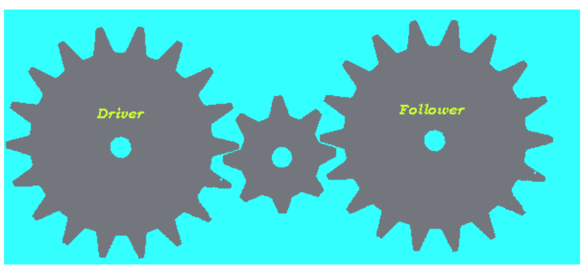
Gear Trains

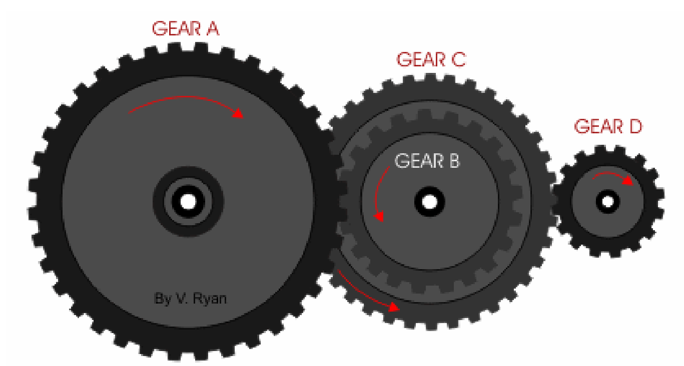
Introduction

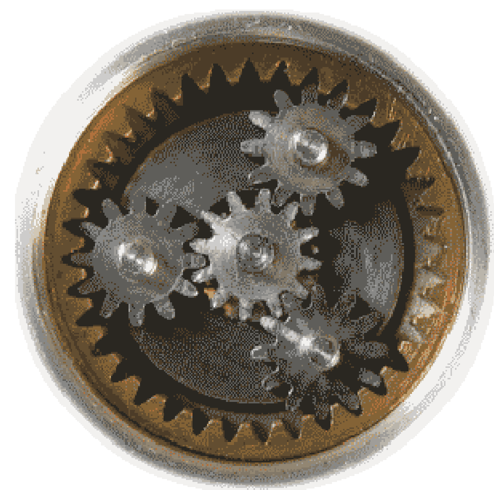
- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train or train of toothed wheels.***
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts.
- A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

- Following are the different types of gear trains, depending upon the arrangement of wheels :
  1. Simple gear train
  2. Compound gear train
  3. Reverted gear train
  4. Epicyclic gear train
- In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other.
- But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

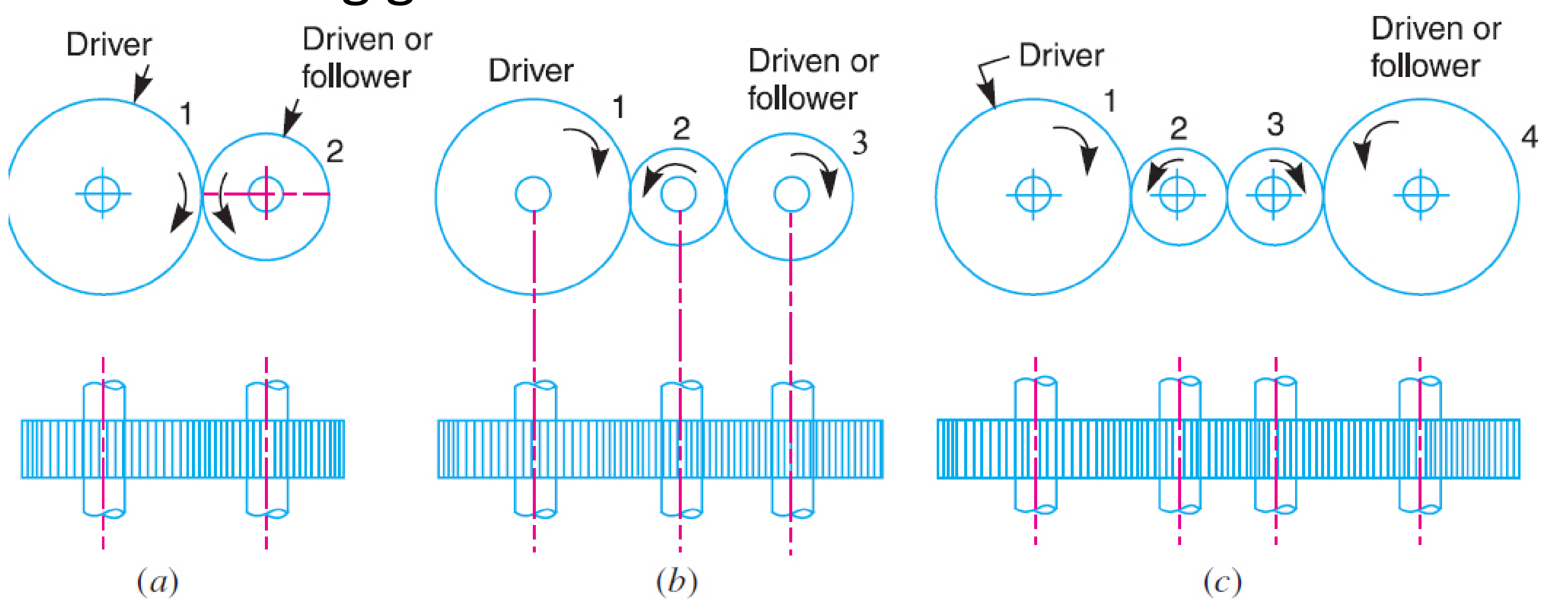


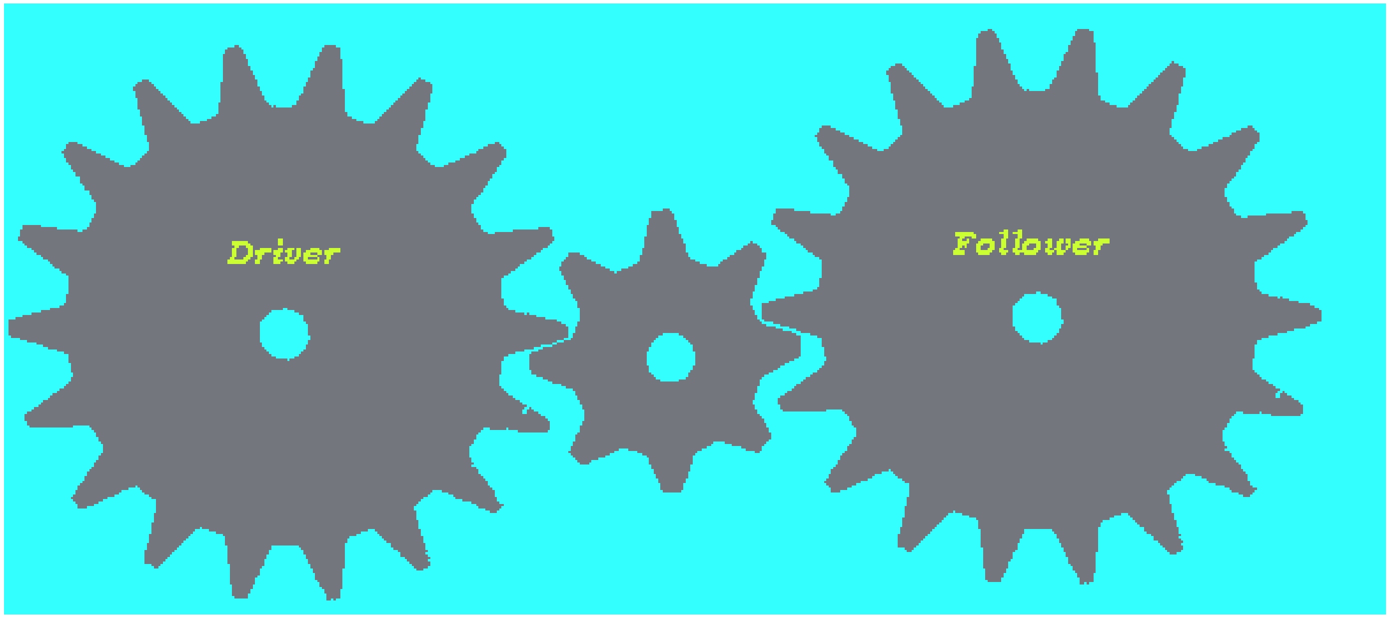


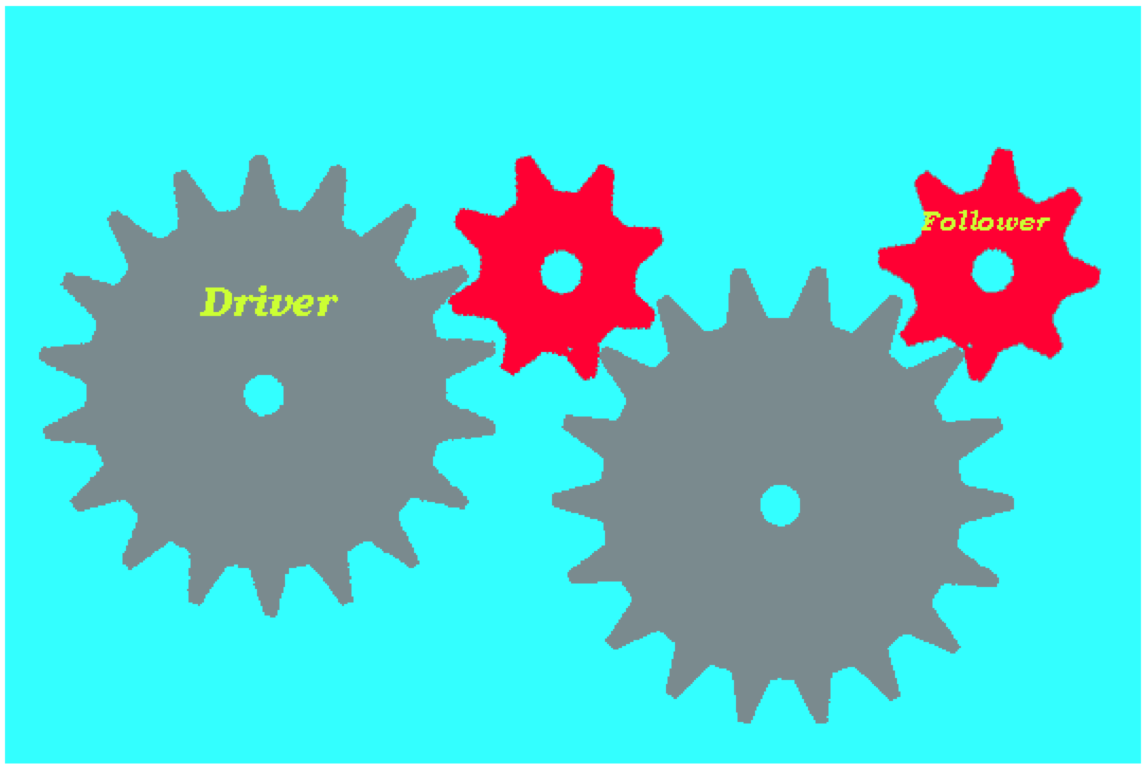


Simple Gear Train

- When there is only one gear on each shaft, as shown in Fig., it is known as *simple gear train.*
- *The gears are* represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. (*a).*
- *Since the gear 1 drives the gear* 2, therefore gear 1 is called the *driver and the gear 2 is called the driven or follower.*
- *It may be noted* that the motion of the driven gear is opposite to the motion of driving gear.




Driver
Follower


Driver
Follower

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*).

But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*).

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \text{...}(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \text{...}(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations (*i*) and (*ii*).

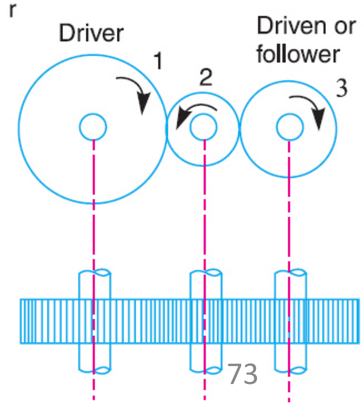
$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

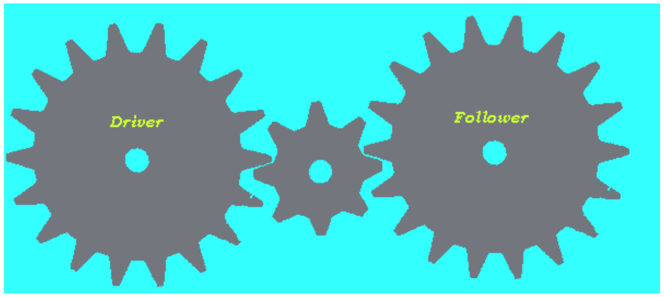


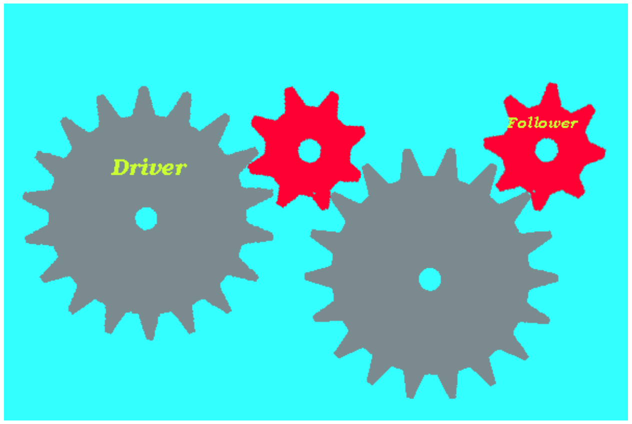
(*b*)

Similarly, it can be proved that the ıbove equation holds good even if there are ıny number of intermediate gears. From ıbove, we see that the speed ratio and the rain value, in a simple train of gears, is in-lependent of the size and number of inter-nediate gears. These intermediate gears are ;alled ***idle gears***, as they do not effect the ;peed ratio or train value of the system. The dle gears are used for the following two pur-poses :

1. To connect gears where a large ;entre distance is required, and

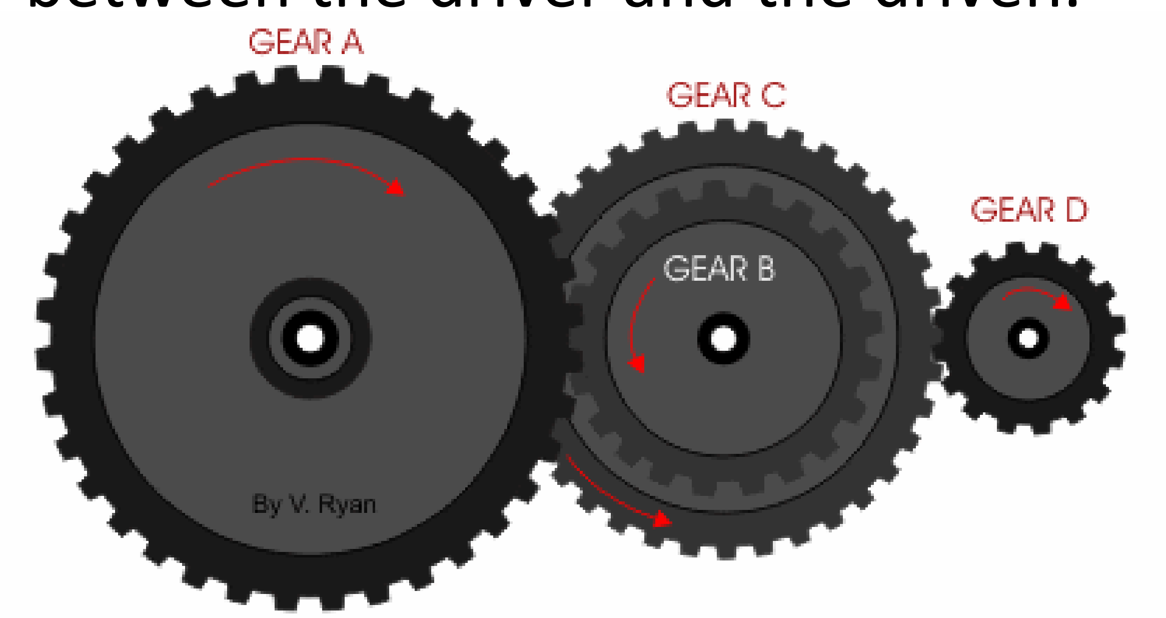
2. To obtain the desired direction of notion of the driven gear (*i.e.* clockwise or ınticlockwise).

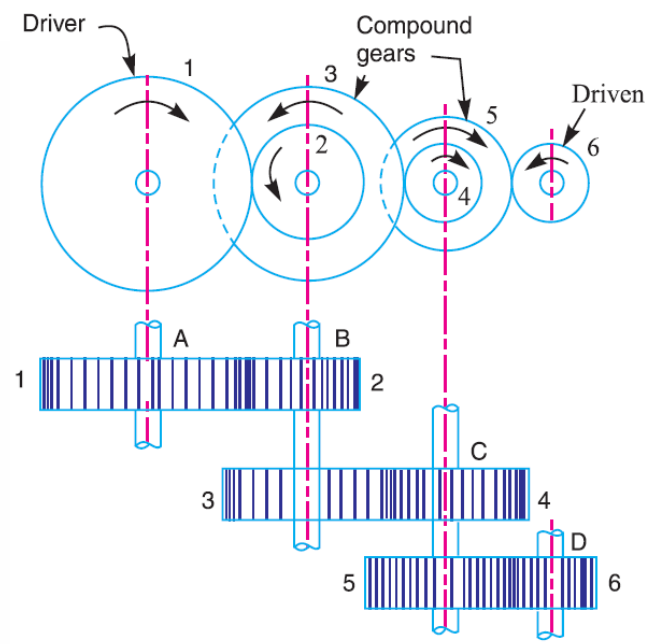




Compound Gear Train

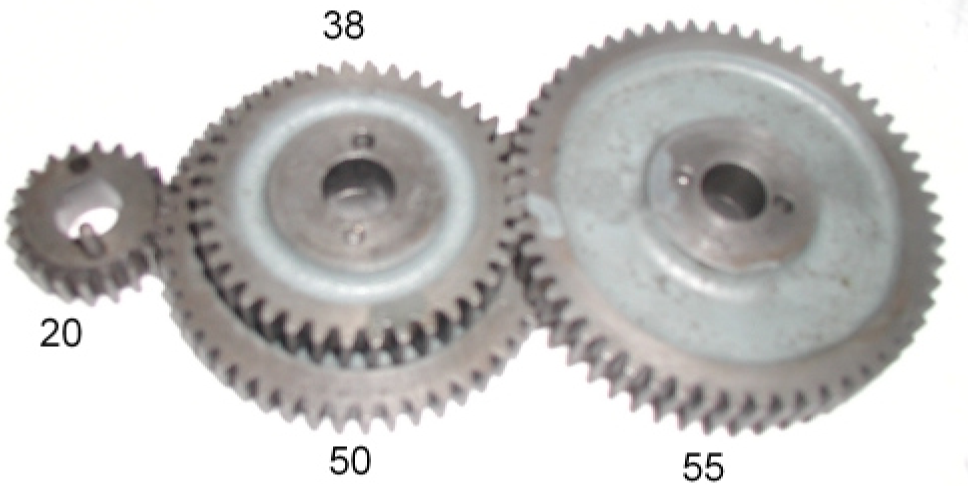
- When there are more than one gear on a shaft, as shown in Fig. , it is called a *compound train of gear.*
- We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system.
- But these gears are useful in bridging over the space between the driver and the driven.





Compound Gear Train (Continued)

- But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is increased by providing compound gears on intermediate shafts.
- In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed.
- One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.



shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

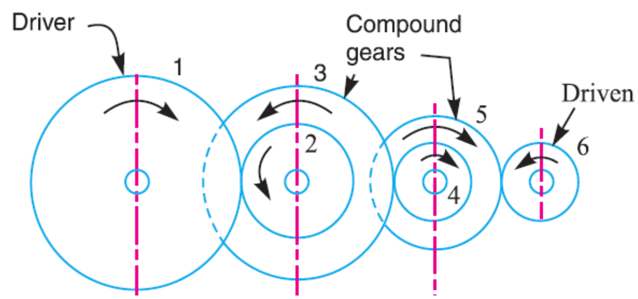
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is



$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^{*}\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

* Since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$.

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

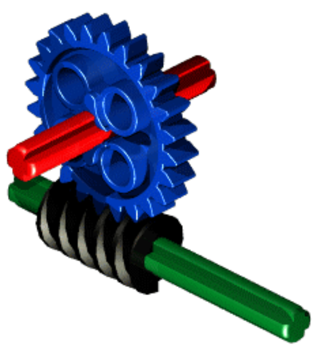
$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

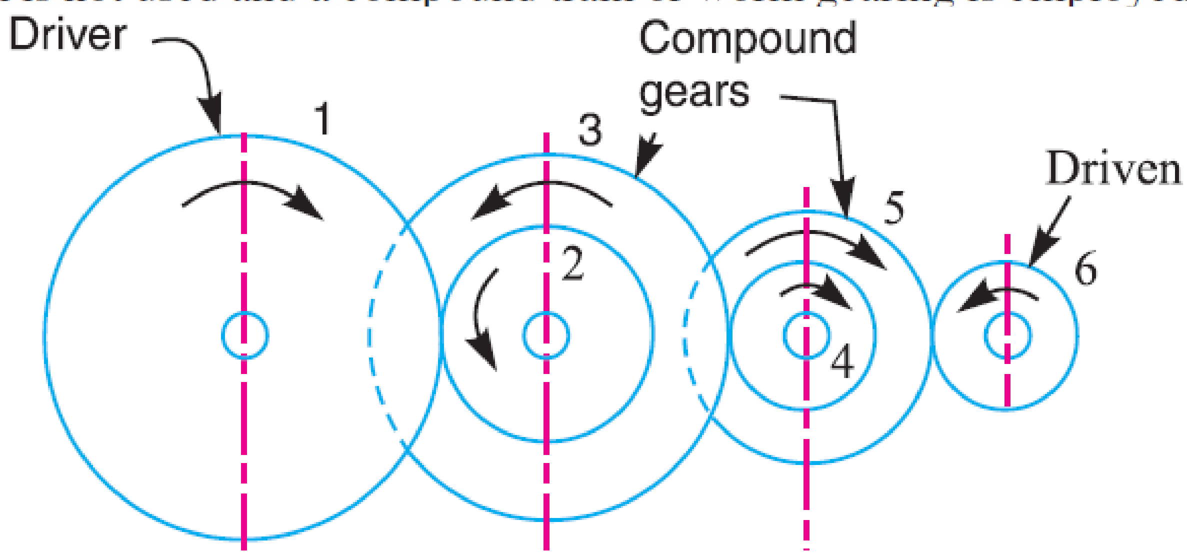
and

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

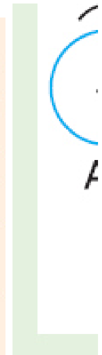
$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

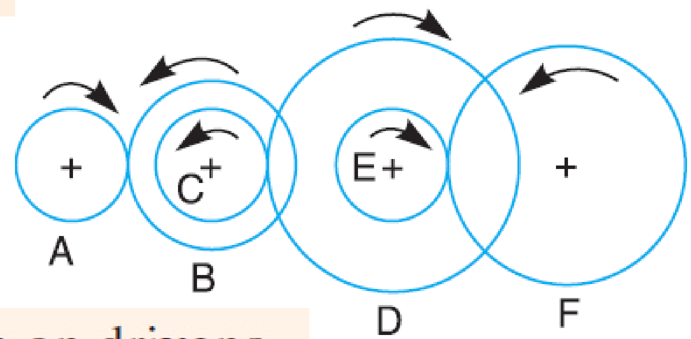




Example 13.1. *The gearing of a machine tool is shown in Fig. 13.3. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :*



Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*



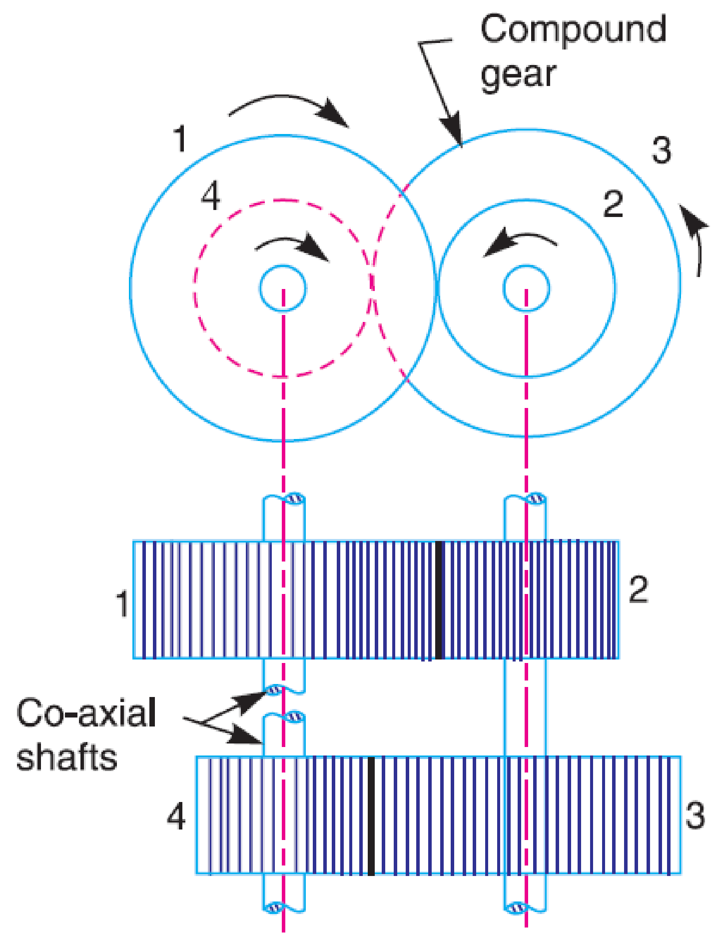
$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivens}}{\text{Product of no. of teeth on drivers}}$$

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26} = 18.75$$

$$N_F = \frac{N_A}{18.75} = \frac{975}{18.75} = 52 \text{ r. p. m.}$$ **Ans.**

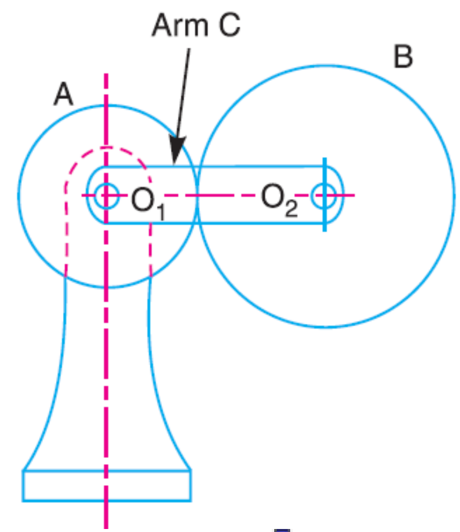
Reverted Gear Train

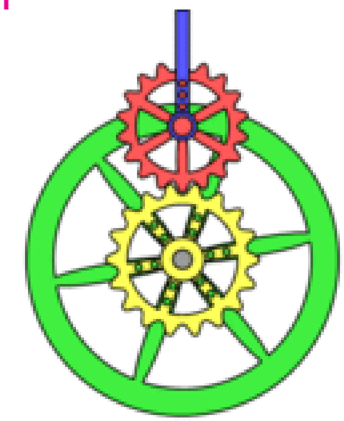
- When the axes of the first gear (*i.e. first driver)* and the last gear (*i.e. last driven or follower) are co-axial,* then the gear train is known as ***reverted gear train as*** shown in Fig.
- We see that gear 1 (*i.e. first driver) drives the* gear 2 (*i.e. first driven or follower) in the opposite direction.*
- Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2.
- The gear 3 (which is now the second driver) drives the gear 4 (*i.e. the last driven or follower) in the same direction as* that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***



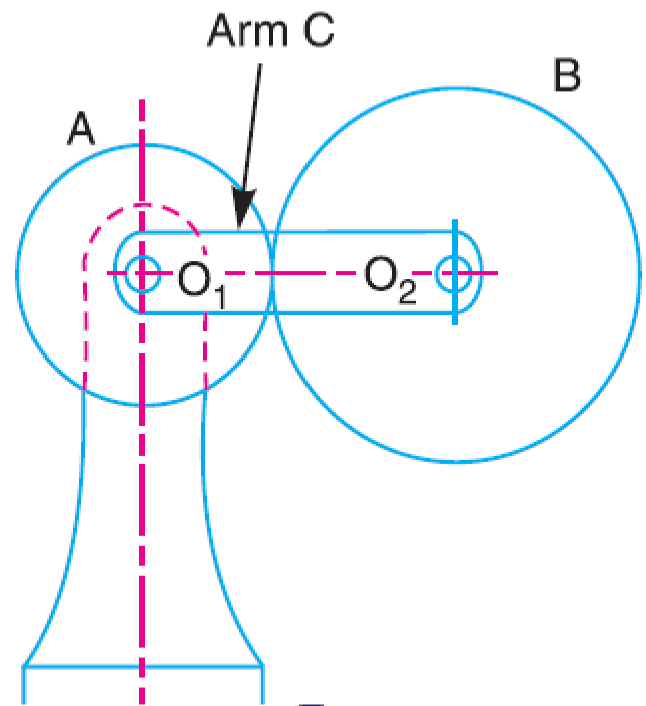
Epicyclic Gear Train

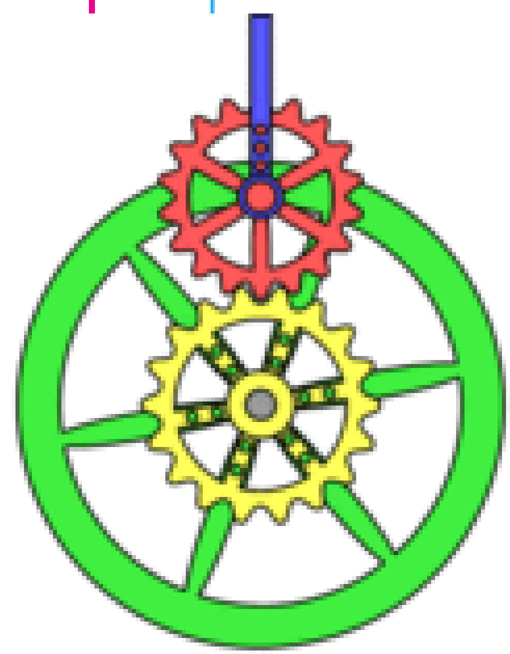
- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- A simple epicyclic gear train is shown in Fig., where a gear *A and the arm C have a common axis at O*1 about which they can rotate.
- The gear *B meshes with gear A and has its axis on the arm at O*2, about which the gear *B can rotate.*





- *If the* arm is fixed, the gear train is simple and gear *A can drive gear B* or *vice- versa, but if gear A is fixed and the arm is rotated about* the axis of gear *A (i.e. O*1), then the gear *B is forced to rotate upon and around gear A.*
- *Such a motion is called epicyclic and* the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as *epicyclic gear trains (epi. means upon and cyclic* means around).
- The epicyclic gear trains may be *simple or compound.*
- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.
- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.





Algebraic method

Let the arm *C* be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear *A* relative to the arm *C*

$$= N_A - N_C$$

and speed of the gear *B* relative to the arm *C*,

$$= N_B - N_C$$

Since the gears *A* and *B* are meshing directly, therefore they will revolve in ***opposite*** directions.

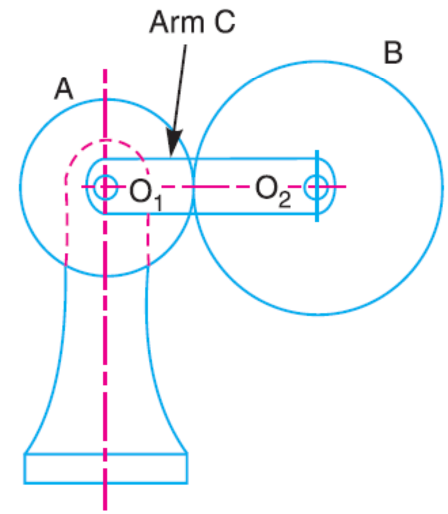
$$\therefore \qquad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm *C* is fixed, therefore its speed, $N_C = 0$.

$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear *A* is fixed, then $N_A = 0$.

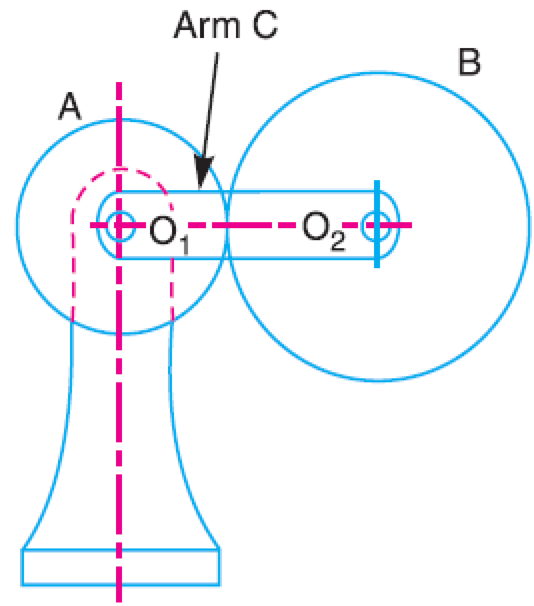
$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \qquad \text{or} \qquad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$



Tabular method.

Consider an epicyclic gear train as shown in Fig.

- Let *T*A = Number of teeth on gear *A, and T*B = Number of teeth on gear *B.*
- First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other.
- When the gear *A makes one* revolution anticlockwise, the gear *B will* make ***T*** A / *T*B revolutions, clockwise.



We know that $N_B / N_A = T_A / T_B$. Since $N_A = 1$ revolution, therefore $N_B = T_A / T_B$.

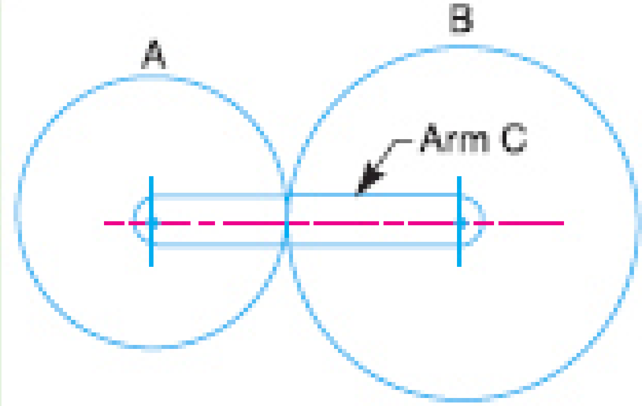
Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Tabular method.

- when gear *A makes + 1* revolution, then the gear *B will make* (– *T*A / *T*B) revolutions. This statement of relative motion is entered in the first row of the table.
- Secondly, if the gear *A makes* + *x revolutions, then the gear B will* make – *x* × *T*A / *T*B revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by *x.*
- Thirdly, each element of an epicyclic train is given + *y revolutions and entered in the third* row.
- Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*



Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$$\therefore \text{ Speed of gear } B, \quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$$

$$= 270 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig. 13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and

2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively.

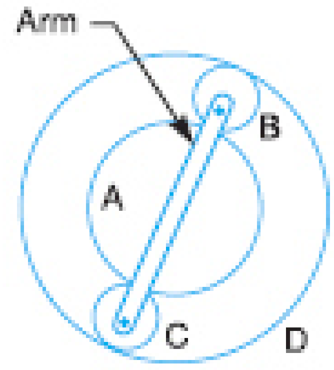


Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because T_B = T_C)$$

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear A rotates through $-$ 1 revolution (*i.e.* 1 rev. clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. ***Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise***

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(i)$$

Also, the gear *D* makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad ...(ii)$$

From equations (*i*) and (*ii*), $x = 1.04$ and $y = -0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise}$$ **Ans.**

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

2. ***Speed of arm when A makes 1 revolution clockwise and D is stationary***

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(iii)$$

Also the gear *D* is stationary, therefore

$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

$$\therefore \quad 40\,x-90\,y=0 \quad \text{or} \quad x-2.25\,y=0 \qquad ...(iv)$$

From equations *(iii)* and *(iv)*,

$$x=0.692 \quad \text{and} \quad y=0.308$$

$\therefore$ Speed of arm $= -y = -0.308 = 0.308$ revolution clockwise **Ans.**

References:

1. Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.
2. Theory of Machines, R S Kurmi, Eurasia Publishing House, 2005