Title

KINEMATICS OF MACHINES

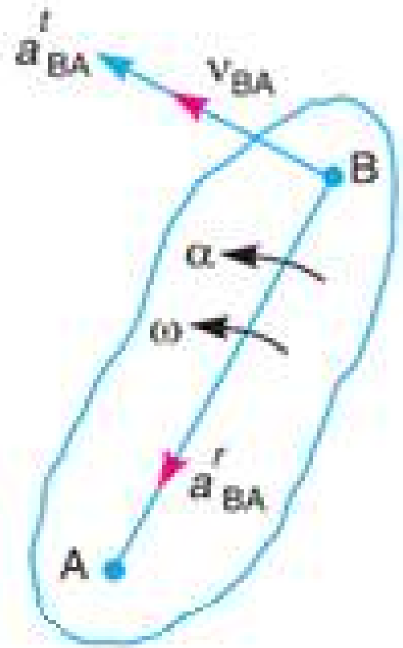
Sub-title

Velocity & Acceleration Analysis

Acceleration in Mechanisms

Acceleration of a particle whose **velocity changes both in magnitude and direction** at any instant has the following two components :

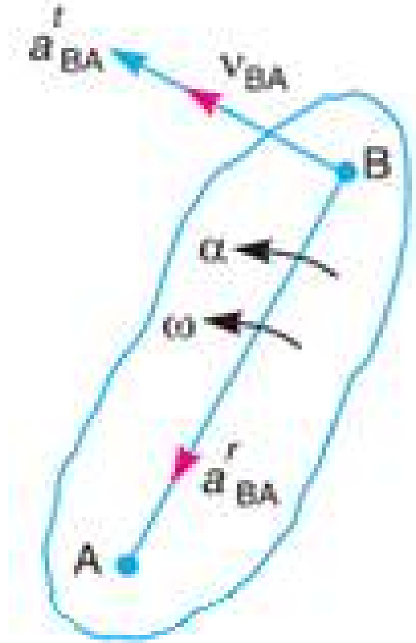
1. The ***centripetal or radial component, which is perpendicular to the velocity of the*** **particle at the given instant.**
2. The ***tangential component, which is parallel to the velocity of the particle at the given*** **instant.**



$$a^r_{BA} = \omega^2 \times \text{Length of link } AB$$

$$= \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because\ \omega = \frac{v_{BA}}{AB}\right)$$

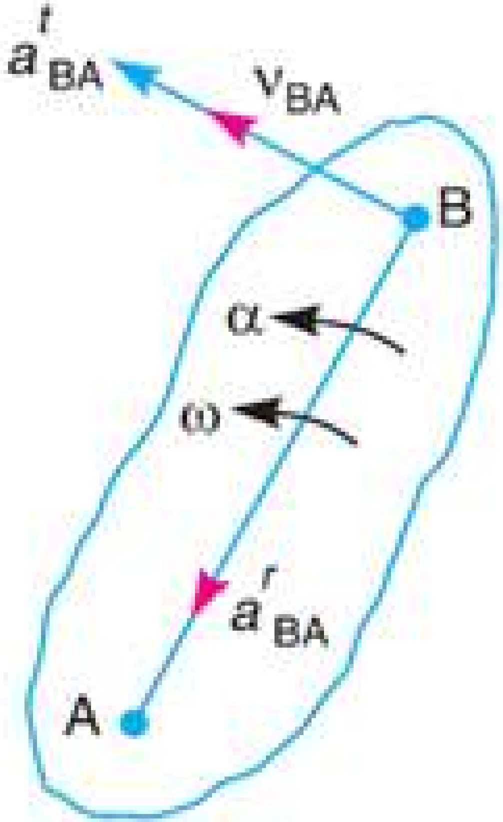
This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel to the link AB.***



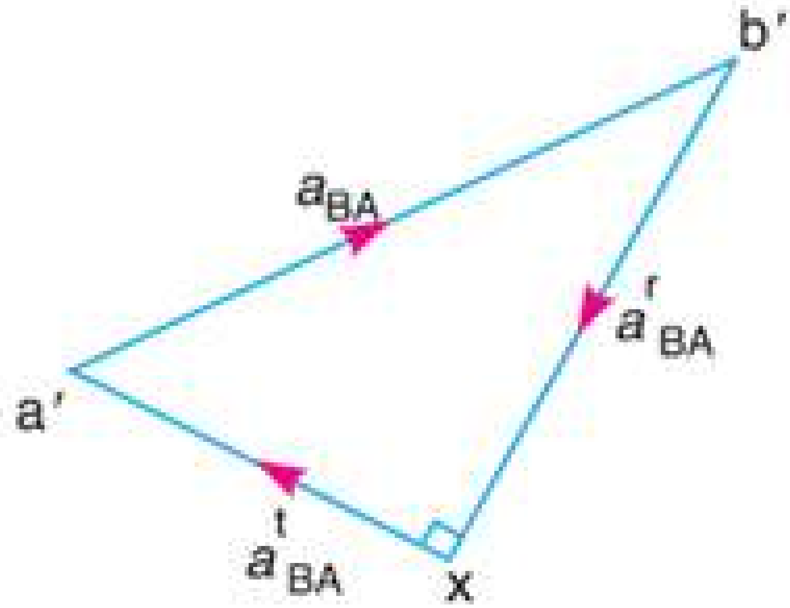
We know that tangential component of the acceleration of B with respect to A,

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB \quad = dv/dt$$

This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts *perpendicular* to the link AB.

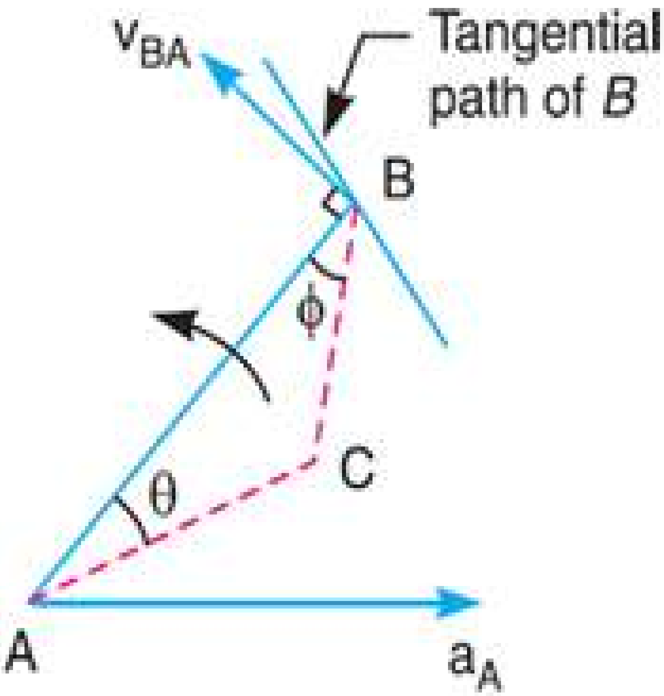


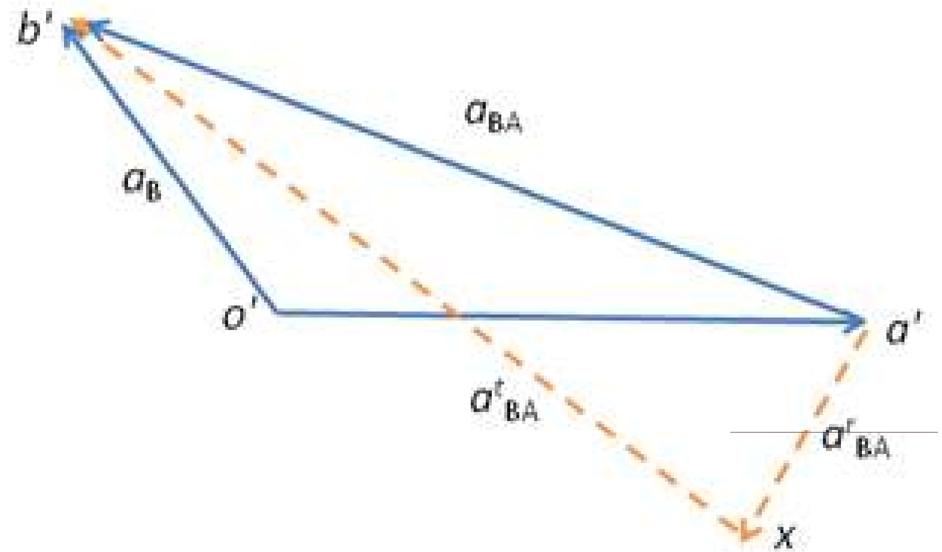
Link.



Acceleration diagram.

Acceleration of a Point on a Link



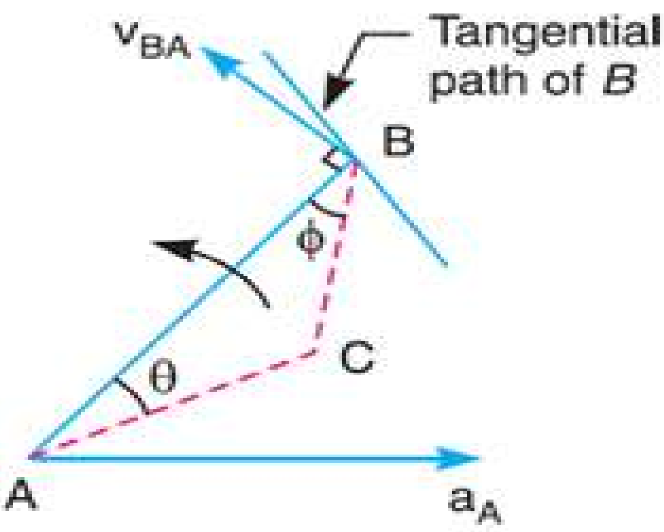


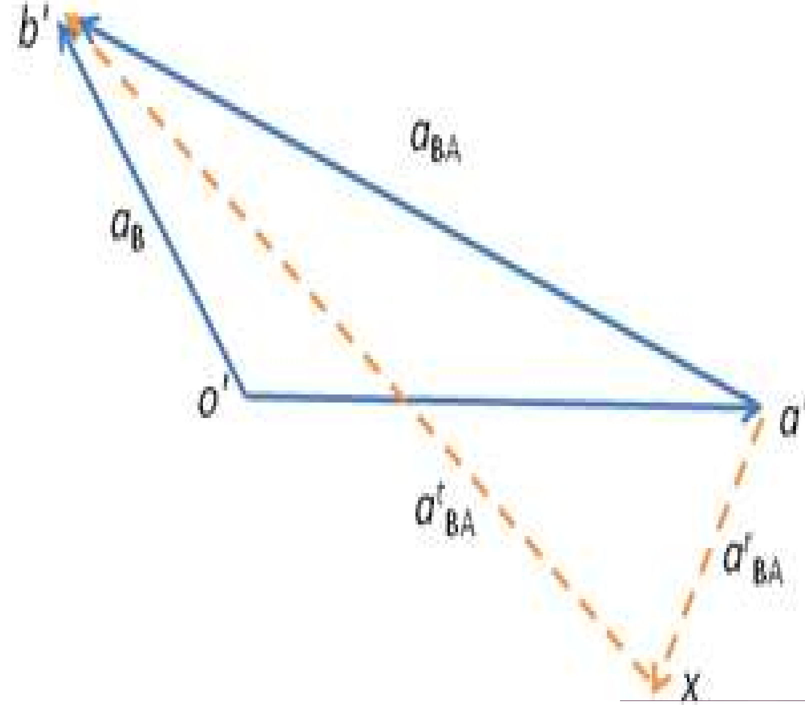
From any point o', *draw vector o'a' parallel to the direction of absolute acceleration at* point *A i.e.* a_A *, to some suitable scale*

Draw vector *a'x parallel to the link AB* (because radial component of the acceleration of *B* with respect to *A will pass through AB), such that*

$$\text{vector } a'x = a^r_{BA} = v^2_{BA} / AB$$

where v_{BA} = Velocity of B with respect to A.





From point x, draw vector xb' perpendicular to $A\,B$ or vector $a'x$ *i.e.* a^t_{BA}

through o' draw a line parallel to the path of B to represent the absolute acceleration of B *i.e.* a_B. The vectors xb' and $o'b'$ intersect at b'. Now the values of a_B and a^t_{BA} may be measured, to the scale.

$$\alpha_{AB} = a^t_{BA} / AB$$

By joining the points *a′ and b′ we may determine the total acceleration of* B *with respect to* A *i.e.* a_{BA}. The vector *a′ b′ is known as acceleration image of the link AB.*

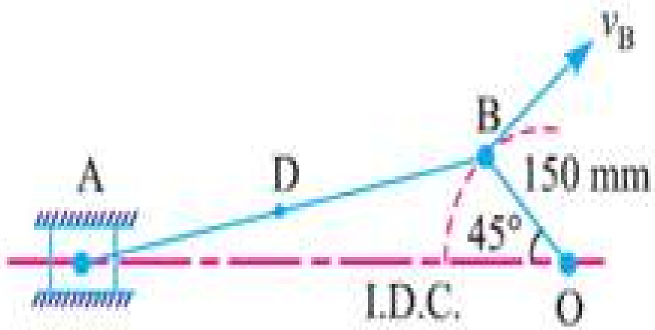
Example 8.1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

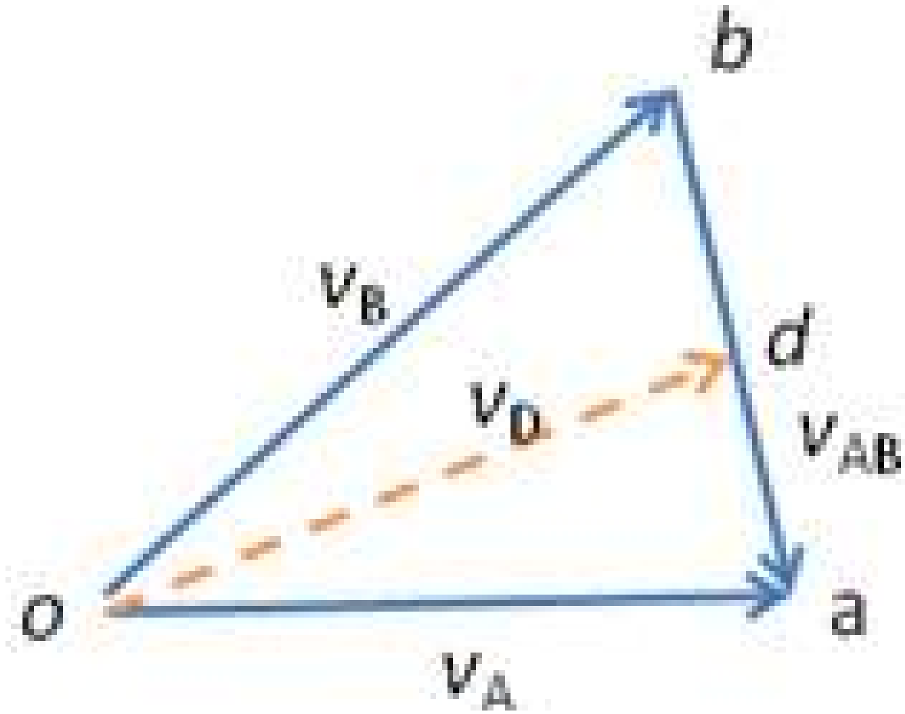
We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

1. First of all draw the space diagram, to some suitable scale; as shown in Fig. (*a*).



(*a*) Space diagram.


b
v_B
d
v_D
v_AB
o
a
v_A

Acceleration of the midpoint of the connecting rod

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleration of *A with respect to B,*

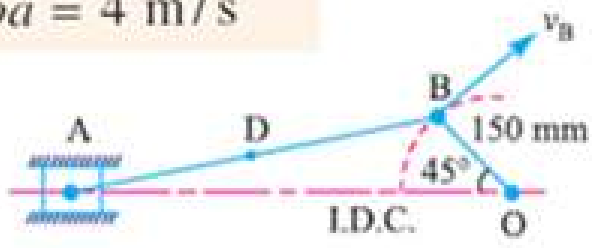
$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

NOTE:1) A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

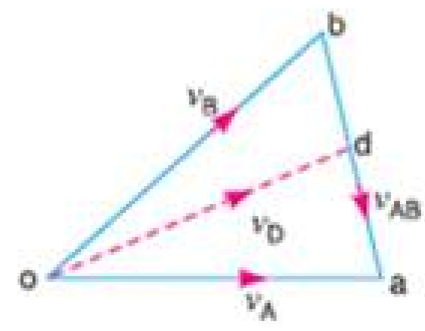
2) When a point moves along a straight line, it has no centripetal or radial component of the acceleration.

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$



(*a*) Space diagram.



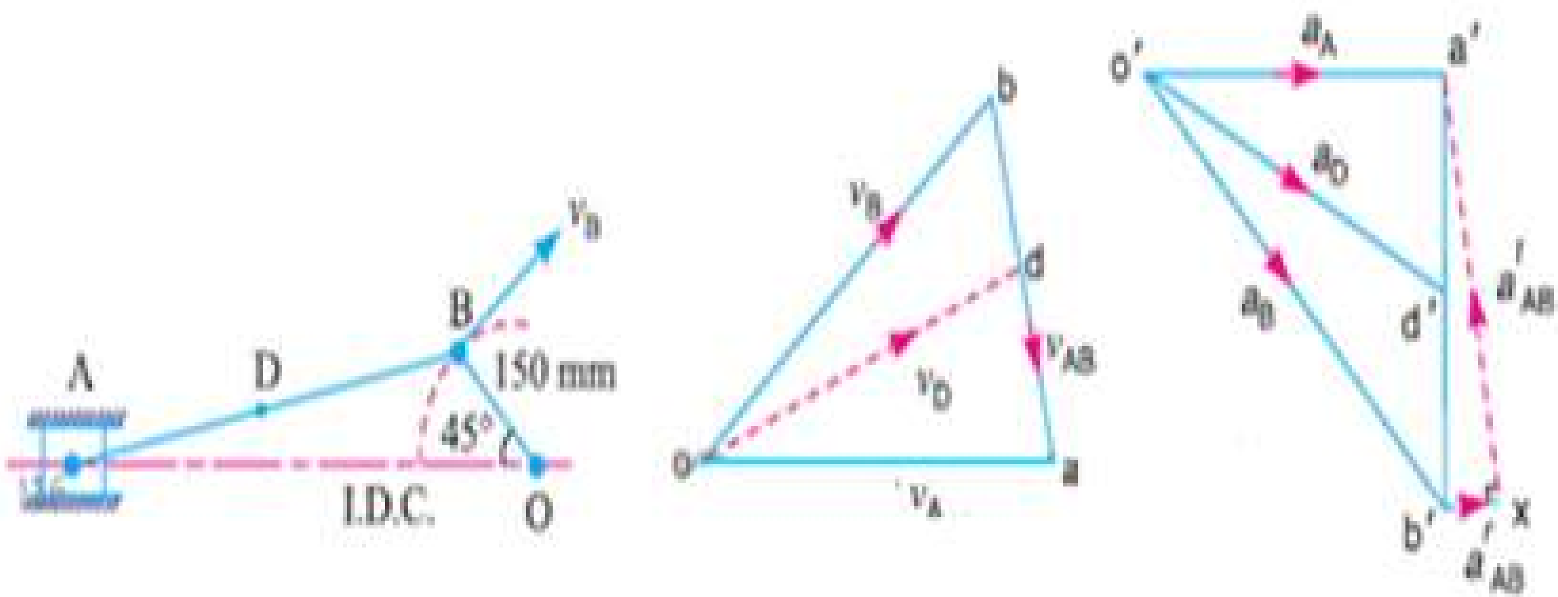
(*b*) Velocity diagram.

Acceleration of the midpoint of the connecting rod

Draw vector $o'b'$ parallel to BO, to some suitable scale,

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank OB rotates at a constant speed, therefore there will be no tangential component of the acceleration of B with respect to O.



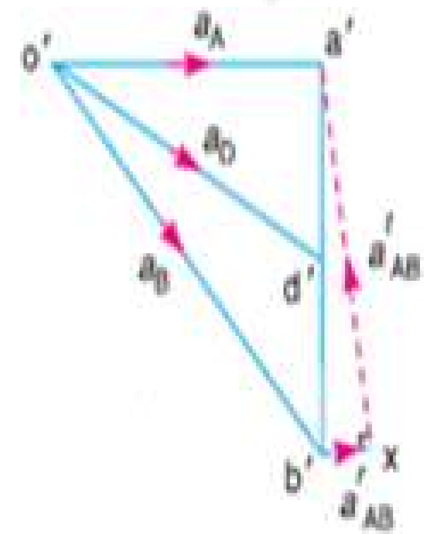
from point b', draw vector $b'x$ parallel to AB to represent $a'_{AB} = 19.3$ m/s^2 and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

Now from o', draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $o'a'$ intersect at a'. Join $a'b'$.

In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector $a'b'$ at d' in the same ratio as D divides AB. In other words

$$b'd' / b'a' = BD / BA$$

Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod *i.e.* a_D.



By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \textbf{ Ans.}$$

Angular velocity of the connecting rod

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}$$ **Ans.**

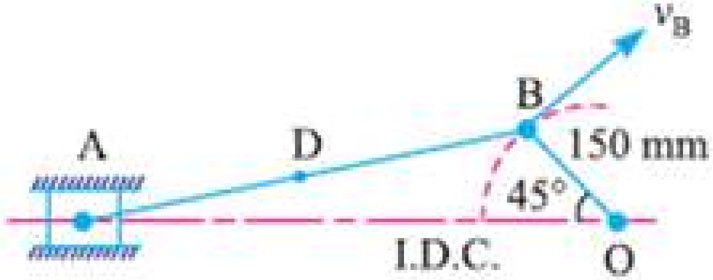
Angular acceleration of the connecting rod

From the acceleration diagram, we find that

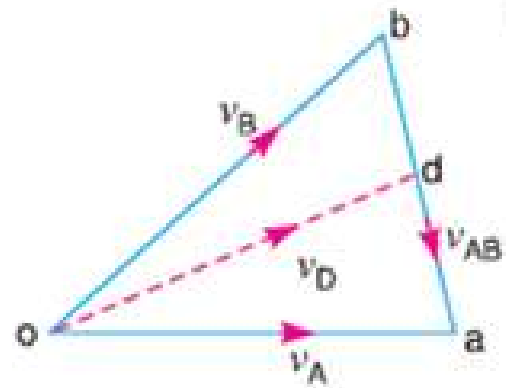
$$a^t_{AB} = 103 \text{ m/s}^2$$ ...(By measurement)

We know that angular acceleration of the connecting rod AB,

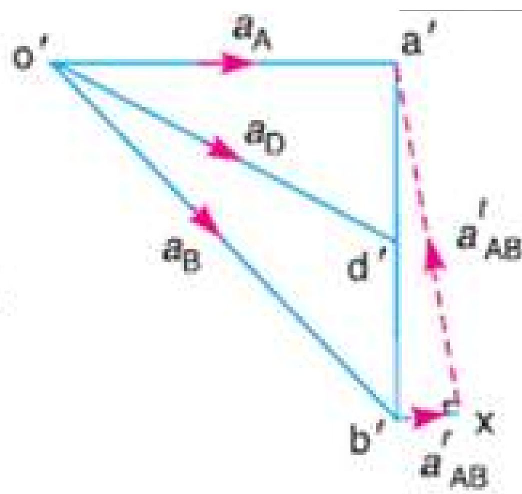
$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2$$ **Ans.**



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

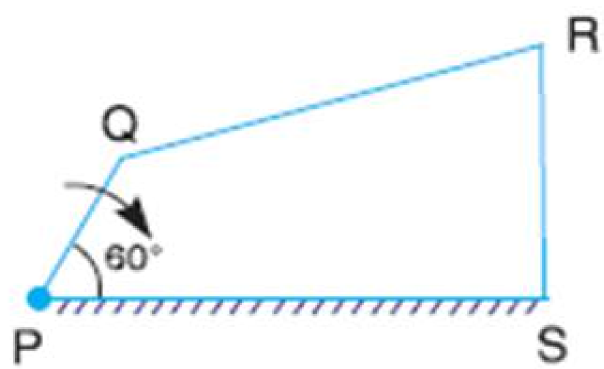
Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale



(*a*) Space diagram.

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q *i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (*i.e.* v_{RQ}) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (*i.e.* v_{RS} or v_R). The vectors qr and sr intersect at r. By measurement, we find that

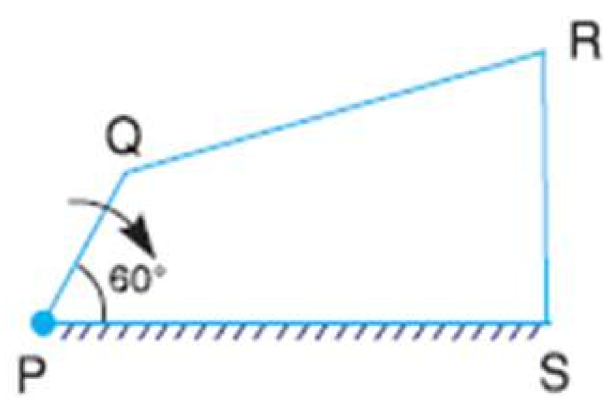
$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

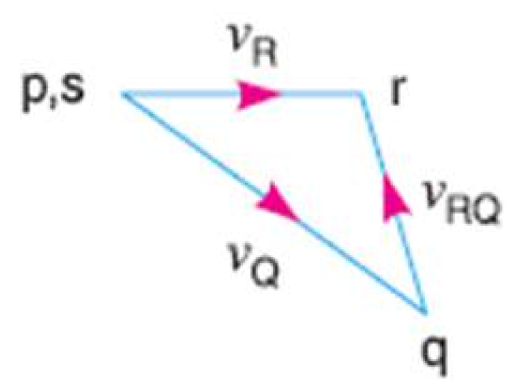
We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

and angular velocity of link RS,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$





Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

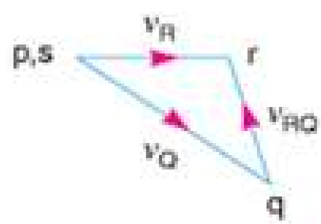
Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

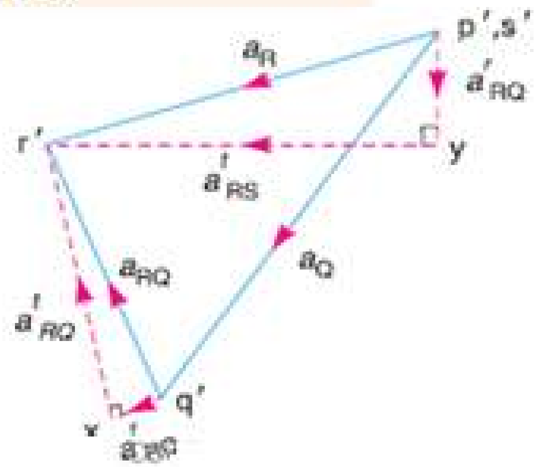
and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

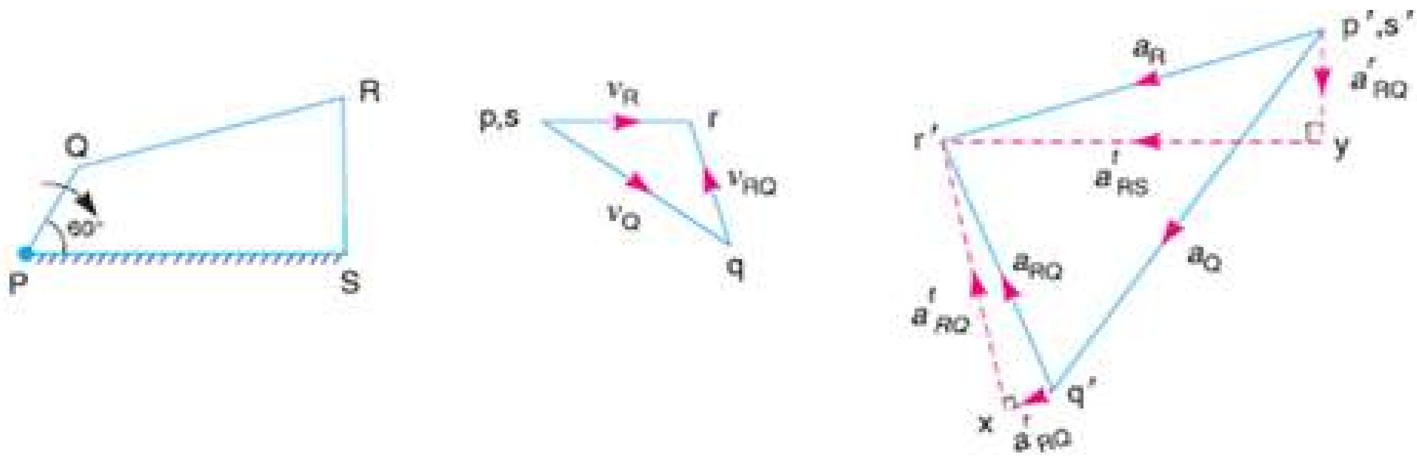
(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. Since *P* and *S* are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector *p'q'* parallel to *PQ*, to some suitable scale, to represent the radial component of acceleration of *Q* with respect to *P* or acceleration of *Q i.e* a^r_{QP} or a_Q such that

$$\text{vector } p'q' = a^r_{QP} = a_Q = 6.25 \text{ m/s}^2$$

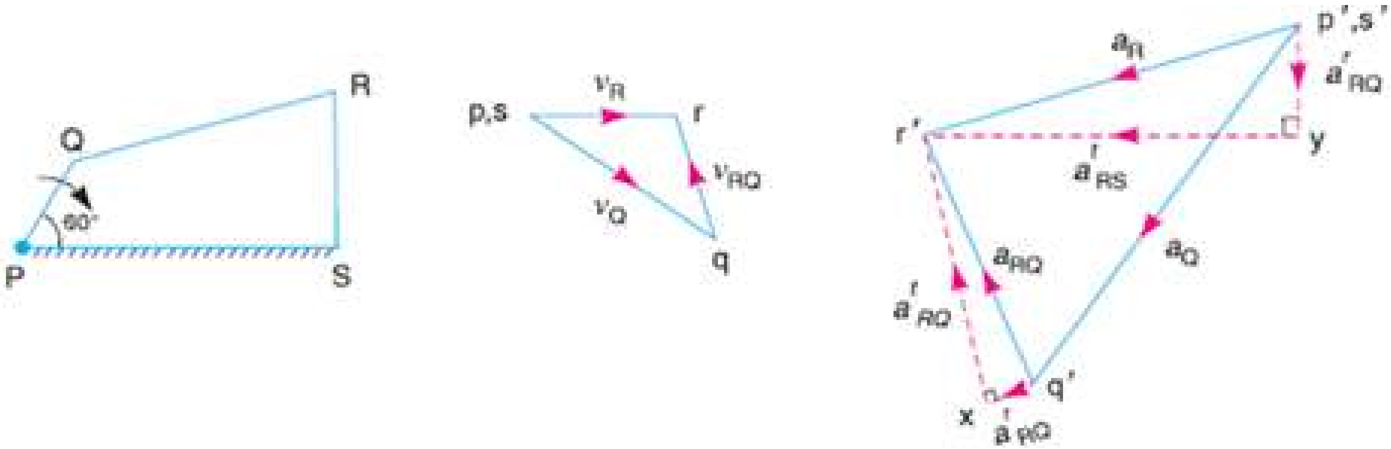
2. From point *q'*, draw vector *q' x* parallel to *QR* to represent the radial component of acceleration of *R* with respect to *Q i.e.* a^r_{RQ} such that

$$\text{vector } q'x = a^r_{RQ} = 0.634 \text{ m/s}^2$$

3. From point *x*, draw vector *xr'* perpendicular to *QR* to represent the tangential component of acceleration of *R* with respect to *Q i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point *s'*, draw vector *s'y* parallel to *SR* to represent the radial component of the acceleration of *R* with respect to *S i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

5. From point *y*, draw vector *yr'* perpendicular to *SR* to represent the tangential component of acceleration of *R* with respect to *S* *i.e.* a^t_{RS}.

6. The vectors *xr'* and *yr'* intersect at *r'*. Join *p'r* and *q'r'*. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link *QR*,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

and angular acceleration of link *RS*,

$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} \text{ Anticlockwise}$$ **Ans.**

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

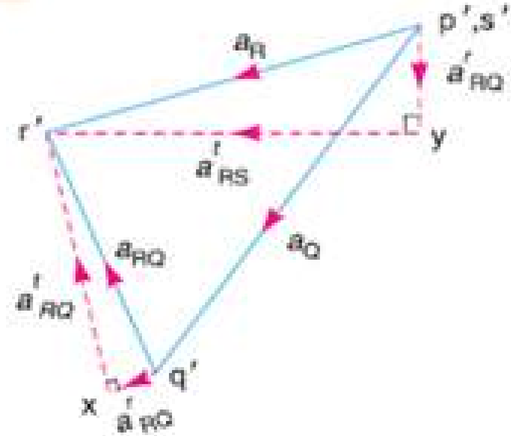
$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

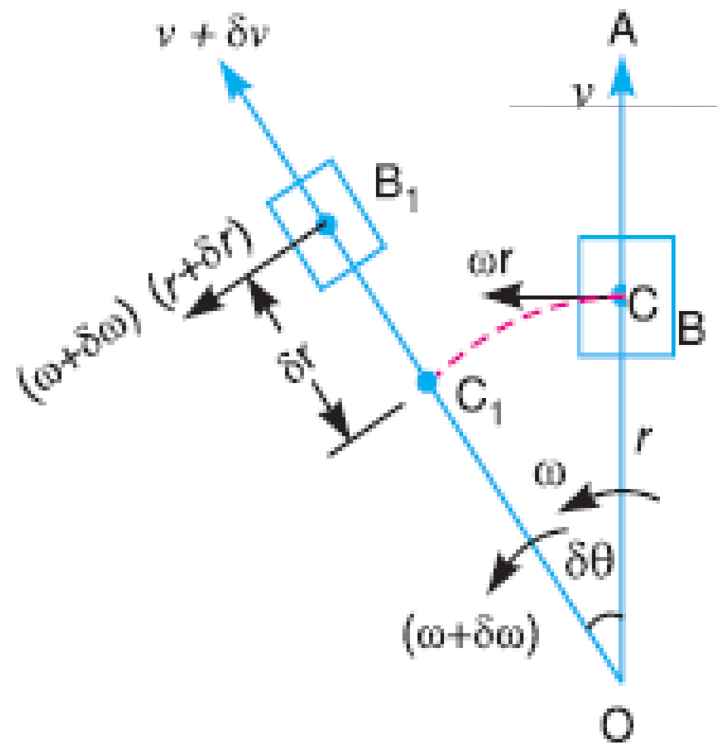


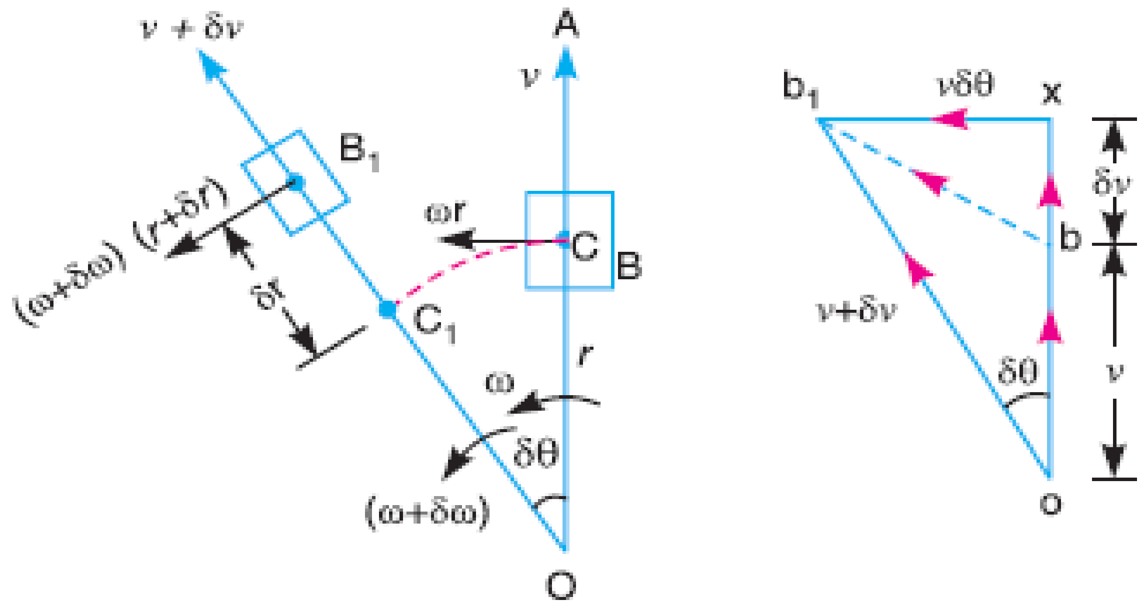
(c) Acceleration diagram.

Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link *OA* and a slider *B* as shown in Fig. The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.





$$bx = ox - ob = (v + \delta v)\cos\delta\theta - v \uparrow$$

Since $\delta\theta$ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$bx = (v + \delta v - v) \uparrow = \delta v \uparrow$$

...(Acting radially outwards)

and $$xb_1 = (v + \delta v)\sin\delta\theta$$

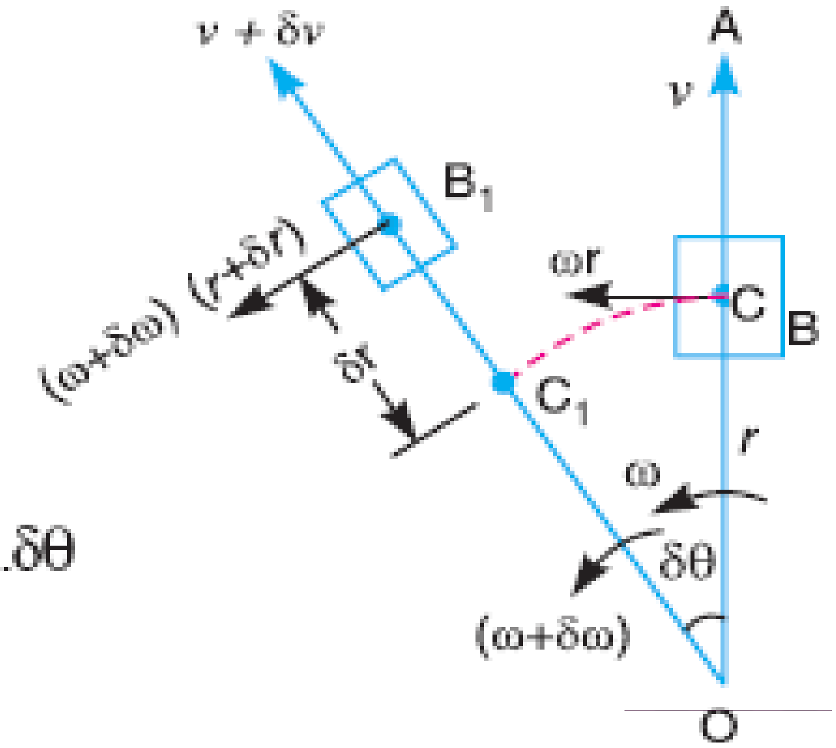
Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\,\delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = v.\overleftarrow{\delta\theta}$$

Fig. shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)(r + \delta r)$ are considered.



$$yb_1 = (\omega + \delta\omega)(r + \delta r)\sin\delta\theta \downarrow$$

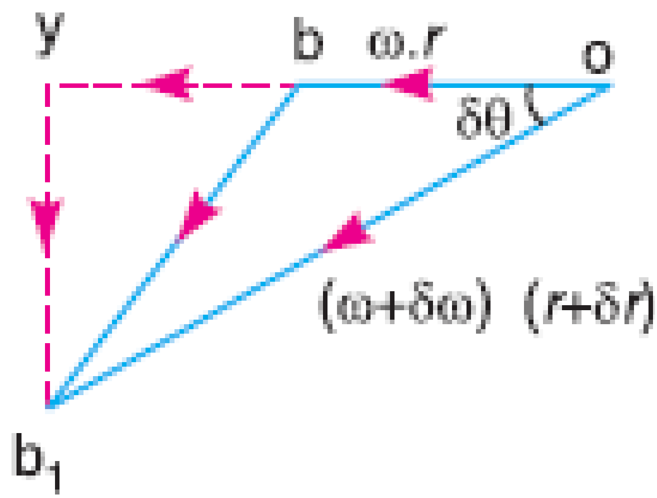
$$= (\omega r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\sin\delta\theta$$

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow \text{, acting radially inwards}$$

$$by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$



$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega$$

total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow$$

∴ Radial component of the acceleration of the slider B with respect to O on the link OA,

$$a_{BO}^{r} = \text{Lt} \frac{\delta v - \omega . r . \delta\theta}{\delta t} = \frac{dv}{dt} - \omega . r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2 . r \uparrow \quad (\because d\theta / dt = \omega)$$

total component of change of velocity along tangential direction,

$$= xb_1 + by = v . \overleftarrow{\delta\theta} + (\omega . \overleftarrow{\delta r} + r . \delta\omega) \text{ (Perpendicular to } OA \text{ and towards left)}$$

Tangential component of acceleration of the slider B acting perpendicular to OA and towards left,

$$a_{BO}^{t} = \text{Lt} \frac{v . \delta\theta + (\omega . \delta r + r . \delta\omega)}{\delta t} = v \frac{d\theta}{dt} + \omega \frac{dr}{dt} + r \frac{d\omega}{dt}$$

$$= v . \omega + \omega . v + r . \alpha = (2 v . \overleftarrow{\omega} + r . \alpha) \quad (\because dr / dt = v, \text{ and } d\omega / dt = \alpha)$$

$$a_{BO}^{r} = \frac{dv}{dt} - \omega^2 . r \uparrow \qquad a_{BO}^{t} = (2\, v.\overleftarrow{\omega} + r.\alpha)$$

$$a_{CO}^{r} = \omega^2 . r \uparrow \qquad a_{CO}^{t} = \overleftarrow{\alpha}.r \uparrow$$

$$a_{BC}^{r} = a_{BO}^{r} - a_{CO}^{r} = \left(\frac{dv}{dt} - \omega^2 . r\right) - \left(-\omega^2 . r\right) = \frac{dv}{dt} \uparrow$$

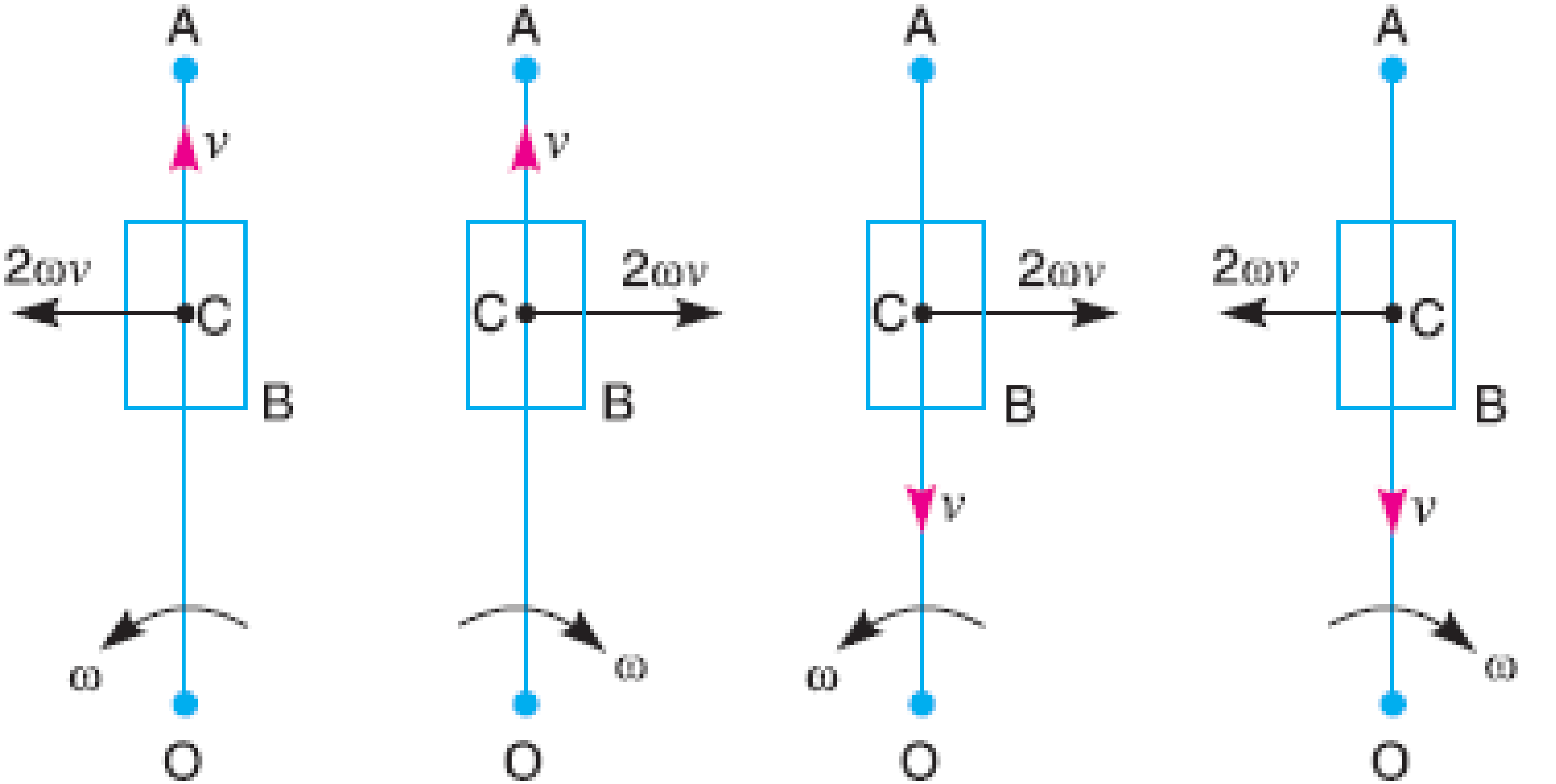
$$a_{BC}^{t} = a_{BO}^{t} - a_{CO}^{t} = \left(2\,\omega.v + \alpha.r\right) - \alpha.r = 2\,\overleftarrow{\omega.v}$$

Coriolis component of the acceleration of B with respect of C,

$$a_{BC}^{c} = a_{BC}^{t} = 2\,\omega.v$$

ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.



Direction of coriolis component of acceleration.

Thank You