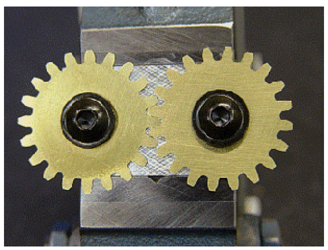
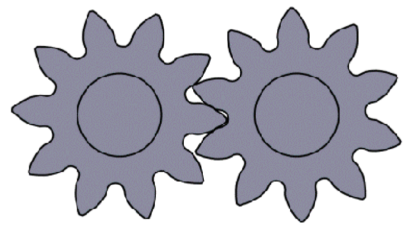
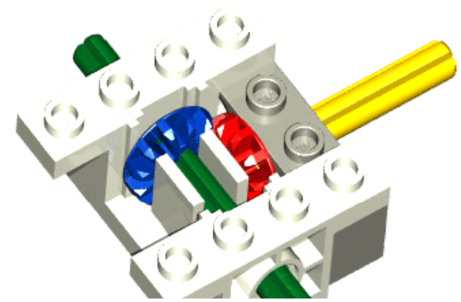
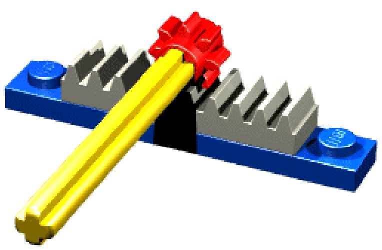
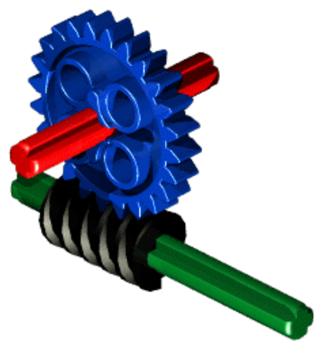
Gears

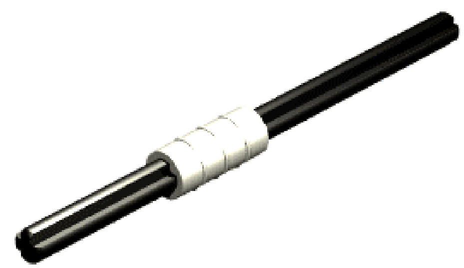












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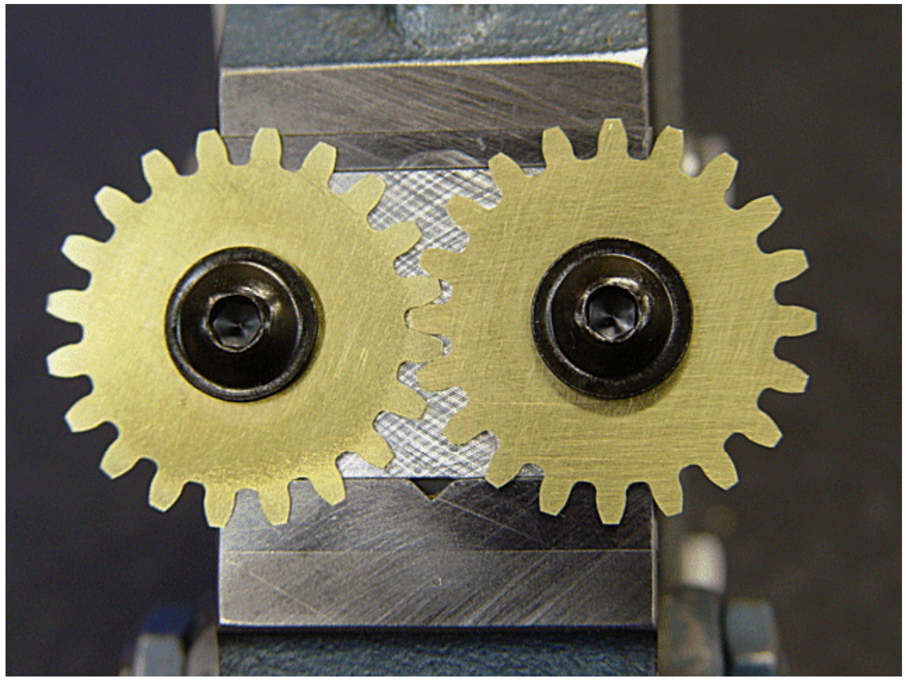
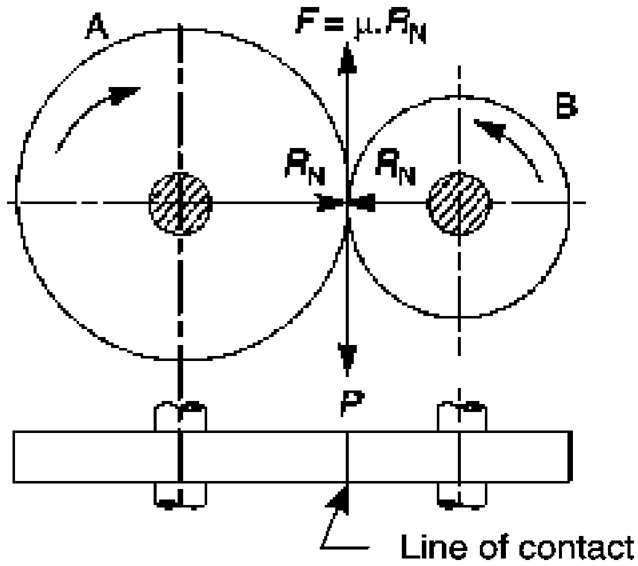
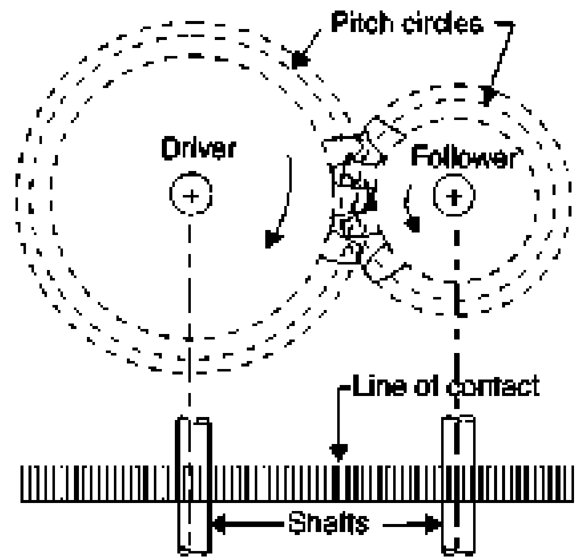
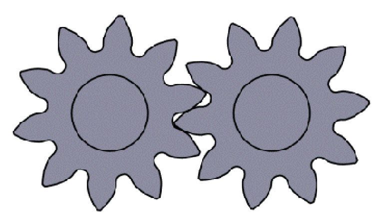
SPUR GEARS







(*b*) Toothed wheels.



TYPES OF GEARS

1. According to the position of axes of the shafts.

a. Parallel

   1.Spur Gear

   2.Helical Gear

   3.Rack and Pinion

b. Intersecting

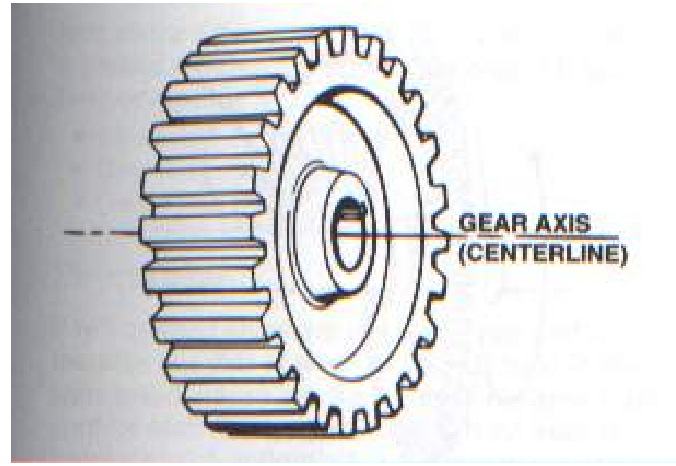
   Bevel Gear

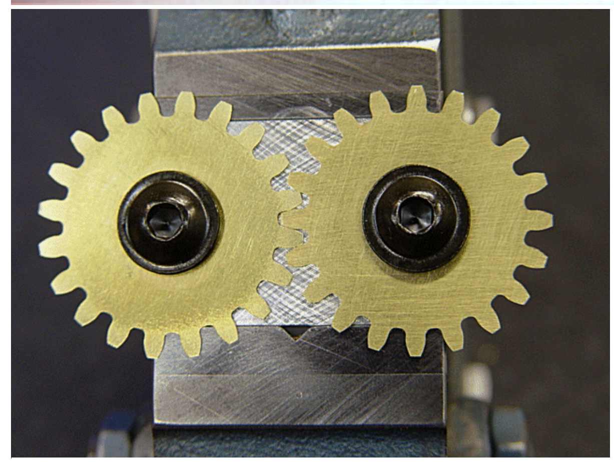
c. Non-intersecting and Non-parallel

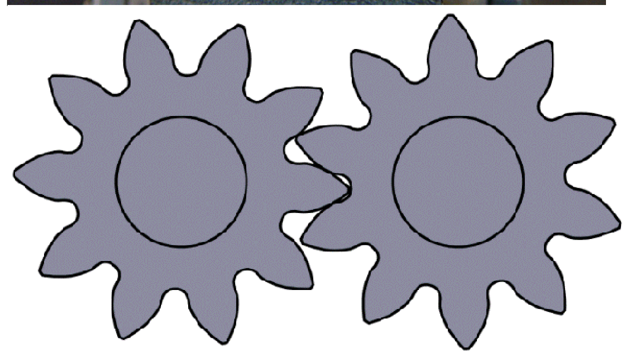
   worm and worm gears

SPUR GEAR

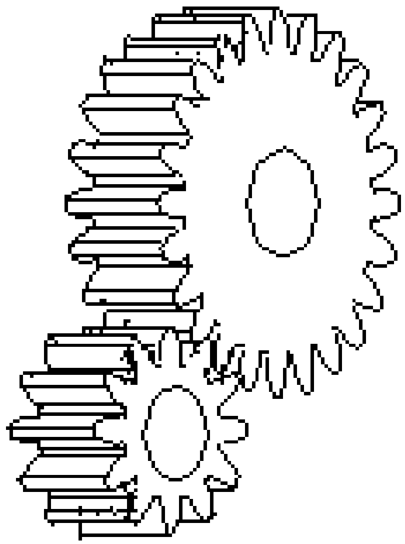
- **Teeth is parallel to axis of rotation**
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer

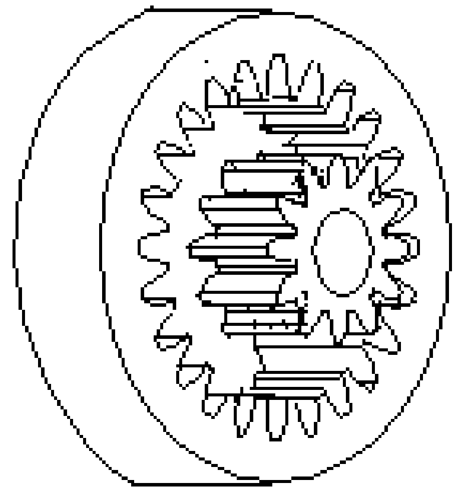






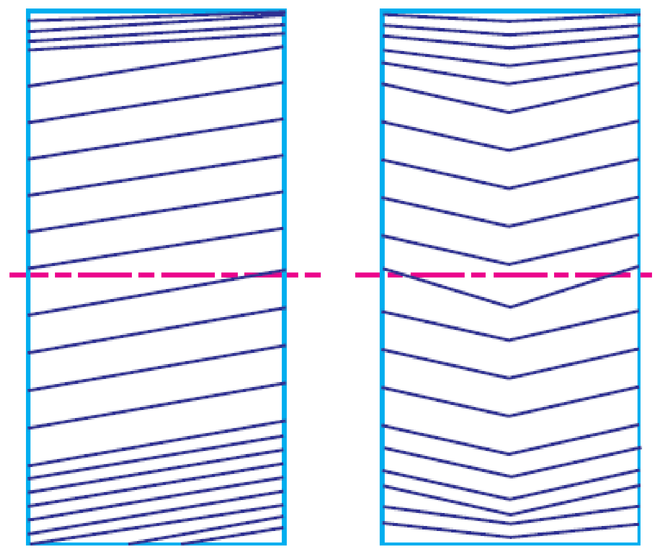
External and Internal spur Gear…





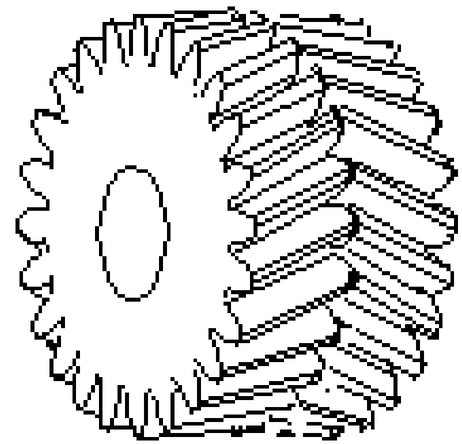
Helical Gear

- **The teeth on helical gears are cut at an angle to the face of the gear**
- This gradual engagement makes helical gears operate much more **smoothly and quietly** than spur gears
- One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle



Herringbone gears

- **To avoid axial thrust, two helical gears of opposite hand can be mounted side by side,** to cancel resulting thrust forces

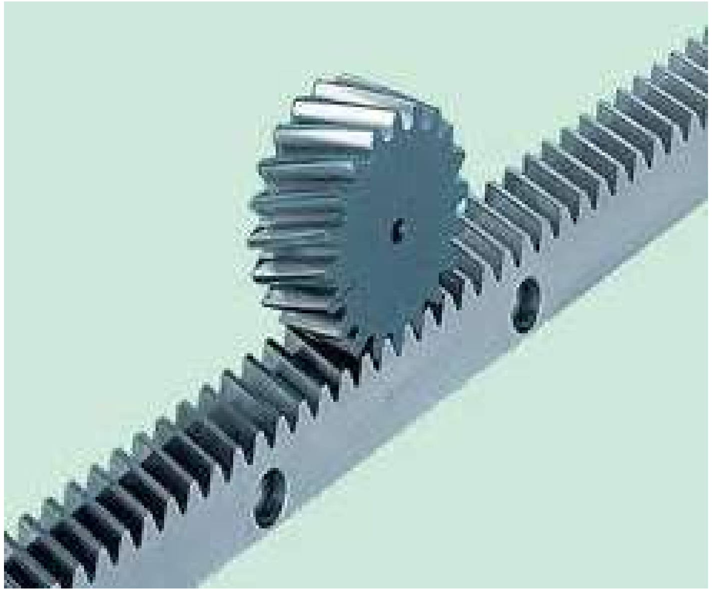


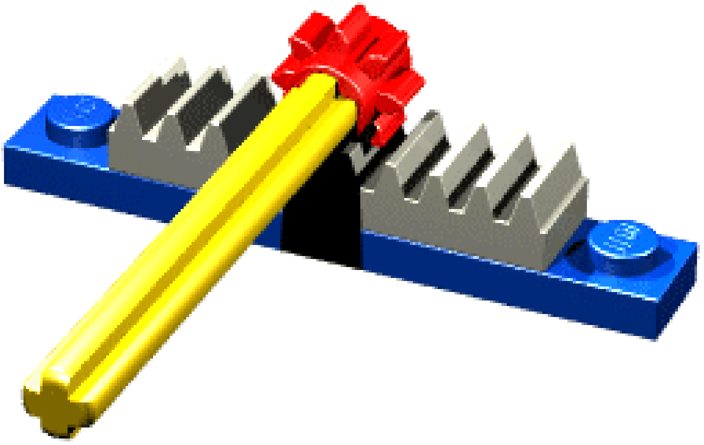
- Herringbone gears are mostly used on **heavy machinery.**

Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

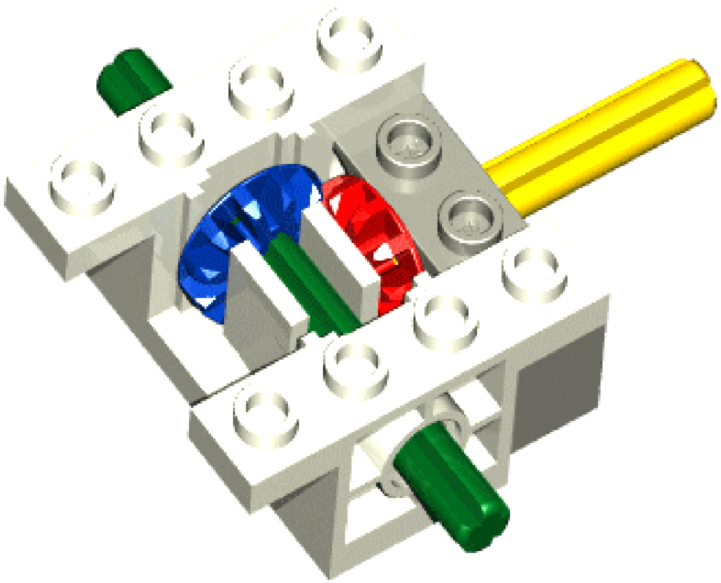
- A perfect example of this is the steering system on many cars

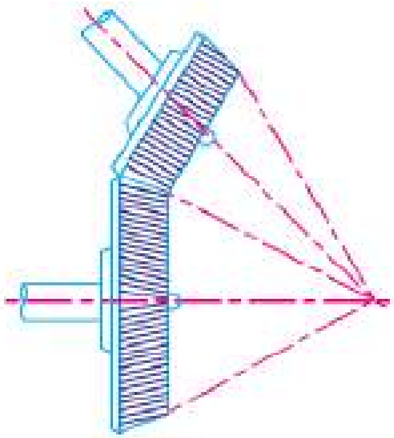




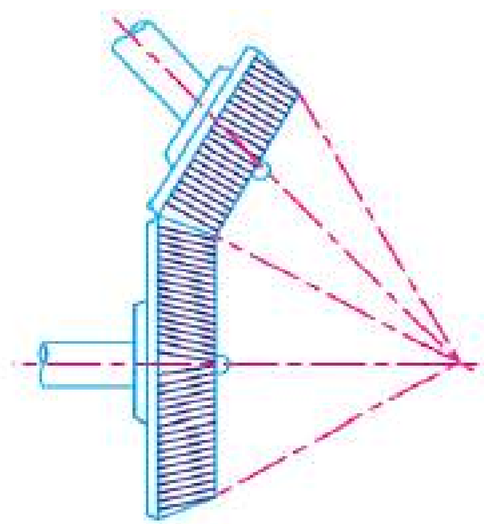
Bevel Gears

- **Bevel gears** are useful when the **direction of a shaft's rotation needs to be changed**
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral** or **hypoid**
- locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.



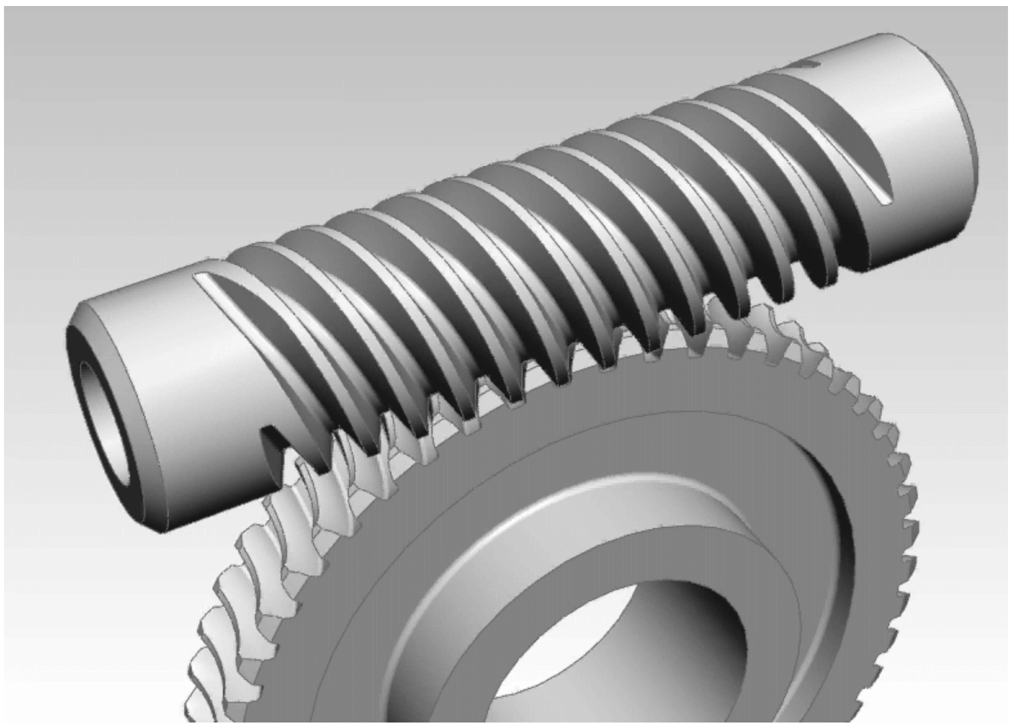


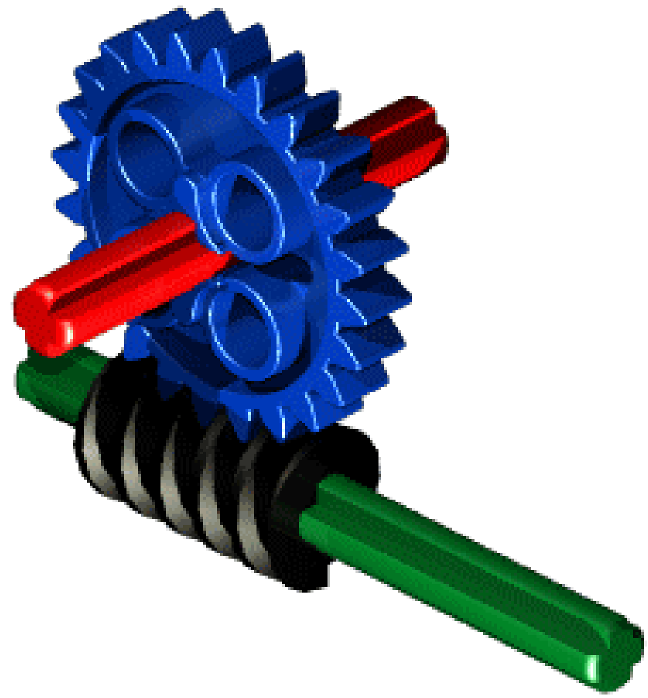
Straight and Spiral Bevel Gears



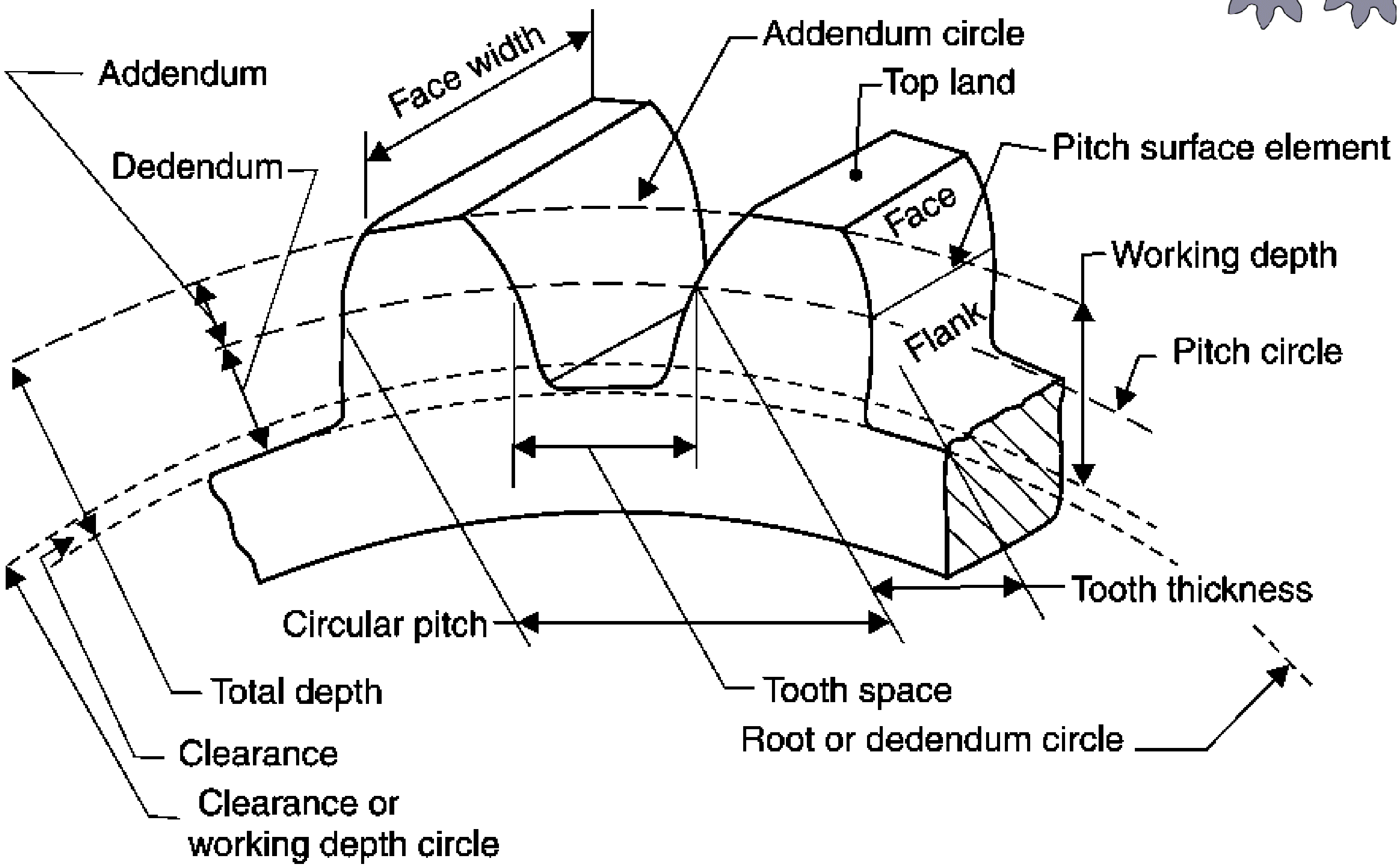
WORM AND WORM GEAR

- **Worm gears** are used when **large gear reductions** are needed. It is common for worm gears to have reductions of **20:1**, and even up to **300:1** or greater
- Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm
- Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc

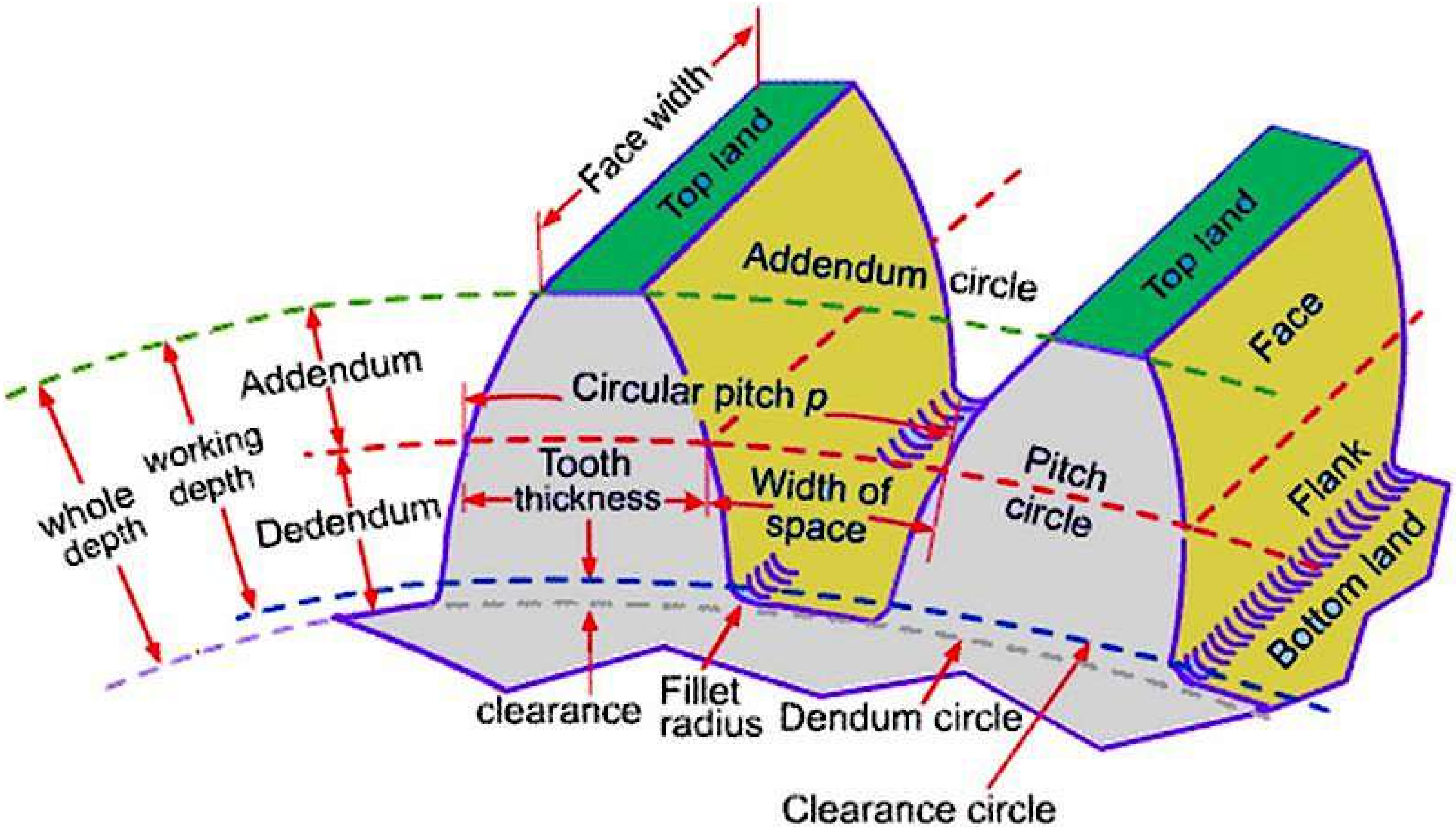




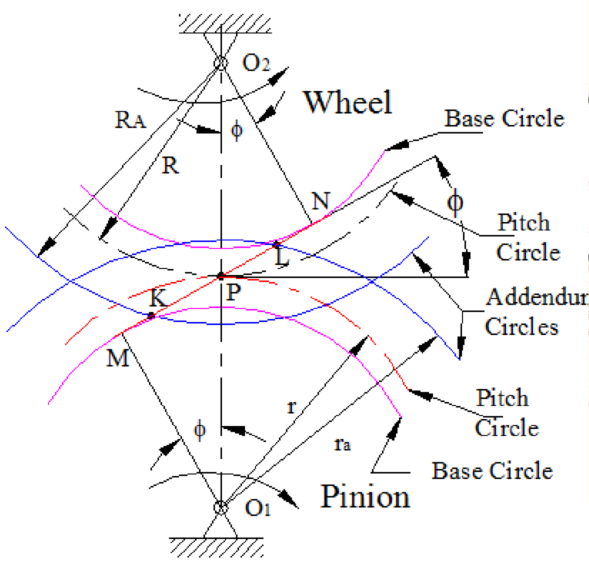
NOMENCLATURE OF SPUR GEARS

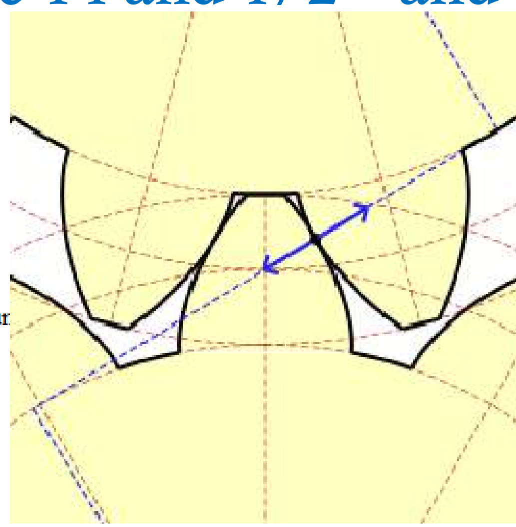


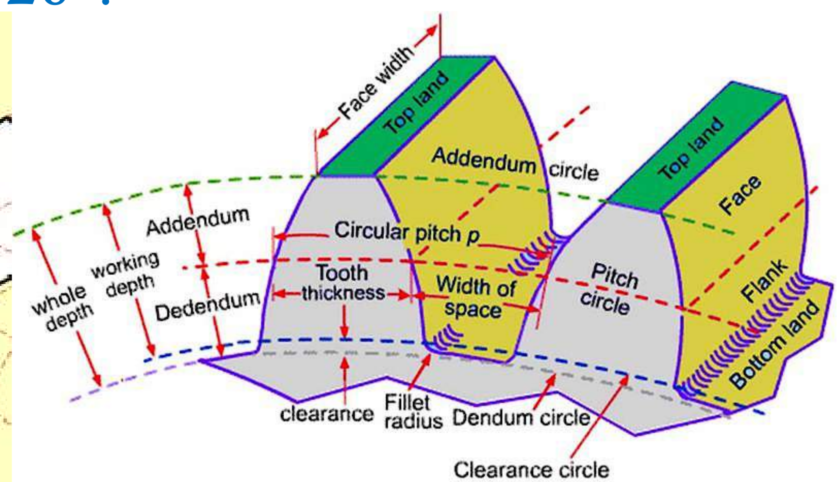
NOMENCLATURE OF SPUR GEARS



- ***Pitch circle:*** *It is an imaginary circle which by pure rolling action would give the same* motion as the actual gear.
- ***Pitch circle diameter:*** *It is the diameter of the pitch circle. The size of the gear is usually* specified by the pitch circle diameter. It is also known as *pitch diameter.*
- ***Pitch point:*** *It is a common point of contact between two pitch circles.*
- ***Pressure angle or angle of obliquity:*** *It is the angle between the common normal to two* gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are 14 and 1/2 ° and 20°.

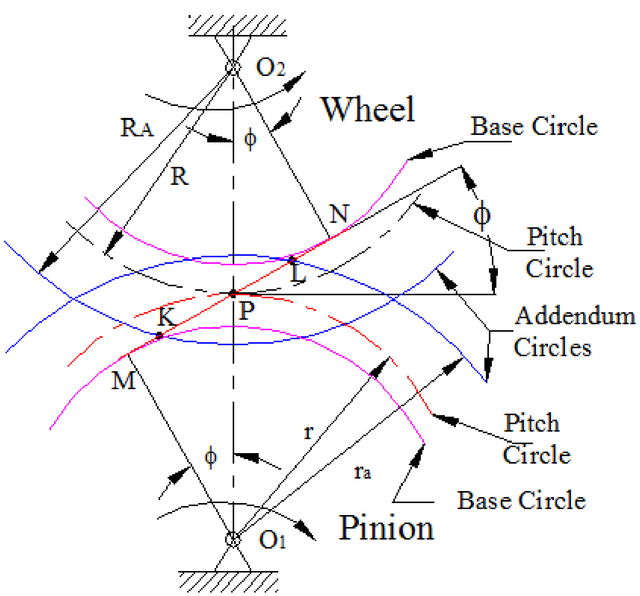


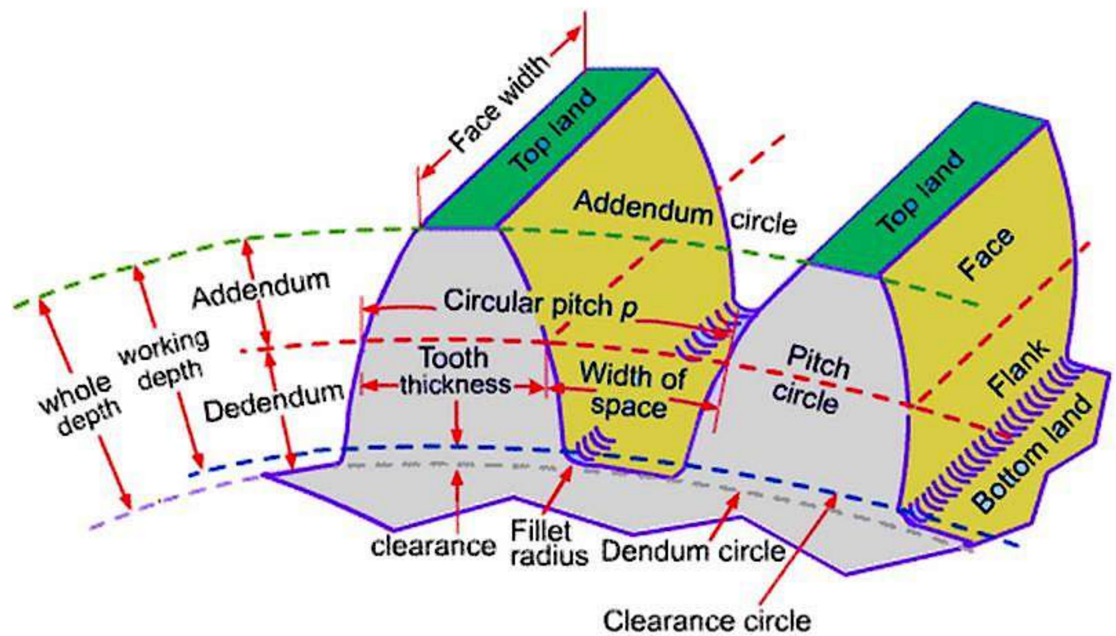


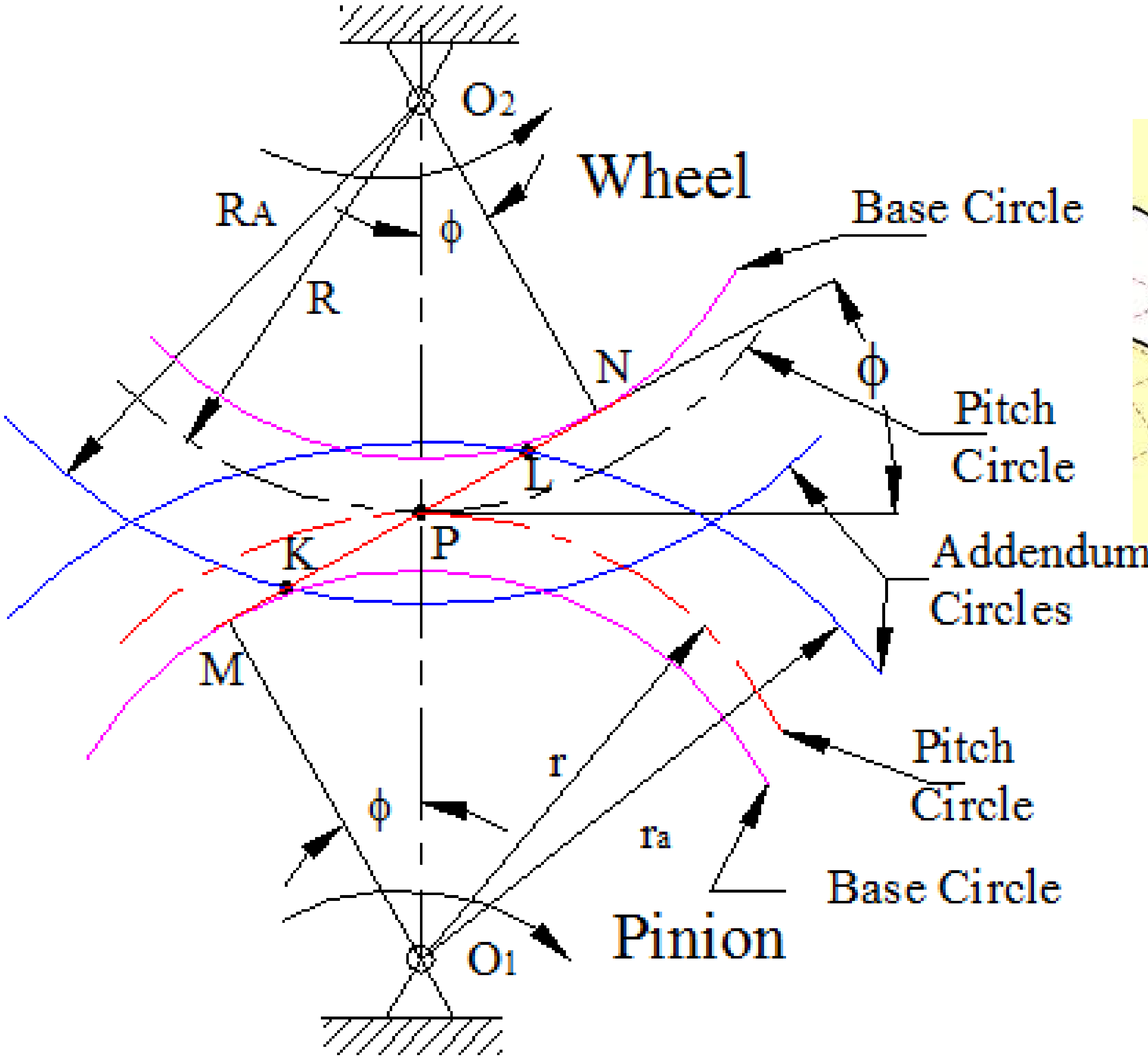


- ***Addendum:*** *It is the radial distance of a tooth from the pitch circle to the top of the tooth.*
- ***Dedendum:*** *It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.*
- ***Addendum circle:*** *It is the circle drawn through the top of the teeth and is concentric with* the pitch circle.
- ***Dedendum circle:*** *It is the circle drawn through the bottom of the teeth. It is also called* root circle.

Note : Root circle diameter






O2
Wheel
RA
R
ϕ
Base Circle
N
ϕ
Pitch
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L
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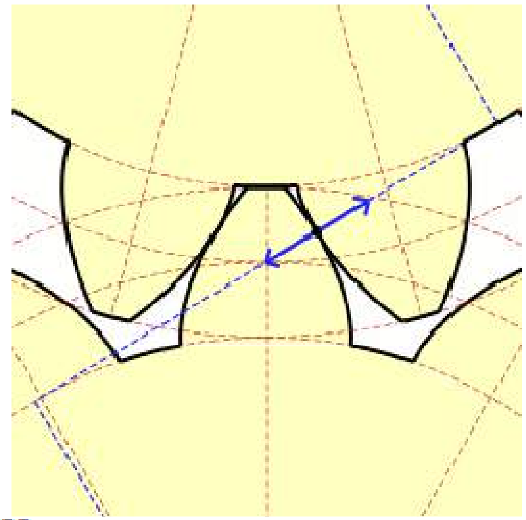
Circle
Base Circle
Pinion
O1



Circular pitch: *It is the distance measured on the circumference of the pitch circle from* a point of one tooth to the corresponding point on the next tooth. It is usually denoted by P_c ,Mathematically,

Circular pitch, $p_c = \pi D/T$

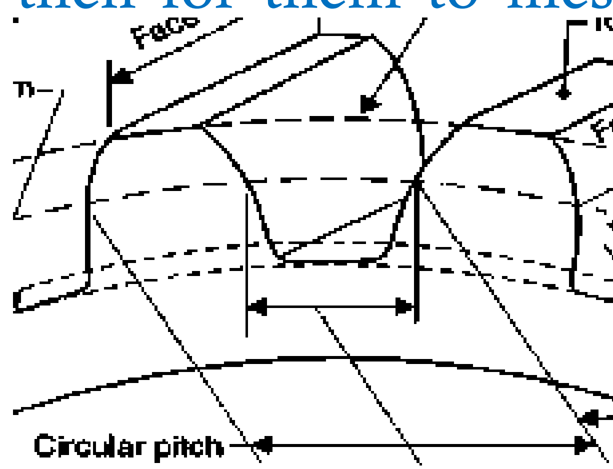
where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the **two gears will mesh together correctly, if the two wheels have the same circular pitch.**

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$



Diametral pitch: *It is the ratio of number of teeth to the pitch circle diameter in millimetres.* It is denoted by p_d. Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because\ p_c = \frac{\pi D}{T} \right)$$

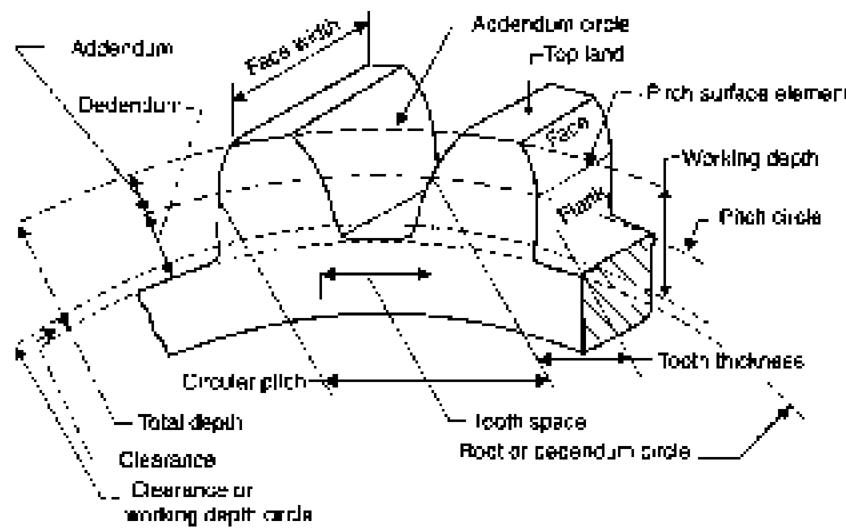
Module: *It is the ratio of the pitch circle diameter in millimeters to the number of teeth.* It is usually denoted by *m.*

Mathematically, $\text{Module, } m = D/T$

Clearance: *It is the radial distance from the top of the tooth to the bottom of the tooth, in* a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

Total depth: *It is the radial distance between the addendum and the dedendum circles of* a gear.

It is equal to the sum of the addendum and dedendum.

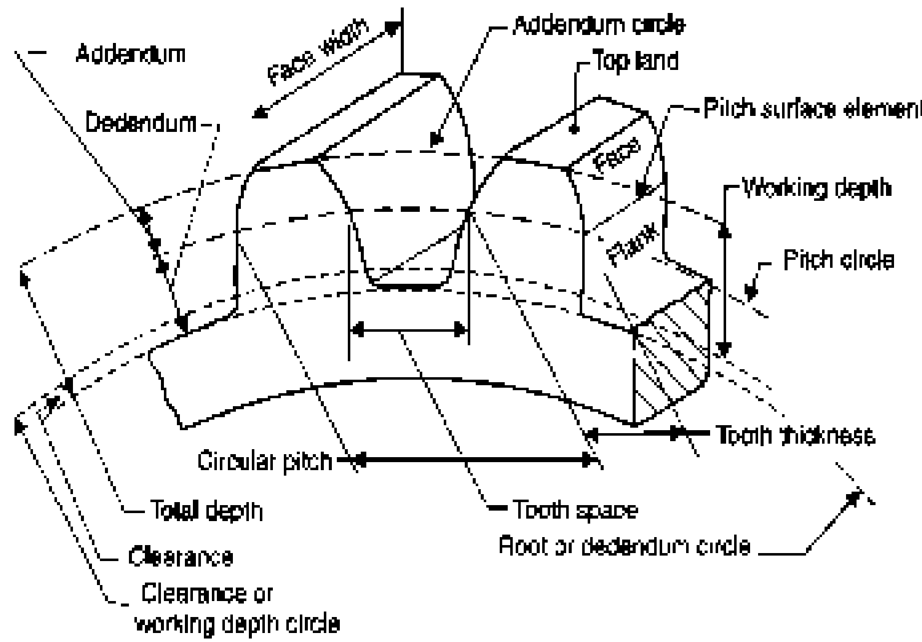


Working depth: *It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.*

Tooth thickness: *It is the width of the tooth measured along the pitch circle.*

Tooth space: *It is the width of space between the two adjacent teeth measured along the pitch circle.*

Backlash: *It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.*



Face of tooth: *It is the surface of the gear tooth above the pitch surface.*

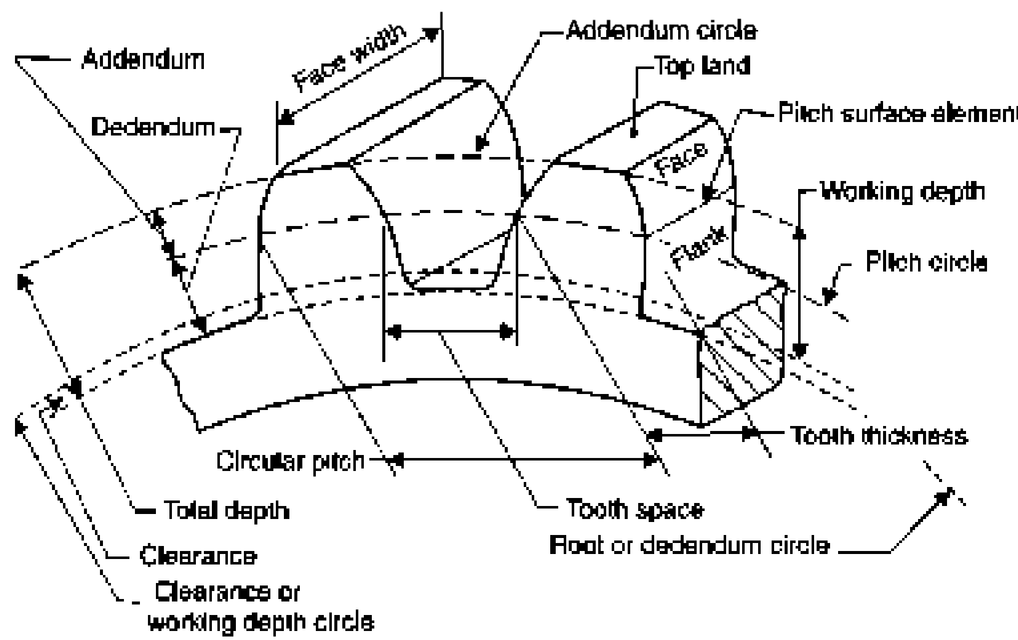
Flank of tooth: *It is the surface of the gear tooth below the pitch surface.*

Top land: *It is the surface of the top of the tooth.*

Face width: *It is the width of the gear tooth measured parallel to its axis.*

Profile: *It is the curve formed by the face and flank of the tooth.*

Fillet radius: *It is the radius that connects the root circle to the profile of the tooth.*



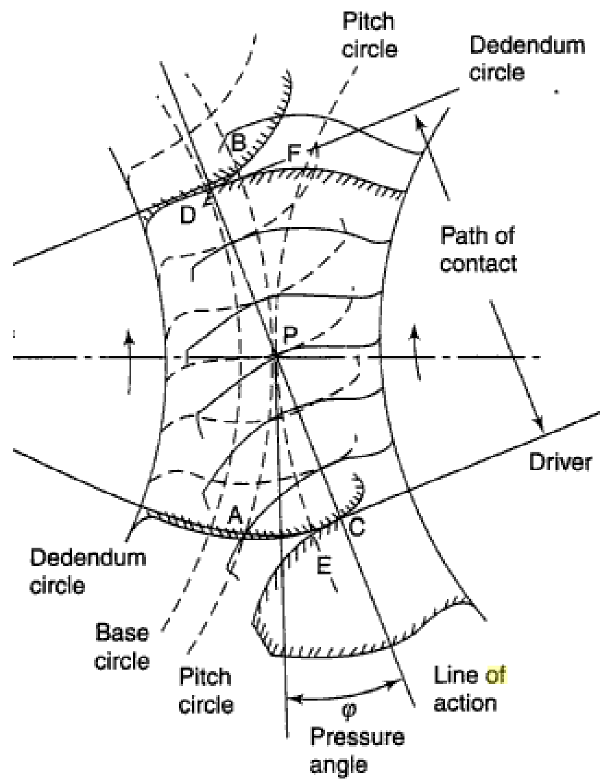
Path of contact: *It is the path traced by the point of contact of two teeth from the* beginning to the end of engagement.

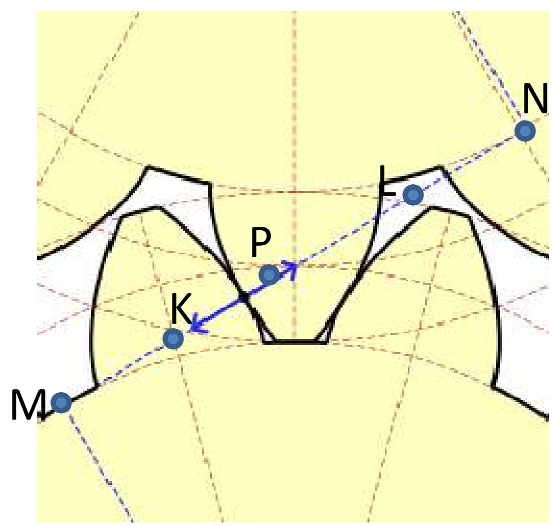
Length of the path of contact: *It is the length of the common normal cut-off by the* addendum circles of the wheel and pinion.

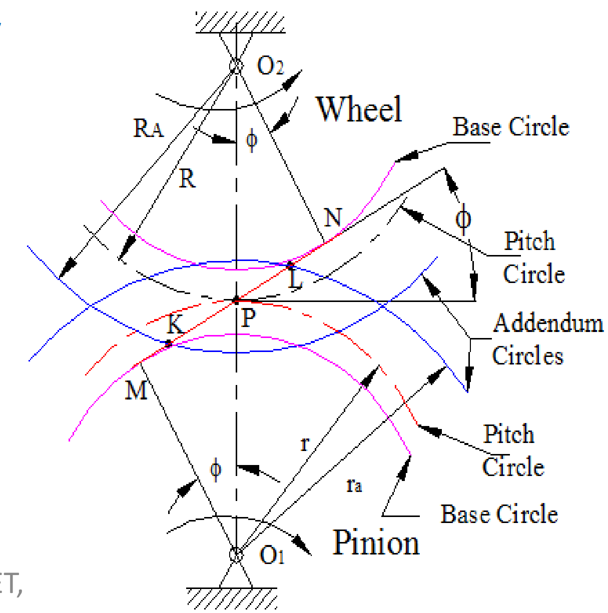
Arc of contact: *It is the path traced by a point on the pitch circle from the beginning* to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(a) Arc of approach. *It is the portion of the arc of contact from the beginning of the* engagement to the pitch point.

(b) Arc of recess: *It is the portion of the arc of contact from the pitch point to the end of the* engagement of a pair of teeth.







Forms of Teeth

- In actual practice following are the two types of teeth commonly used

  1. *Cycloidal teeth ; and 2. Involute teeth.*

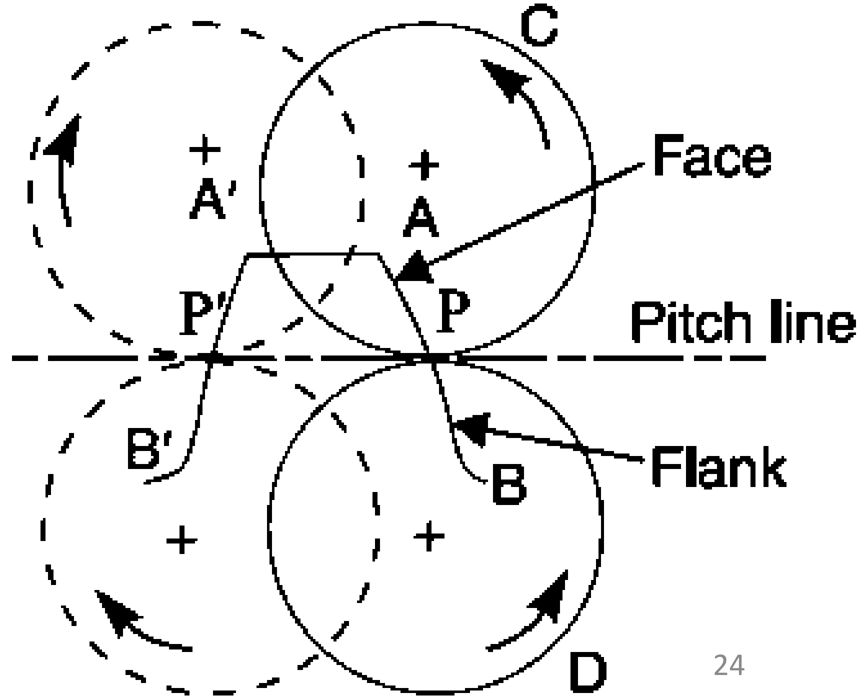
Cycloidal Teeth

- A *cycloid is the curve traced by a point on the circumference of a circle which rolls without* slipping on a fixed straight line.
- When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***.
- *On the other* hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid*.**

Construction of cycloidal teeth for Rack

- *In Fig. (a), the fixed line or pitch line of a rack is shown. When the circle C rolls without slipping above the pitch line in the direction as indicated in Fig, then the point P on the circle traces epi-cycloid PA. This represents the face of the cycloidal tooth profile.*
- *When the circle D rolls without slipping below the pitch line, then the point P on the circle D traces hypo-cycloid PB, which represents the flank of the cycloidal tooth. The profile BPA is one side of the cycloidal rack tooth.*

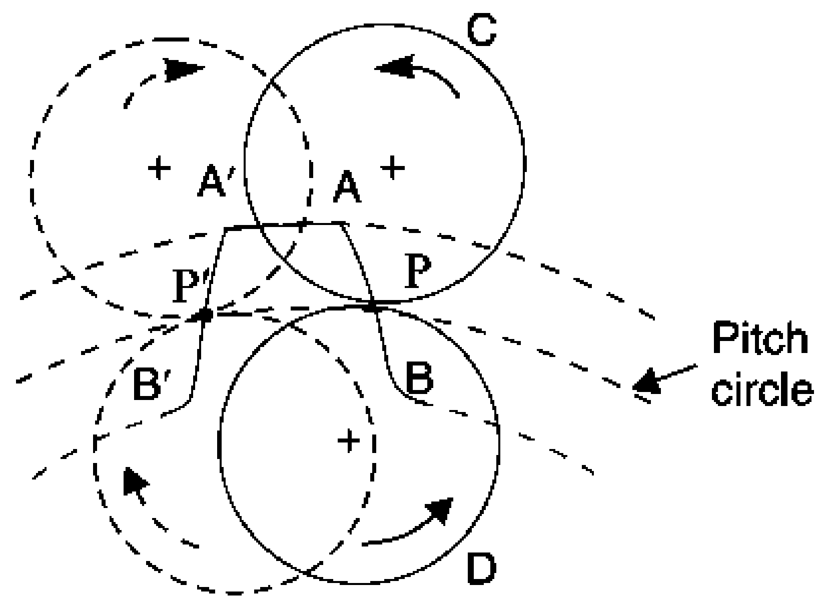
Similarly, the two curves P' A' and P'B' forming the opposite side of the tooth profile are traced by the point *P' when the circles C and D roll in the opposite directions.*



Construction of cycloidal teeth for gear

- The cycloidal teeth of a gear may be constructed as shown in Fig.
- The circle *C is rolled without slipping on the outside of the pitch circle and the point P on the circle C traces epi-cycloid PA, which represents the face of the cycloidal tooth.*
- *The circle D is rolled on the* inside of pitch circle and the point *P on the circle D traces hypo-cycloid PB, which represents the* flank of the tooth profile.

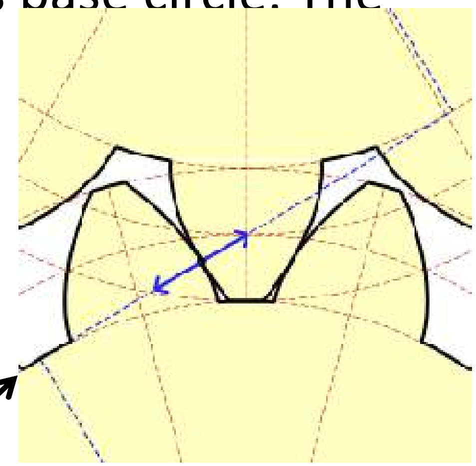
The profile *BPA is one side of the cycloidal tooth. The opposite side of the* tooth is traced as explained above.



(b)

Involute Teeth

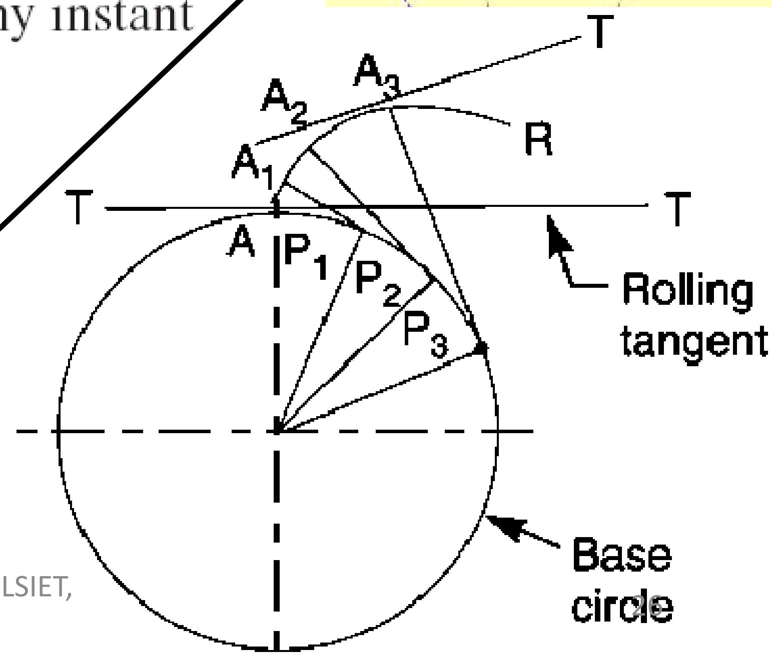
- An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping as shown in Fig.
- In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :



Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant

A_3, the tangent A_3T *to the involute is perpendicular to* P_3A_3 and P_3A_3 is the normal to the involute.

In other words, ***normal at any point of an involute is a tangent to the base circle.***



Comparison Between Involute and Cycloidal Gears

- In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears

- The most important advantage of the involute gears is that **the centre distance for a pair of involute gears can be varied within limits** without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.
- **In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant**. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

- The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (*i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively.* Thus the **involute teeth are easy to manufacture than cycloidal teeth**. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.
- *Note :* ***The only disadvantage of the involute teeth is that the interference occurs with pinions having smaller number of teeth****. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.*

Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

- Since the cycloidal teeth have wider flanks, therefore the **cycloidal gears are stronger than the involute gears**, for the same pitch.
- In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in **less wear in cycloidal gears as compared to involute gears**. However the difference in wear is negligible.
- **In cycloidal gears, the interference does not occur at all**. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

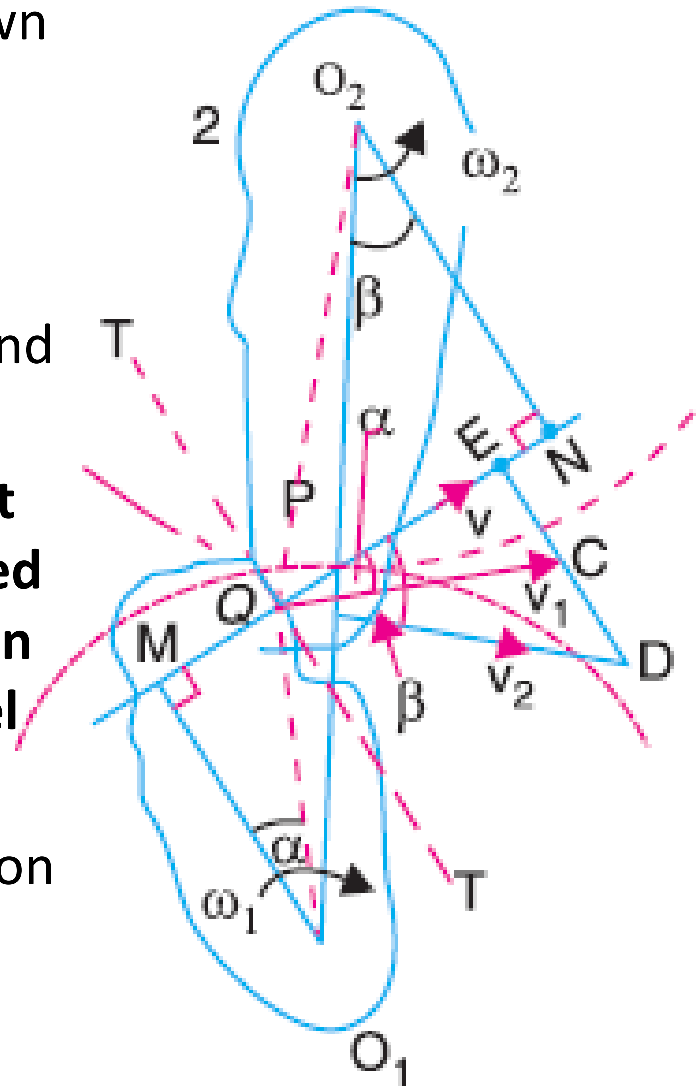
- **Statement :**

- The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant velocity ratio between two gears.
- The condition is that " For constant velocity ratio of the two gears, the common normal at the point of contact the two mating teeth must pass through the pitch point.

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

- Let the two teeth come in contact at point Q, and the wheel rotates in the direction as shown in fig.
- **Let , TT = common tangent**

  MN= common normal
- Now, from the centers O_1 and O_2, draw O_1M and O_2N perpendicular to MN.
- **A little consideration will show that the point Q moves in the direction QC, when considered as a point on the wheel 1, and in the direction QD, when considered as a point on the wheel 2.**
- Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2. If the velocities along the common normal MN must be equal.



Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Therefore, from fig.,

$$\therefore v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

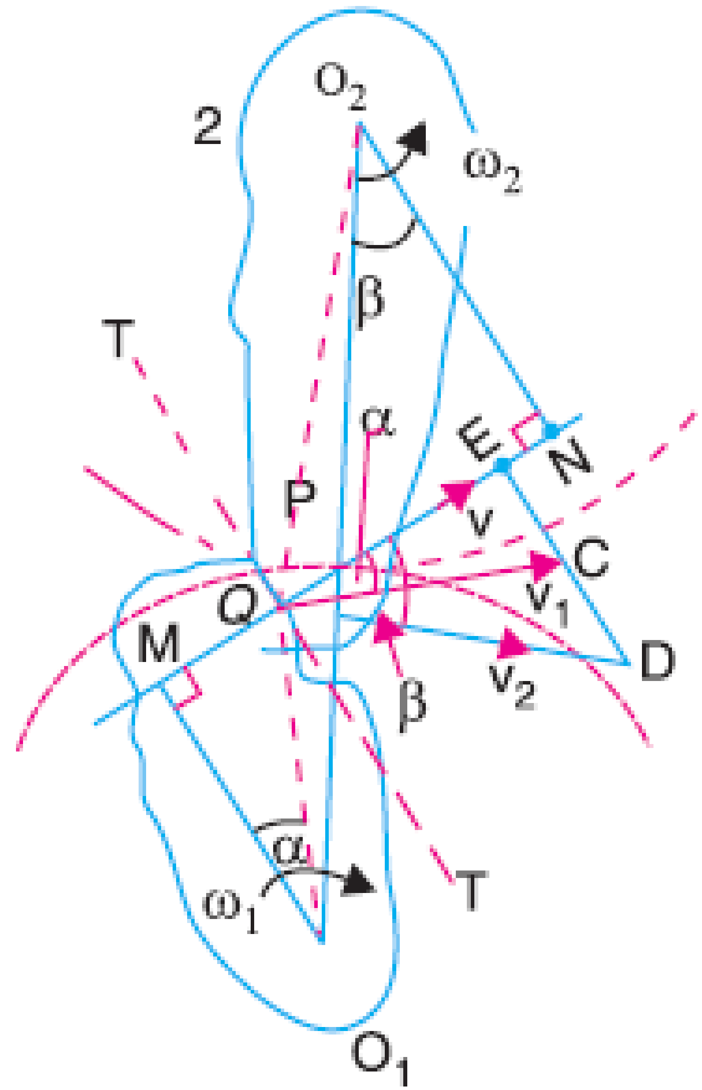
$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q}$$

$$(\omega_1 \times O_1M) = (\omega_2 \times O_2N)$$

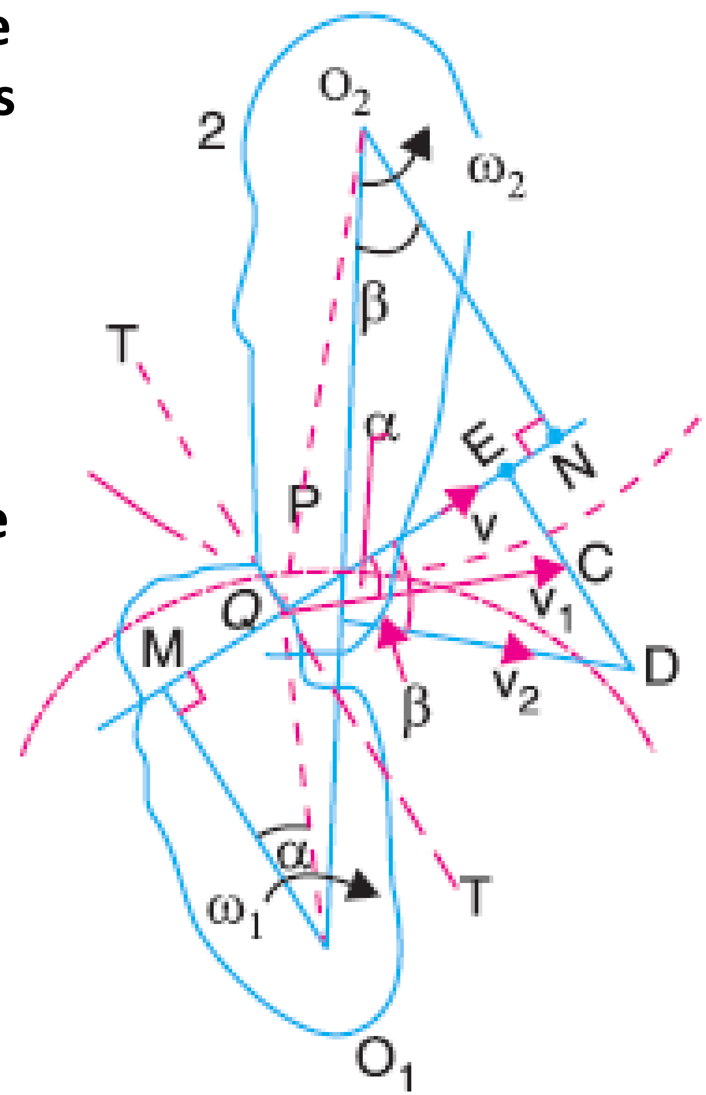
$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

As similar $\triangle O_1MP$ and $\triangle O_2NP$,

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$



Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing



- From above, we see that the angular velocity ratio is inversely proportional to the ratio of the two surfaces at the point of contact Q intersects the line of centers at point P which divides the centre distance inversely as the ratio of angular velocities.

- Therefore in order to have constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point for the two wheels.

- In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point.

- It is known as law of gearing.

VELOCITY OF SLIDING

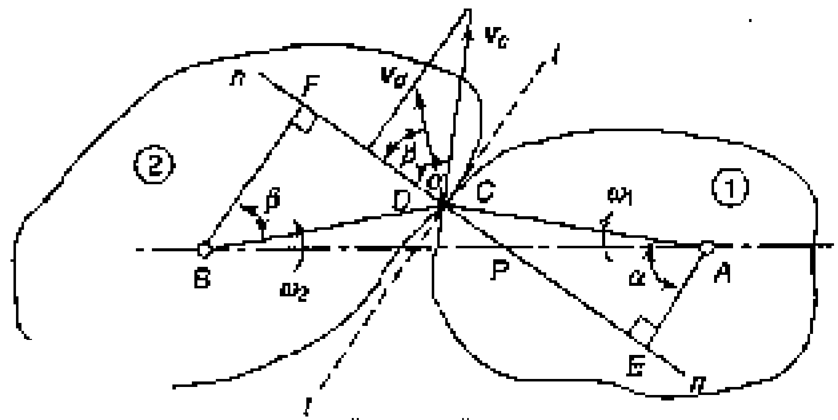
If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent t-t at C or *D.*

Component of v_2 along t - t = $v_2 \sin\alpha$

Component of v_1 along t - t= $v_1 \sin\beta$

Relative motion along $n - n$

$$= v_2 \sin\alpha - v_1 \sin\beta$$



AC and $v_d = \omega_2$ BD

$$= \omega_1 AC\frac{EC}{AC} - \omega_2 BD\frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1(EP + PC) - \omega_2(FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

(*C* and *D* are the coinciding points)

$$= (\omega_1 + \omega_2)PC + \omega_1 EP - \omega_2 FP$$

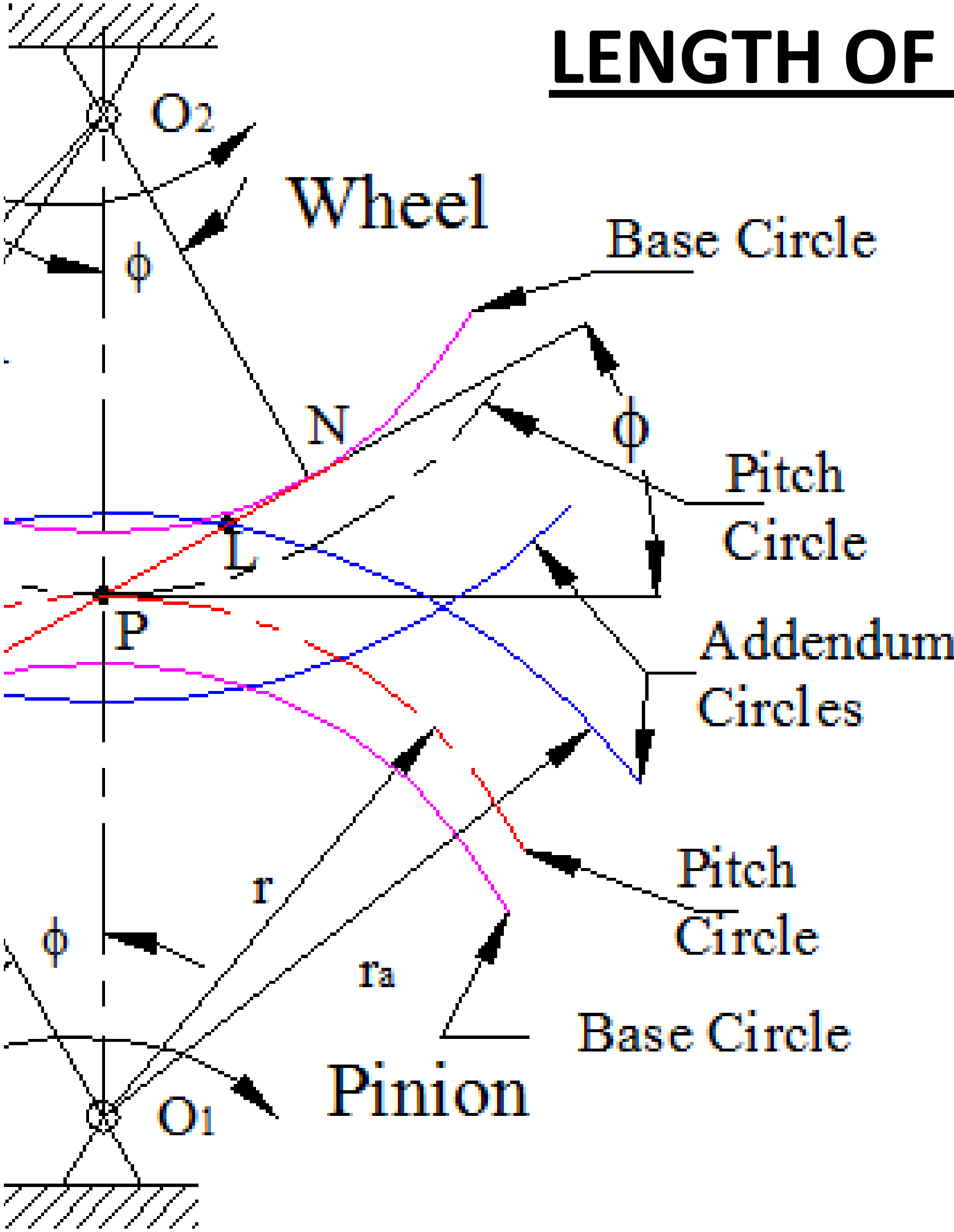
$$= (\omega_1 + \omega_2)PC \qquad [\omega_1 EP = \omega_2 FP,$$

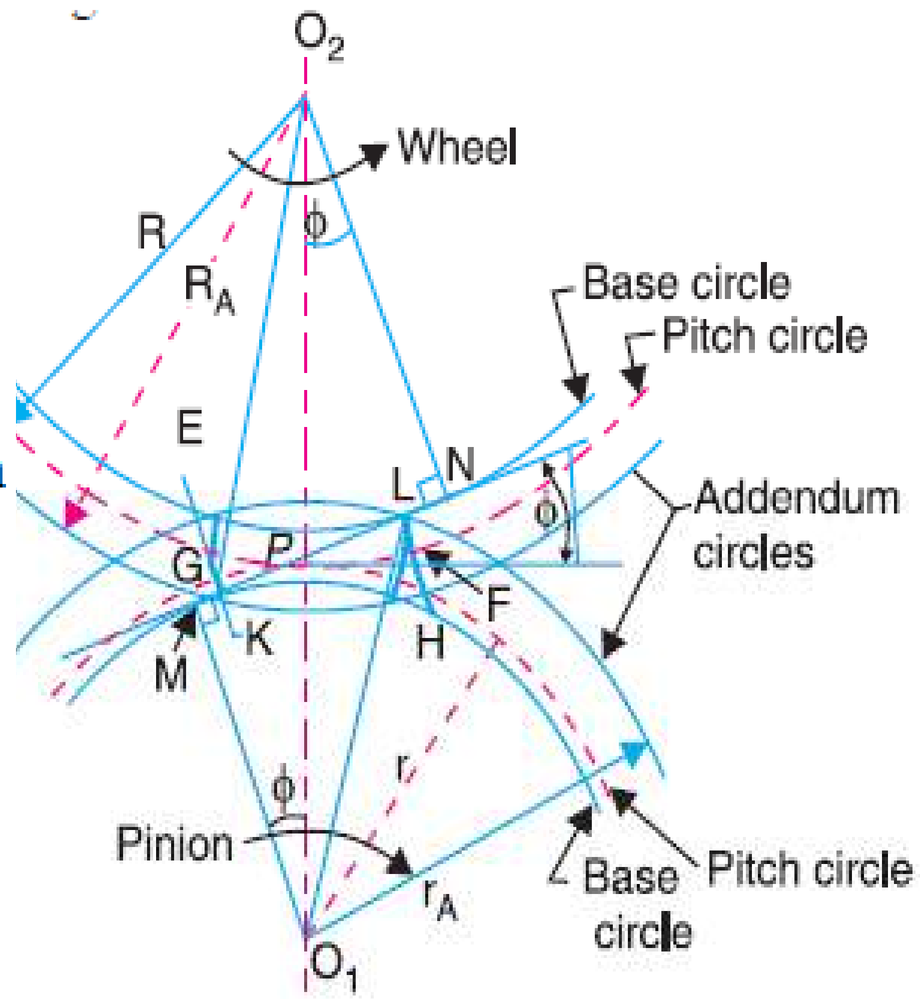
= sum of angular velocities × distance between the pitch point and the point of contact

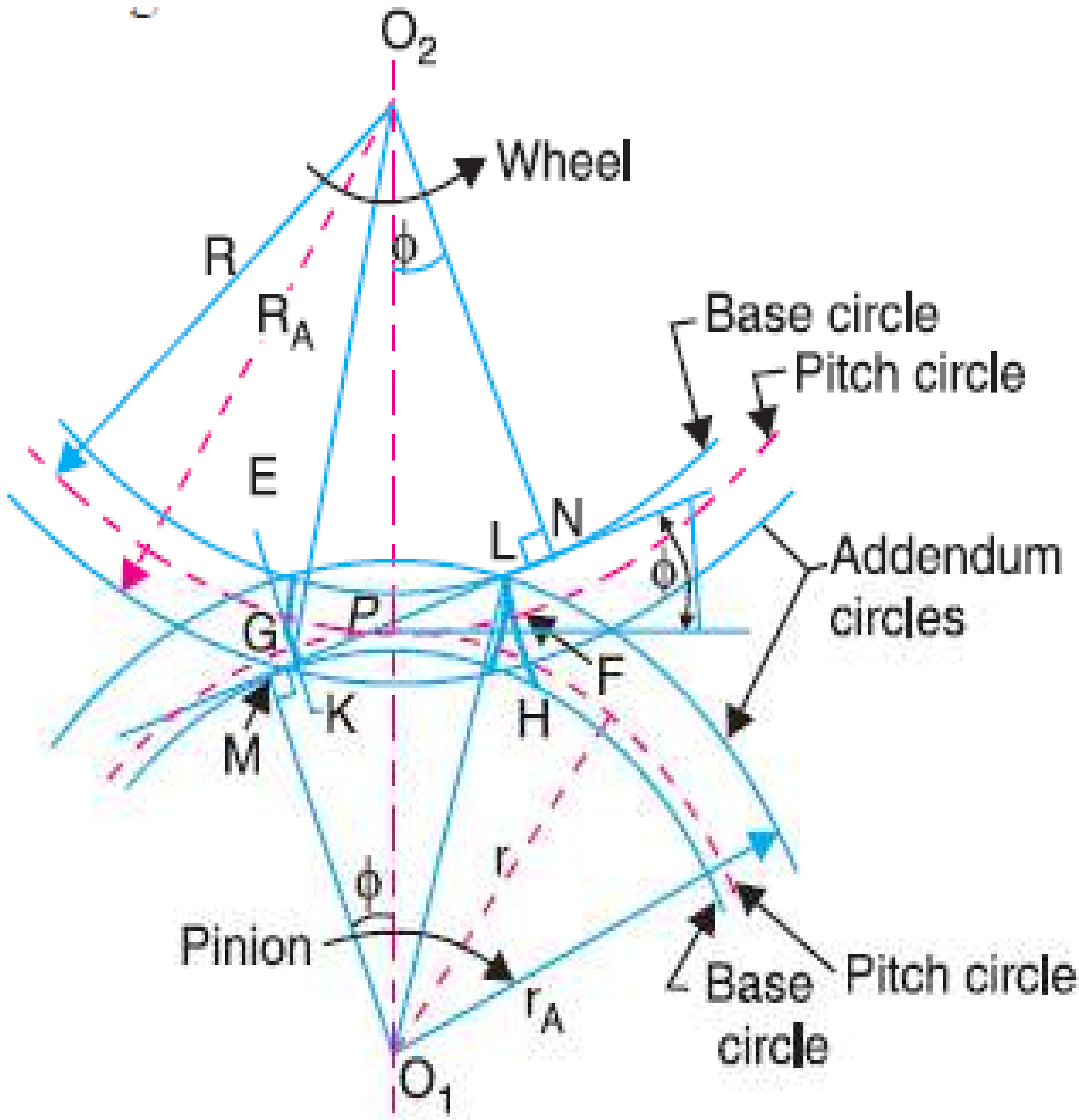
$$\frac{BP}{AP} = \frac{FP}{EP}$$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP} = \frac{FP}{EP}$$

LENGTH OF PATH OF CONTACT



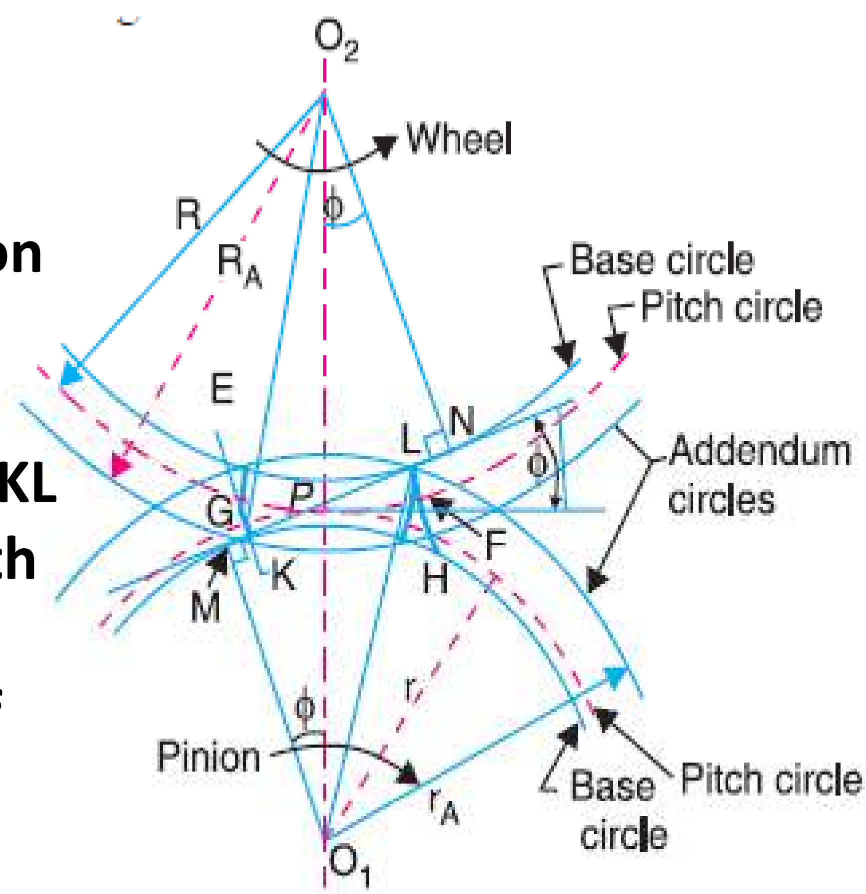



O2
Wheel
R
RA
ϕ
Base circle
Pitch circle
E
L
N
ϕ
Addendum circles
G
P
F
K
H
M
r
ϕ
Pinion
rA
Base circle
Pitch circle
O1

•MN is the common normal at the point of contact and the common tangent to the base circles.

•The point K is the intersection of the addendum circle of the wheel and the common tangent. The point L is the addendum circle of pinion and common tangent.

•Thus, the length of path of contact is KL which is the sum of the part of the path of contact KP and PL. The part of the path of contact KP is known as path of approach and the part of the path of contact PL is known as path of recess.



Let,

$$r_A = O_1L = \text{Radius of addeendum circle of pinion,}$$

$$R_A = O_2K = \text{Radius of addeendum circle of wheel,}$$

$$r = O_1P = \text{Radius of pitch circle of pinion,}$$

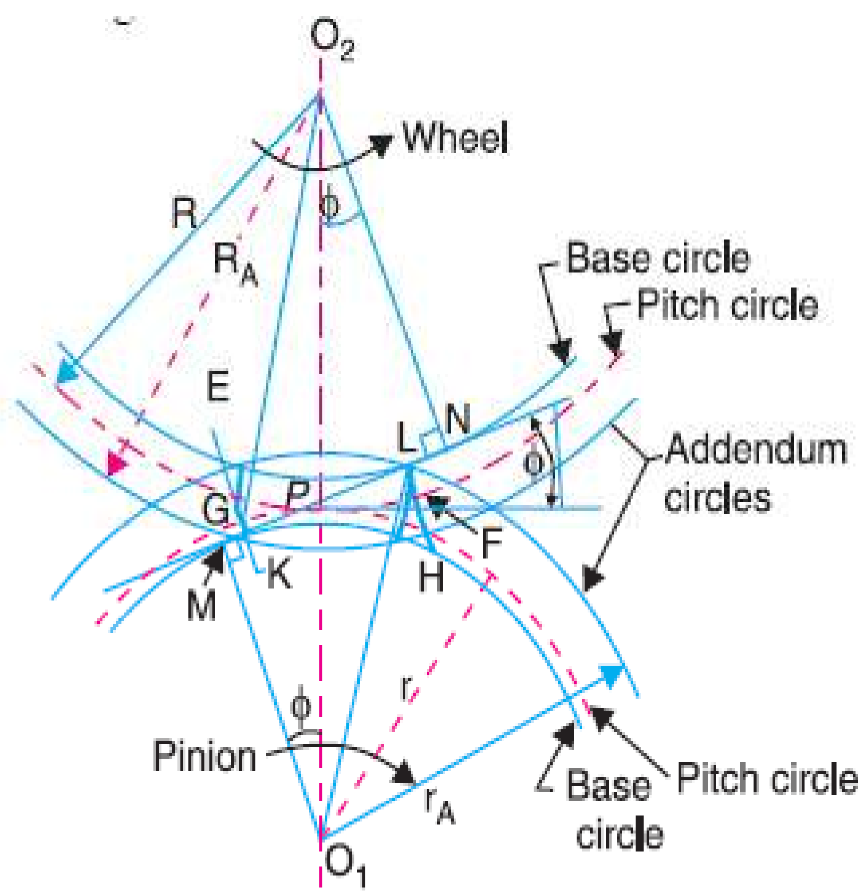
$$R = O_2p = \text{Radius of pitch circle of wheel,}$$

From the fig. radius of the base circle of pini

$$O_1M = O_1P\cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$



Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 + (R\cos\phi)^2}$$

and $\quad PN = O_2P\sin\phi = R\sin\phi$

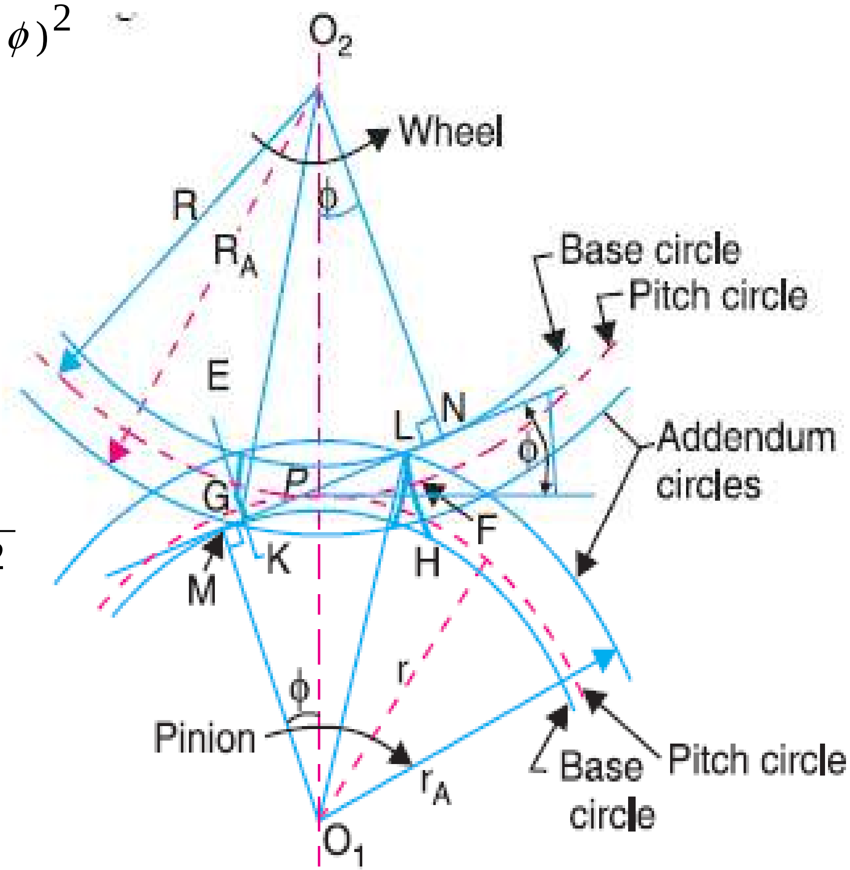
$\therefore$ The path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - (R\cos\phi)^2} - R\sin\phi$$

Similarly from right angled triangles O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - (r\cos\phi)^2}$$

$$MP = O_1P\sin\phi = r\sin\phi$$



$\therefore$ The path of recess,

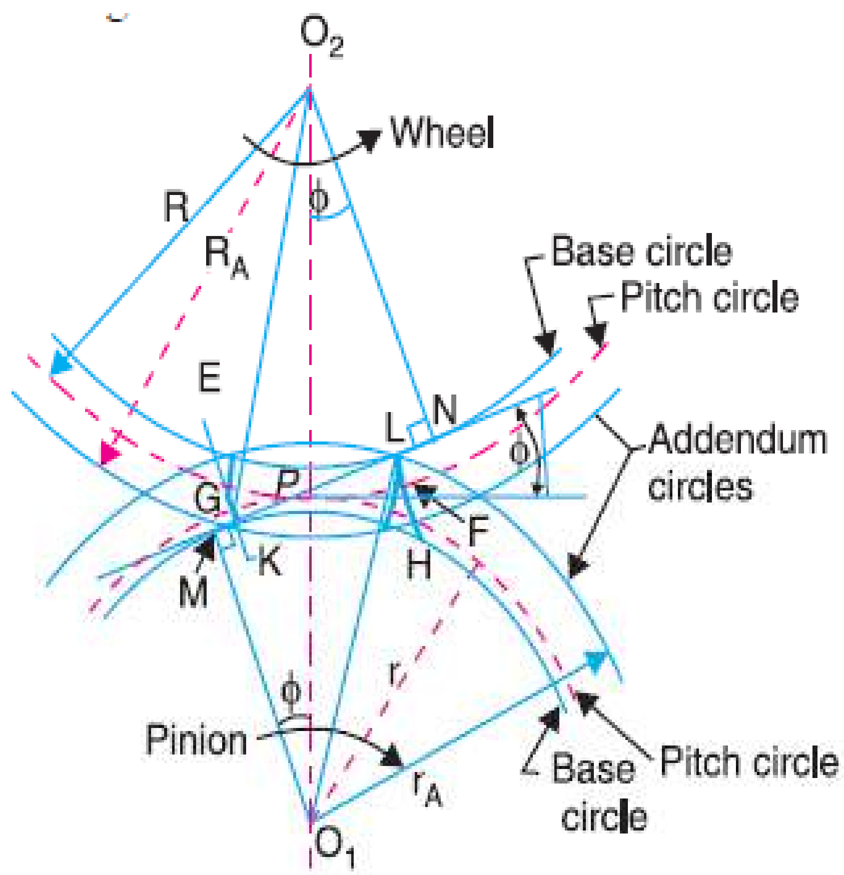
$$PL = ML - MP = \sqrt{(r_A)^2 - (r\cos\phi)^2} - r\sin\phi$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - (R\cos\phi)^2} + \sqrt{(r_A)^2 - (r\cos\phi)^2} - (R + r)\sin\phi$$

LENGTH OF ARC OF CONTACT

- The arc of contact is the path traced by a point on the pitch circle from the beginning of to the end of engagement of a given pair of teeth.
- The arc of contact is GPH, where GP is arc of approach and PH is arc of recess.



We know that,

length of arc of contact(arc GP)

$$= \frac{\text{length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

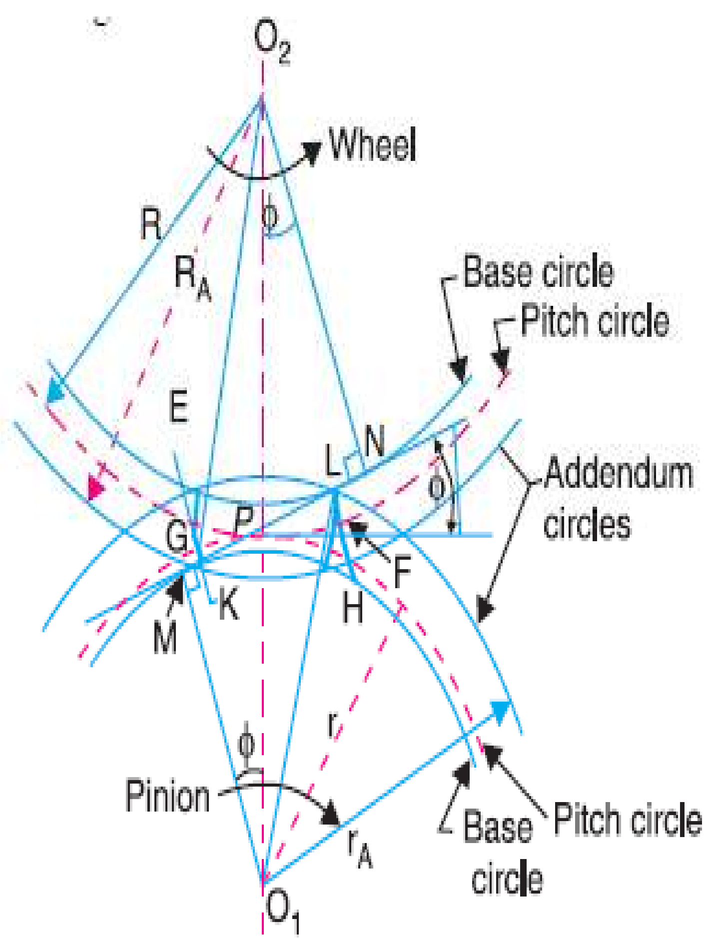
and length of arc of recess(arc PH)

$$= \frac{\text{length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Therefore, length of arc of contact

$$= \text{arc GP} + \text{arc PH} = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{length of path of contact}}{\cos\phi}$$



NUMBER OF PAIRS OF TEETH IN CONTACT (CONTACT RATIO)

- **The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth.**
- **Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other wheel.**

Therefore, the number of teeth in contact =

$$\frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{path of contact}}{\cos\phi} \times \frac{1}{p_c} \quad \text{Where } P_c = \pi D/T$$

- **As the ratio of the arc of contact to the circular pitch is also the *contact ratio,* the number of teeth is also expressed in terms of contact ratio.**
- **For continuous transmission of motion, at least one tooth of one wheel must be in contact with another tooth of the second wheel.**
- **Therefore, *n* must be greater than unity.**

Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Solution $\varphi = 20°;\ t = T = 48;\ m = 8$ mm;

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192 \text{ mm};\ R_a = r_a$$

Arc of contact = 2.25 × Circular pitch = 2.25π m = $2.25\pi \times 8$ = 56.55 mm

$$p_c = \pi D/T$$

Path of contact = 56.55 × cos 20° = 53.14 mm

or $$\left(\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right) = 53.14$$

or $$2\left(\sqrt{R_a^2 - 192^2\cos^2 20^\circ} - 192\sin 20^\circ\right) = 53.14 \quad \text{or} \quad R_a = 202.6 \text{ mm}$$

Addendum = $R_a - R$ = 202.6 – 192 = 10.6 mm

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

Solution. Given : $t = 30$; $T = 80$; $\phi = 20°$; $m = 12$ mm ; Addendum = 10 mm

Length of path of contact

We know that pitch circle radius of pinion,

$$r = m.t/2 = 12 \times 30/2 = 180 \text{ mm}$$

and pitch circle radius of gear,

$$R = m.T/2 = 12 \times 80/2 = 480 \text{ mm}$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{ Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20° = 191.5 - 164.2 = 27.3 \text{ mm}$$

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20°} - 180 \sin 20° = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{52.3}{\cos 20°} = 55.66 \text{ mm } \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi . m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5$$

Two involute gears in mesh have 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of the pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity of sliding.

Solution $\varphi = 20°; t = 24; m = 6$ mm;

$T = 24 \times 3 = 72;$

$$r = \frac{mt}{2} = \frac{6 \times 24}{2} = 72 \text{ mm};$$

$R = 72 \times 3 = 216$ mm; $r_a = 72 + 6 = 78$ mm ; $R_a = 216 + 6 = 222$ mm

$$\text{Path of contact} = \left(\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right)$$

$$= \left(\sqrt{222^2 - 216^2\cos^2 20^\circ} - 216\sin 20^\circ\right) + \left(\sqrt{78^2 - 72^2\cos^2 20^\circ} - 72\sin 20^\circ\right)$$

$$= 16.04 + 14.18 = 30.22 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi} = \frac{30.22}{\cos 20^\circ}$$

$= 32.16$ mm

$$\text{Angle of action} = \frac{\text{Arc of contact}}{r} = \frac{32.16}{72}$$

$= 0.4467 \text{ rad} = 0.4467 \times 180/\pi = 25.59°$

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of approach

$$= \left(\frac{v}{r} + \frac{v}{R}\right) \times \text{Path of approach}$$

$$= \left(\frac{1500}{72} + \frac{1500}{216}\right) \times 16.04 = 445.6 \text{ mm/s}$$

Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addenda on pinion and gear wheels are equal to one module, find the (i) contact ratio (ii) angle of action of the pinion and the gear wheel (iii) ratio of the sliding to rolling velocity at the (a) beginning of contact (b) pitch point (c) end of contact

Solution $\varphi = 20°$; $T = 57$; $t = 23$; $m = 8$ mm;

addendum = m = 8 mm

$$R = \frac{mT}{2} = \frac{8 \times 57}{2} = 228 \text{ mm};$$

$$R_a = R + m = 228 + 8 = 236 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{8 \times 23}{2} = 92 \text{ mm};$$

$$r_a = r + m = 92 + 8 = 100 \text{ mm}$$

(i) $$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\frac{\text{Path of contact}}{\cos\varphi}\right)$$

$$\times \frac{1}{\pi m} = \frac{\text{Path of approach} + \text{Path of recess}}{\cos\varphi \times \pi m}$$

$$= \frac{\left[\begin{array}{l}\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi \\ +\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\end{array}\right]}{\cos\varphi \times \pi m}$$

$$\frac{\left[\begin{array}{l}\sqrt{(236)^2 - (228^2\cos^2 20^\circ} - 228\sin 20^\circ \\ +\sqrt{(100)^2 - (92)^2\cos^2 20^\circ} - 92\sin 20^\circ\end{array}\right]}{\cos 20^\circ \pi \times 8}$$

$$= \frac{20.97 + 18.79}{\cos 20° \times \pi \times 8} = 42.31 \times \frac{1}{\pi \times 8} = \underline{1.68}$$

Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addenda on pinion and gear wheels are equal to one module, find the (i) contact ratio (ii) angle of action of the pinion and the gear wheel (iii) ratio of the sliding to rolling velocity at the (a) beginning of contact (b) pitch point (c) end of contact

(ii) Angle of action, $\delta_p = \dfrac{\text{Arc of contact}}{r} = \dfrac{42.31}{92}$

$= 0.46$ rad or $0.46 \times 180/\pi = 26.3°$

$$\delta_g = \frac{\text{Arc of contact}}{R} = \frac{42.31}{228} = 0.1856 \text{ rad}$$

or $0.1856 \times 180/\pi = 10.63°$

(iii) (a) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\text{Pitch line velocity } (= \omega_p \times r)}$$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times 20.97}{\omega_p \times 92} = \underline{0.32}$$

(b) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}} = \dfrac{(\omega_p + \omega_g) \times 0}{\text{Pitch line velocity}} = 0$

(c) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times \text{Path of recess}}{\omega_p \times r}$$

$$= \frac{\left(1 + \frac{23}{57}\right) \times 18.79}{92} = \underline{0.287}$$

Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1 and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the lip of the tooth of gear 2. Take addendum equal to one module. Also, find the maximum velocity of sliding.

Solution 1 is the gear wheel and 2 is the pinion.

$\varphi = 20°$; $T = 40$; $N_p = 600$ mm; $t = 24$; $m = 4$ mm

Addendum = 1 module = 4 mm

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}; R_a = 80 + 4 = 84 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{4 \times 24}{2} = 48 \text{ mm}; \ r_a = 48 + 4 = 52 \text{ mm}$$

$$N_g = N_p \times \frac{t}{T} = 600 \times \frac{24}{40} = 360 \text{ rpm}$$

Let pinion (gear 2) be the driver. The tip of the driving wheel is in contact with a tooth of the driven wheel at the end of engagement. Thus, it is required to find the path of recess which is obtained from the dimensions of the driving wheel.

$$\text{Path of recess} = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$= \sqrt{(52)^2 - (48\cos 20°)^2} - 48\sin 20°$$

$= 9.458$ mm

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of recess

$= 2\pi\,(N_p + N_g) \times 9.458$

$= 2\pi\,(600 + 360) \times 9.458$

$= 57\,049$ mm/min

$=$ <u>950.8 mm/s</u>

(ii) In case the gear wheel is the driver, the tip of the pinion will be in contact with the flank of a tooth of the gear wheel at the beginning of contact. Thus, it is required to find the distance of the point of contact from the pitch point, i.e.. path of approach. The path of approach is found from the dimensions of the driven wheel which is again pinion.

Thus, path of approach $= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

=9.458mm, same as before and velocity of sliding = 950.8 mm/s

Thus, it is immaterial whether the driver is the gear wheel or the pinion, the velocity of sliding is the same when the contact is at the tip of the pinion. The maximum velocity of sliding will depend upon the larger path considering any of the wheels to be the driver. Consider pinion to be the driver. Path of recess = 9.458 mm

Path of approach $= \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$

$$= \sqrt{(84)^2 - (80\cos 20°)} - 80\sin 20°$$

$$= 10.117 \text{ mm}$$

This is also the path of recess if the wheel becomes the driver.

Maximum velocity of sliding

$$= (\omega_p + \omega_g) \times \text{Maximum path}$$

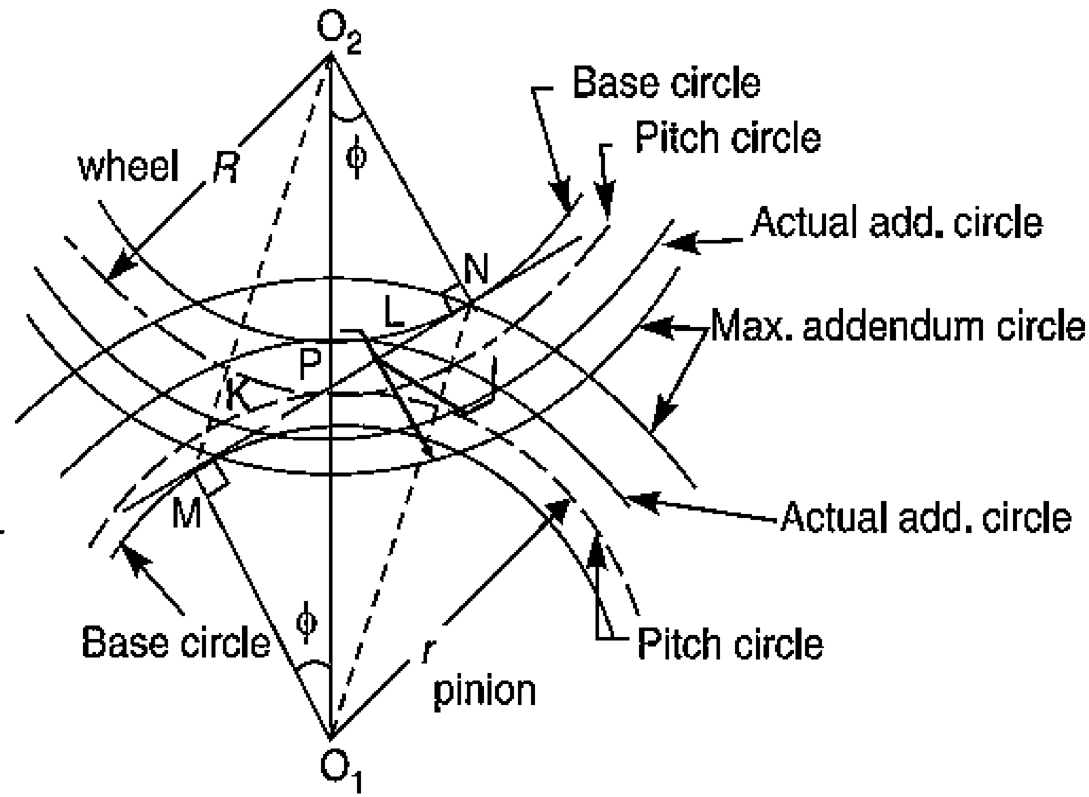
$$= 2\pi(600 + 360) \times 10.117$$

$$= 61024 \text{ mm/min}$$

$$= \underline{1017.1 \text{ mm/s}}$$

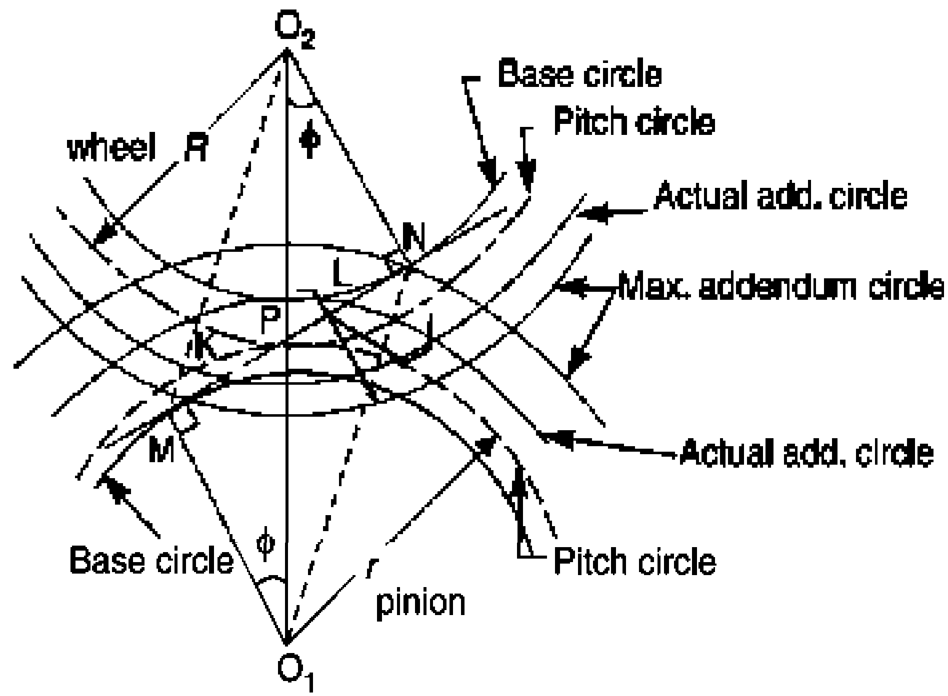
Interference in Involute Gears

- Fig. shows a pinion with centre *O*1, in mesh with wheel or gear with centre *O*2.
- *MN is* the common tangent to the base circles and *KL is the path of contact between the two mating teeth.*



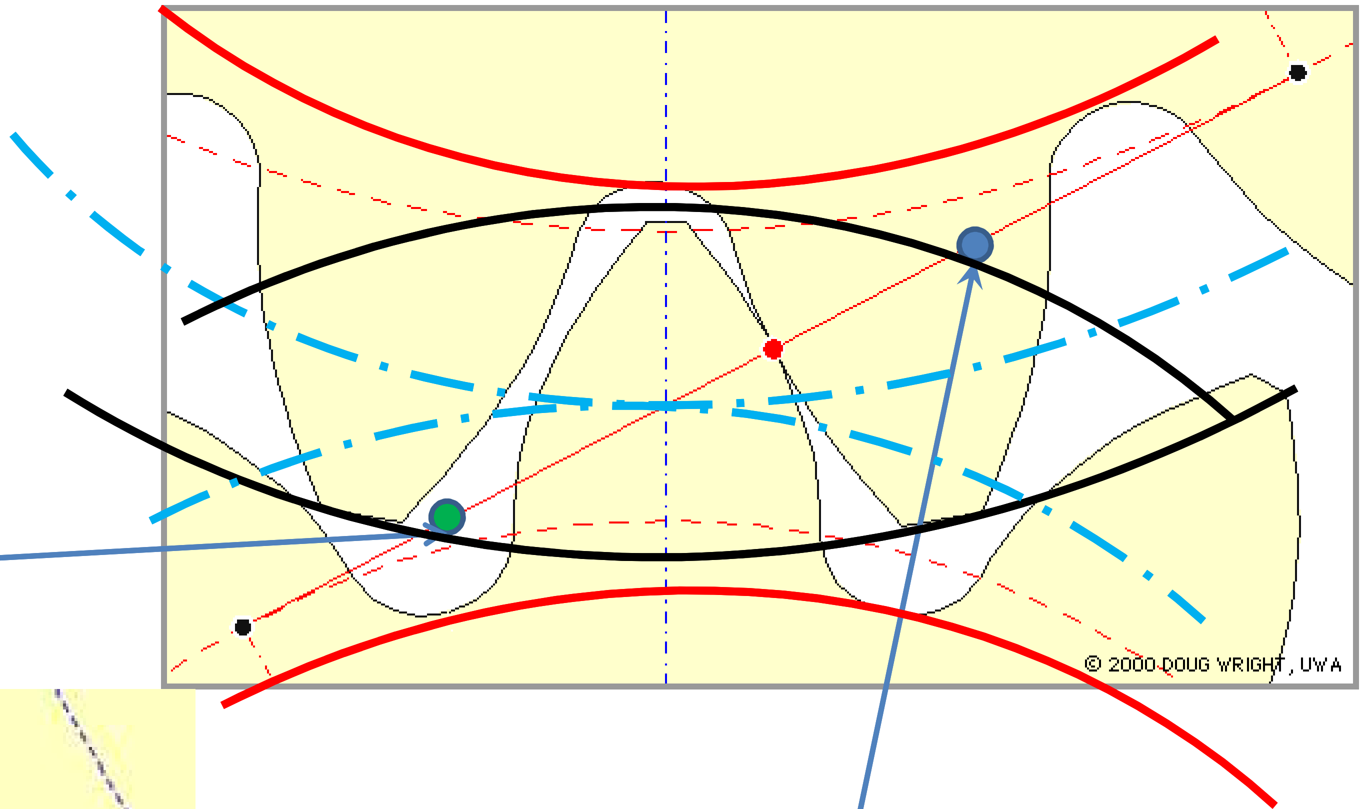
  - A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1 *N, the point of contact L will move from L to N.*
  - *When this radius is further increased,* the point of contact *L will be on the inside of base circle of wheel and not on the involute profile of* tooth on wheel.
- The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as *interference, and* occurs when the teeth are being cut.
- In brief, *the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.*

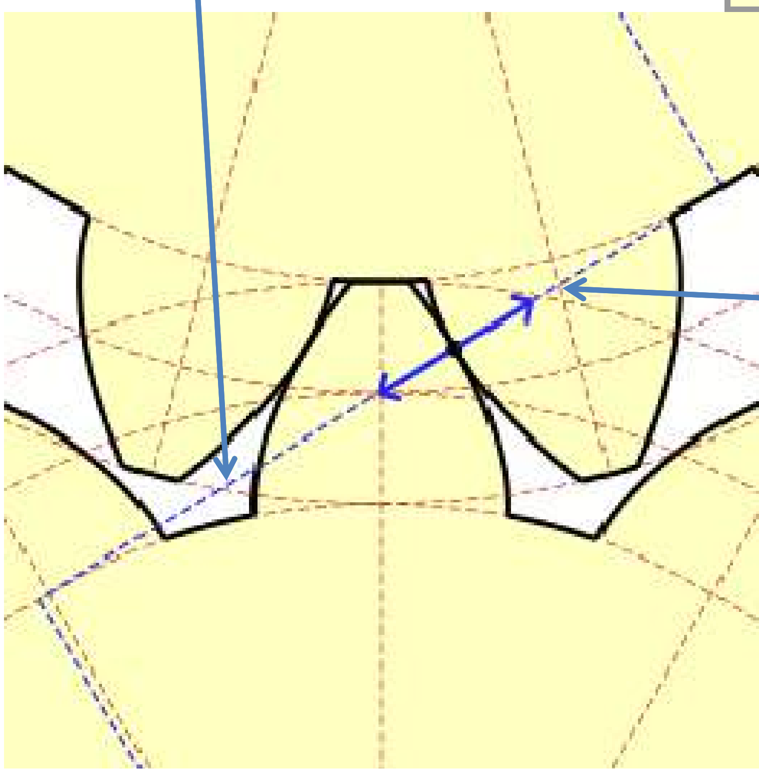
- Similarly, if the radius of the addendum circle of the wheel increases beyond *O*2*M, then the* tip of tooth on wheel will cause interference with the tooth on pinion. The points *M and N are called interference points.*

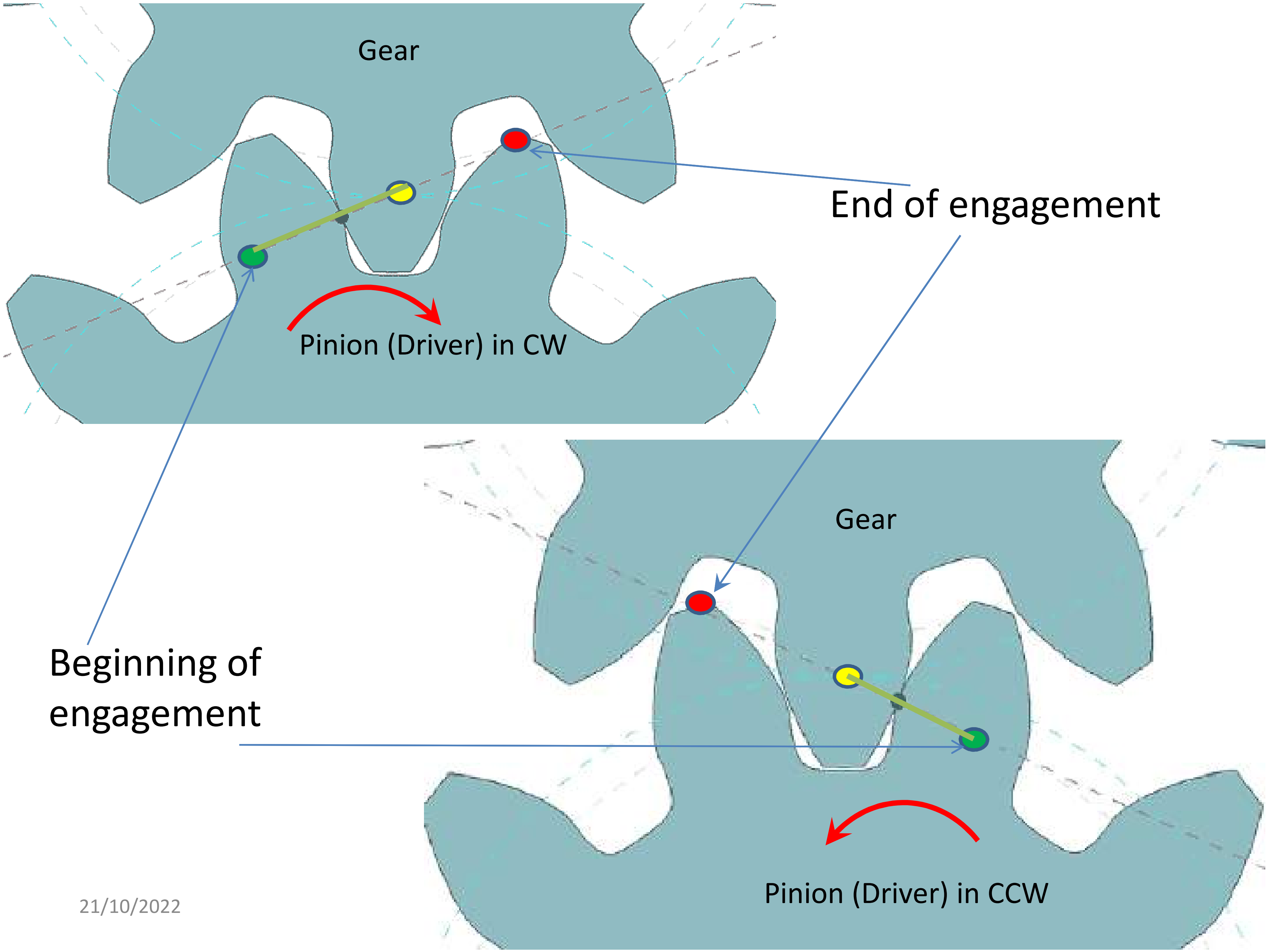


Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is *O*1*N and of the wheel is O*2*M.*

- From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, *interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.*
- When interference is just avoided, the maximum length of path of contact is *MN when the* maximum addendum circles for pinion and wheel pass through the points of tangency *N and M respectively* as shown in Fig.


© 2000 DOUG WRIGHT , UWA
Beginning of engagement
End of engagement




Gear
Pinion (Driver) in CW
End of engagement
Beginning of engagement
Gear
Pinion (Driver) in CCW

Methods of elimination of Gear tooth Interference

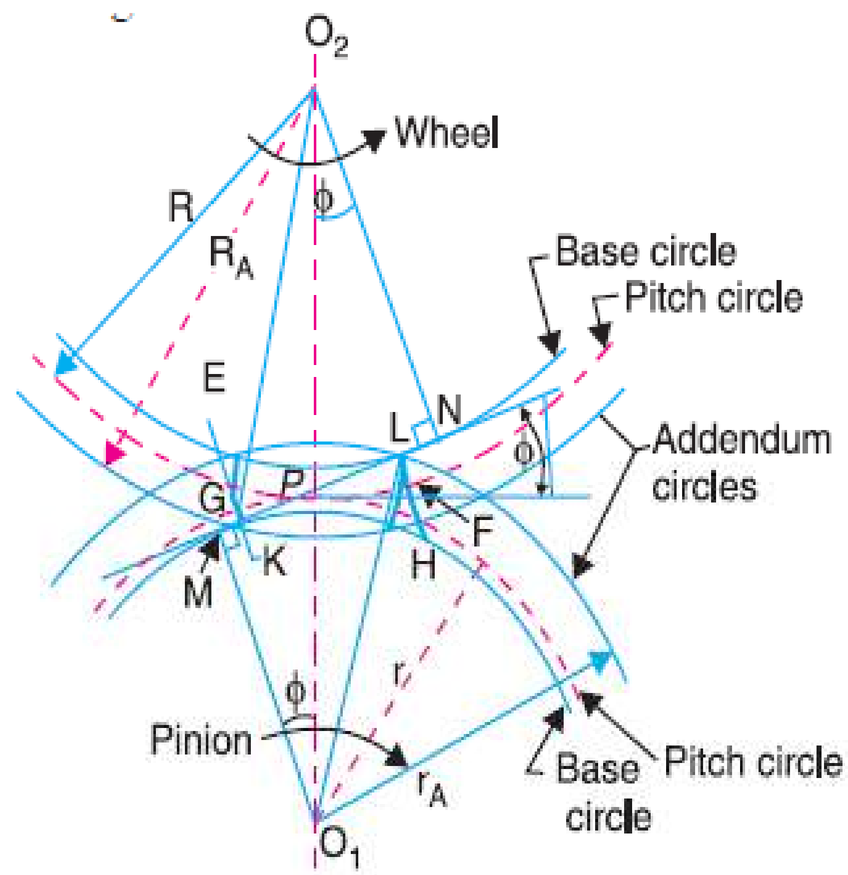
In certain spur designs if interference exists, it can be overcome by:

1. Removing the cross hatched tooth tips i.e., using stub teeth.
2. Increasing the number of teeth on the mating pinion.
3. Increasing the pressure angle
4. Tooth profile modification or profile shifting
5. Increasing the centre distance.

MINIMUM No. OF TEETH ON THE PINION TO AVOID INTERFERENCE :

- In order to avoid interference, the addendum circle for the two mating gears must cut the common tangent to the base circles between the point of tangency.

- The limiting condition reaches, when the addendum of pinion and wheel pass though point M and N.



Let,

t = No. of teeth on the pinion,

T = No. of teeth on the wheel,

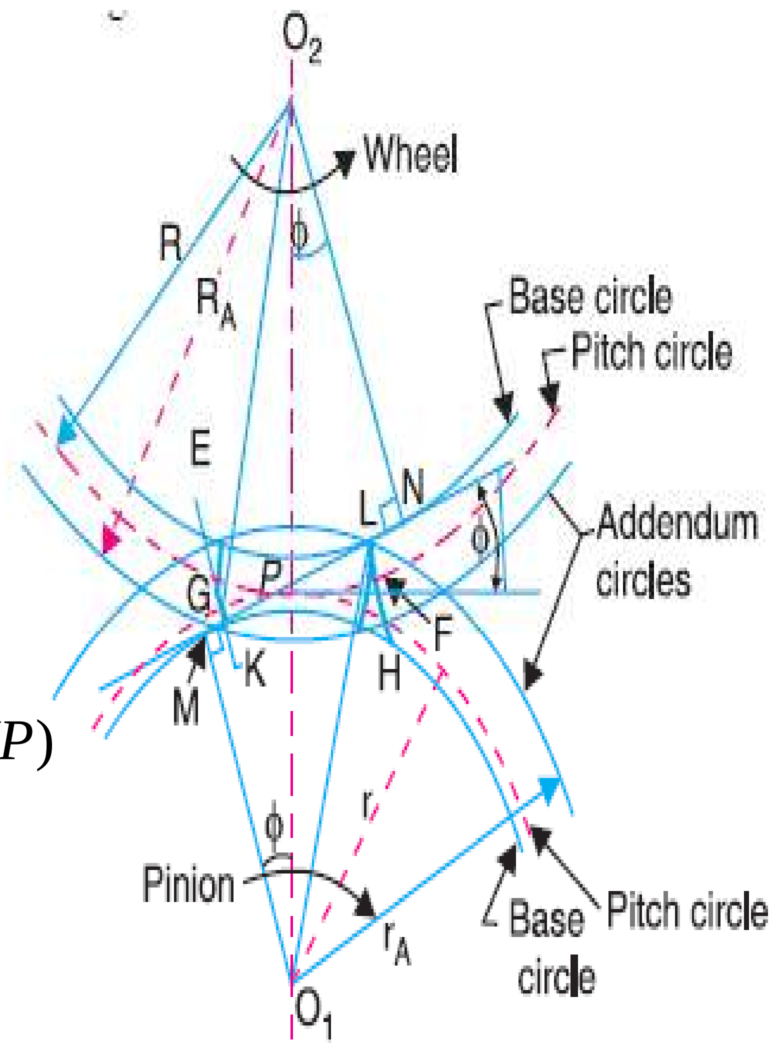
m = Module of the teeth,

r = Pitch circle radius of pinion = $mt/2$

G = Gear ratio = $T/t = R/r$

ϕ = Pressure angle.

From $\triangle O_1NP$,



$$O_1N^2 = O_1P^2 + PN^2 - 2OP \times PN\cos(O_1NP)$$

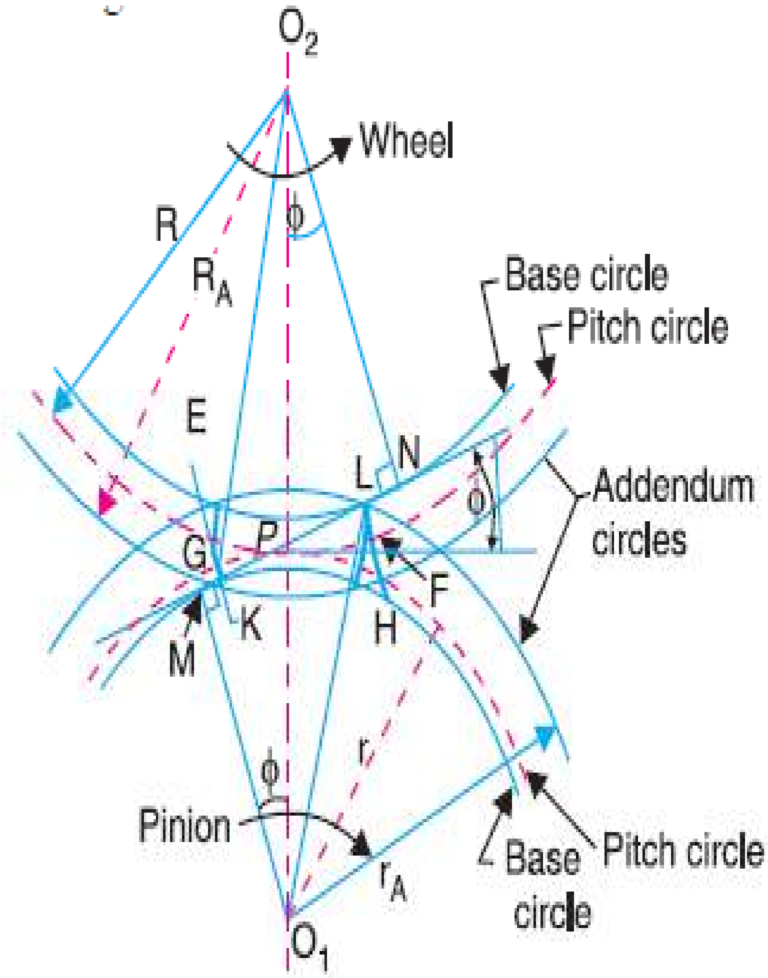
$$= r^2 + (R\sin\phi)^2 + 2rR\sin^2\phi$$

$$= r^2\left[1 + \frac{(R\sin\phi)^2}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$= r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$\therefore\ O_1N = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi} = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}$$

Let $A_P m$ = Addendum of pinion, where A_P is a fraction by which the standard addendum of 1 module for the pinion should be multiplied in order to avoid interference.



Addendum of pinion = $O_1P - O_1P$

$$\therefore\ A_P m = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{mt}{2}$$

$$A_P m = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$t = \frac{2A_P}{\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]}$$

$$t = \frac{2A_P}{\left[\sqrt{1+G\,(G+2)\sin^2\phi} - 1\right]}$$

Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine (i) the minimum number of teeth on each wheel to avoid interference (ii) the number of pairs of teeth in contact.

Solution $\varphi = 20°$ $N_p = 120$ rpm

$VR = 3$ Addendum = 1.1 m

$m = 3$ mm $a_w = 1.1$

(i) $$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

$$= \frac{2\times1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°}-1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio,

i.e., $T = 51$ and $t = \frac{51}{3} = \underline{17}$

(ii) Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \left(\frac{\text{Path of contact}}{\cos\varphi}\right)\times\frac{1}{\pi m}$$

or

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi + \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi}{\cos\varphi\times\pi m}$$

We have, $R = \frac{mT}{2} = \frac{3\times51}{2} = 76.5$ mm

$R_a = R + 1.1\,m = 76.5 + 1.1\times3 = 79.8$ mm

$r = \frac{mt}{2} = \frac{3\times17}{2} = 25.5$ mm

$r_a = 25.5 + 1.1\times3 = 28.8$ mm

Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine (i) the minimum number of teeth on each wheel to avoid interference (ii) the number of pairs of teeth in contact.

$$n = \frac{\left[\begin{array}{l}\sqrt{(79.8)^2 - (76.5\cos 20°)^2} - 76.5\sin 20° \\ +\sqrt{(28.8)^2 - (25.5\cos 20°)^2} - 25.5\sin 20°\end{array}\right]}{\cos 20° \times \pi \times 3}$$

$$= \frac{34.646 - 26.165 + 15.977 - 8.720}{\cos 20° \times \pi \times 3}$$

$$= 1.78$$

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact.

Two 20° involute spur gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth. Will the gear interfere with the pinion?

Solution

$\varphi = 20°$, $T = 40$, $m = 10$ mm, $t = 20$

Addendum = 1 m = 10 mm

$$R = \frac{mT}{2} = \frac{10 \times 40}{20} = 200 \text{ mm}$$

$$R_a = 200 + 10 = 210 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{10 \times 20}{2} = 100 \text{ mm}$$

$$r_a = 100 + 10 = 110 \text{ mm}$$

Let pinion be the driver (Refer Fig.10.24),

$$\text{Path of approach, } PC = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$$

$$= \sqrt{(210)^2 - (200 \times \cos 20°)^2} - 200 \sin 20°$$

$$= 25.3 \text{ mm}$$

To avoid interference, maximum length of the path of approach can be PE.

$$PE = r \sin \varphi = 100 \sin 20° = 34.2 \text{ mm}$$

Since the actual path of approach is within the maximum limit, no interference occurs.

A pair of spur gears with involute teeth is to give a gear ratio of 4: 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch.

Solution. *Given* : $G = T/t = R/r = 4$; $\phi = 14.5°$

1. ***Least number of teeth on each wheel***

We know that the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r \sin\phi}{\cos\phi} = r \tan\phi$$

$$\text{circular pitch,} \qquad p_c = \pi m = \frac{2\pi r}{t}$$

Since the arc of approach is not to be less than the circular pitch, therefore

$$r \tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25$$

$$T = G.t = 4 \times 25 = 100 \text{ **Ans.**}$$

A pair of spur gears with involute teeth is to give a gear ratio of 4: 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch.

2. *Addendum of the wheel*

We know that addendum of the wheel

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5°} - 1\right]$$

$$= 50\,m \times 0.017 = 0.85\,m = 0.85 \times p_c / \pi = 0.27\,p_c$$

Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occurs, to what value should the pressure angle be changed to eliminate interference?

Solution $\varphi = 20°$; $T = 50$; $m = 10$ mm; $t = 13$;

Addendum = 1 m = 10 mm

$$R = \frac{mT}{2} = \frac{10 \times 50}{2} = 250 \text{ mm};$$

$$R_a = 250 + 10 = 260 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{10 \times 13}{2} = 65 \text{ mm}$$

Maximum addendum radius can also be found using the relation

$$R_{a\,\max} = R\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi}$$

$$= 250\sqrt{1 + \frac{13}{50}\left(\frac{13}{50} + 2\right)\sin^2\varphi} = 258.45 \text{ mm}$$

The actual addendum radius R_a is more than the maximum value $R_{a\,max}$, and therefore, interference occurs.

The new value of φ can be found by taking $R_{a\,max}$ equal to R_a.

i.e., $$260 = \sqrt{(250\cos\varphi)^2 + (315\sin\varphi)^2}$$

or $$(260)^2 = (250)^2\cos^2\varphi + (315)^2(1 - \cos^2\varphi)$$

$$= (250)^2\cos^2\varphi + (315)^2 - (315)^2\cos^2\varphi$$

or $$\cos^2\varphi = \frac{(315)^2 - (260)^2}{(315)^2 - (250)^2} = 0.861$$

$$\cos\varphi = 0.928 \text{ or } \varphi = 21.88° \text{ or } 21°52'$$

Thus, if the pressure angle is increased to 21°52′, the interference is avoided.

The following data relate to two meshing involute gears: Number of teeth on the gear wheel = 60 ; pressure angle = 20º ; Gear ratio = 1.5; Speed of the gear wheel = 100 rpm; Module = 8 mm; The addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of the arc of contact.

Solution $R = \frac{mT}{2} = \frac{8 \times 60}{2} = 240 \text{ mm};$

$$r = \frac{mT}{2} = \frac{8 \times (60/1.5)}{2} = 160 \text{ mm}$$

Refer Fig. 10.24 and let the pinion be the driver.

Maximum possible length of path of approach = $r \sin \varphi$

Actual length of path of approach = $0.4 \times r \sin \varphi$

Similarly, actual length of path of recess = $0.4 R \sin \varphi$

Thus, we have

$$0.4 r \sin \varphi = \sqrt{R_a^2 - (R \cos \varphi)^2} - R \sin \varphi$$

$$0.4 \times 160 \sin 20° = \sqrt{R_a^2 - (240 \cos 20°)^2} - 240 \sin 20°$$

$$R_a^2 - 50\,862 = 10\,809.8$$

$$R_a^2 = 61671.8$$

$$R_a = 248.3 \text{ mm}$$

Addendum of the wheel = 248.3 – 240 = <u>8.3 mm</u>

Also, $0.4 R \sin \varphi = \sqrt{r_a^2 - (r \cos \varphi)^2} - r \sin \varphi$

$$0.4 \times 240 \sin 20° = \sqrt{r_a^2 - (160 \cos 20°)^2} - 160 \sin 20°$$

or $r_a^2 - 22\,605 = 7666$

or $r_a^2 = 30\,271$

or $r_a = 174$ mm

Addendum of the pinion = 174 – 160 = <u>14 mm</u>

A pinion of 20° involute teeth rotating at 275 rpm meshes with a gear and provides a gear ratio of 1.8. The number of teeth on the pinion is 20 and the module is 8 mm. If the interference is just avoided, determine (i) the addenda on the wheel and the pinion (ii) the path of contact, and (iii) the maximum velocity of sliding on both sides of the pitch point.

Solution $\varphi = 20°$; $VR = 1.8$; $m = 8$ mm; $t = 20$;
$G = 1.8$; $T = 20 \times 1.8 = 36$; $N = 275$ rpm

$$R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144 \text{ mm}; r = \frac{144}{1.8} = 80 \text{ mm}$$

Maximum addendum of the wheel,

$$a_{w\max} = R\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$

$$= 144\left[\sqrt{1 + \frac{1}{1.8}\left(\frac{1}{1.8} + 2\right)\sin^2 20^\circ} - 1\right]$$

$= 144\,(1.08 - 1) = 11.5$ mm

Maximum addendum of the pinion,

$$a_{p\max} = r\left[\sqrt{1 + G(G+2)\sin^2\varphi} - 1\right]$$

$$= 80\left[\sqrt{1 + 1.8\,(1.8 + 2)\sin^2 20^\circ} - 1\right] = 27.34 \text{ mm}$$

Path of contact when the interference is just avoided

= maximum length of path of approach + maximum length of path of recess

$= r \sin\varphi + R \sin\varphi = 80 \sin 20° + 144 \sin 20°$

$= 27.36 + 49.24 = 76.6$ mm

$$\omega_p = \frac{2\pi \times 275}{60} = 28.8 \text{ rad/s}; \omega_g = \frac{28.8}{1.8} = 16 \text{ rad/s}$$

Velocity of sliding on one side $= (\omega_p + \omega_g) \times$ Path of approach

$= (28.8 + 16) \times 27.36 = 1226$ mm/s or 1.226 m/s

Velocity of sliding on other side $= (\omega_p + \omega_g) \times$ Path of recess

$= (28.8 + 16) \times 49.24 = 2206$ mm/s or 2.206 m/s