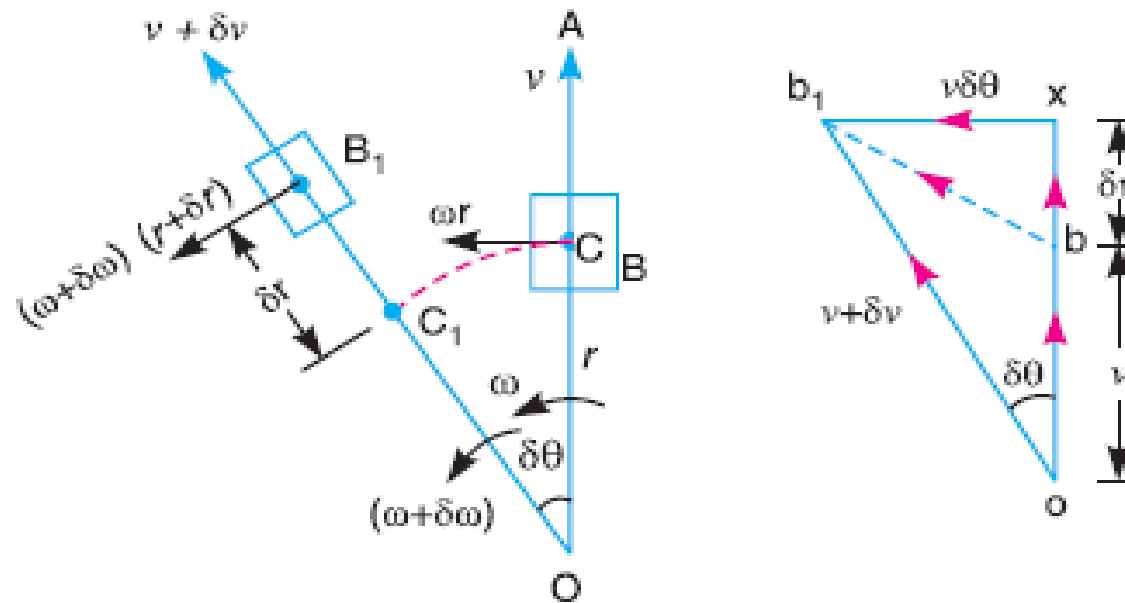
Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link OA and a slider B as shown in Fig. link OA . The point C is the coincident point on the link OA .

The slider B moves along the





$$bx = ox - ob = (v + \delta v) \cos \delta\theta - v \uparrow$$

Since $\delta\theta$ is very small, therefore substituting $\cos \delta\theta = 1$, we have

$$bx = (v + \delta v - v) \uparrow = \delta v \uparrow$$

...(Acting radially outwards)

and

$$xb_1 = (v + \delta v) \sin \delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin \delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v) \delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = v.\delta\theta \leftarrow$$

Fig. shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)(r + \delta r)$ are considered.

$$yb_1 = (\omega + \delta\omega)(r + \delta r) \sin \delta\theta \downarrow$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r) \sin \delta\theta$$

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow, \text{ acting radially inwards}$$

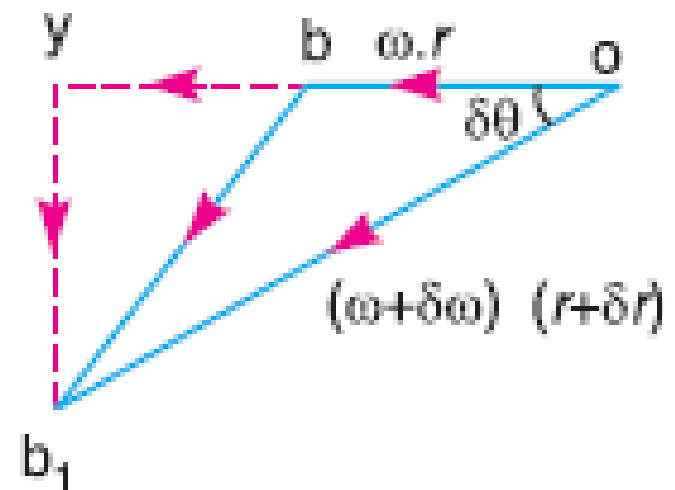
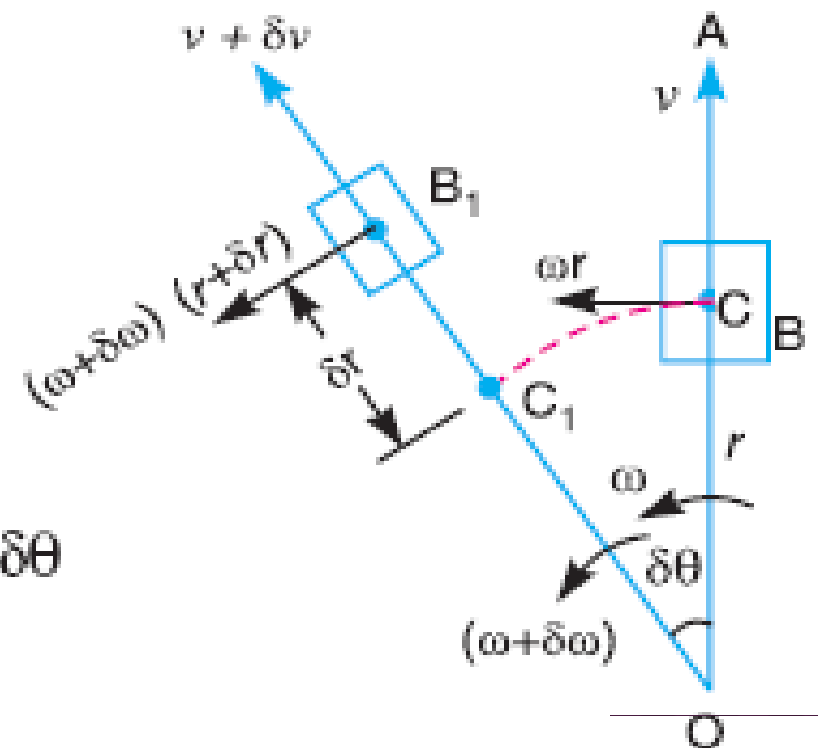
$$by = oy - ob = (\omega + \delta\omega)(r + \delta r) \cos \delta\theta - \omega.r$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r) \cos \delta\theta - \omega.r$$

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega$$

total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow$$



∴ Radial component of the acceleration of the slider B with respect to O on the link OA ,

$$a_{BO}^r = \text{Lt} \frac{\delta v - \omega r \cdot \delta \theta}{\delta t} = \frac{dv}{dt} - \omega r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2 r \quad \uparrow \quad (\because d\theta/dt = \omega)$$

total component of change of velocity along tangential direction,

$$= x b_1 + b y = v \cdot \overset{\leftarrow}{\delta \theta} + (\omega \cdot \overset{\leftarrow}{\delta r} + r \cdot \delta \omega) \quad (\text{Perpendicular to } OA \text{ and towards left})$$

Tangential component of acceleration of the slider B acting perpendicular to OA and towards left,

$$a_{BO}^t = \text{Lt} \frac{v \cdot \delta \theta + (\omega \cdot \delta r + r \cdot \delta \omega)}{\delta t} = v \frac{d\theta}{dt} + \omega \frac{dr}{dt} + r \frac{d\omega}{dt}$$

$$= v \cdot \omega + \omega \cdot v + r \cdot \alpha = (2v \cdot \omega + r \cdot \alpha) \quad (\because dr/dt = v, \text{ and } d\omega/dt = \alpha)$$

$$a_{BO}^r = \frac{dv}{dt} - \omega^2 \cdot r \uparrow$$

$$a_{BO}^t = (2v \cdot \omega + r \cdot \alpha) \leftarrow$$

$$a_{CO}^r = \omega^2 \cdot r \uparrow$$

$$a_{CO}^t = \alpha \cdot r \leftarrow$$

$$a_{BC}^r = a_{BO}^r - a_{CO}^r = \left(\frac{dv}{dt} - \omega^2 \cdot r \right) - \left(-\omega^2 \cdot r \right) = \frac{dv}{dt} \uparrow$$

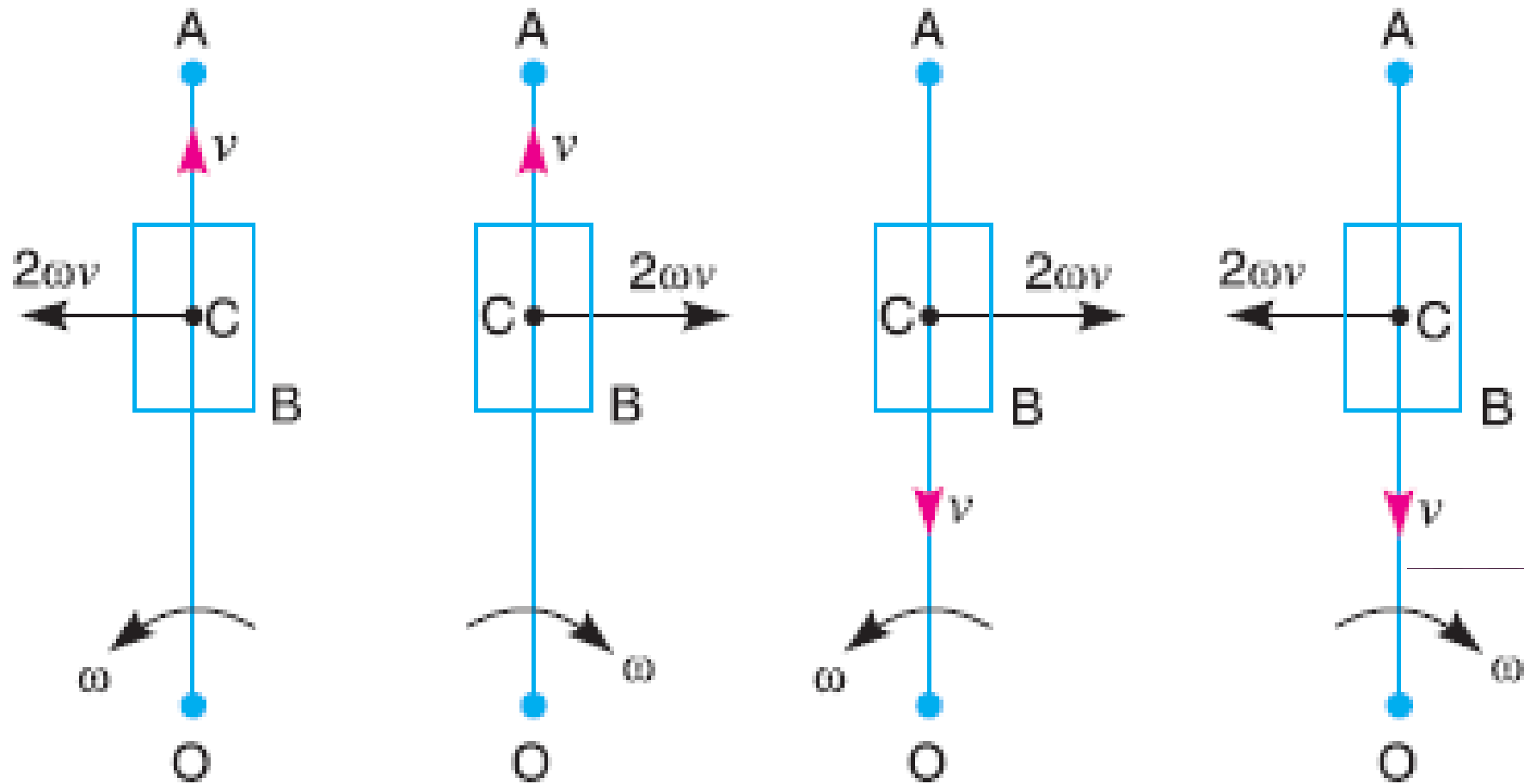
$$a_{BC}^t = a_{BO}^t - a_{CO}^t = (2\omega v + \alpha r) - \alpha r = 2\omega v \leftarrow$$

Coriolis component of the acceleration of B with respect of C ,

$$a_{BC}^c = a_{BC}^t = 2\omega v$$

ω = Angular velocity of the link OA , and

v = Velocity of slider B with respect to coincident point C .



Direction of coriolis component of acceleration.

Thank You