WAVE OPTICS chapter-4

Ques. - Write a note on Huygens' principle.

Ans. - Huygens' Principle - Huygens proposed wave theory of light in 1678. (1) According to Huygens wave theory light travels in the form of longitudinal waves.

(2) For propagations of waves Huygens assumed a hypothetical elastic medium called 'ether'.

(3) The medium 'ether' pervades all space including vacuum.

(4) The source of light creates disturbance periodically which travel through ether.

(5) The continuous locus of all particles vibrating in same phase is known as wave front.

(6) Every particle of medium on wave front acts as a source of secondary wavelets. These secondary wavelets propagates in medium with the speed of primary wavelets.

(7) The envelope of secondary wavelets represents the new position of wave front at that instant of time.

Huygens' wave theory could explain reflection, refraction, and double refraction. This theory could't explain rectilinear propagation of light and polarization.

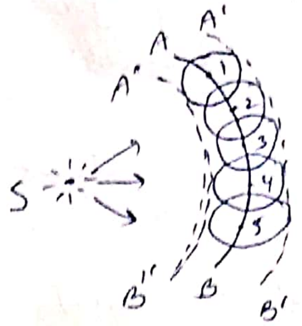
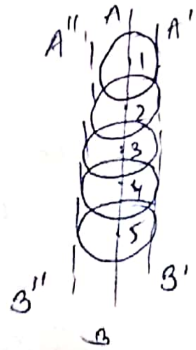


Fig. - Spherical Wavefront



Plane wave front

Ques. - Define coherent source.

Ans. - Two light sources are said to be coherent if they emit light in same phase. i.e. phase difference between two coherent sources is zero or remains constant.

Two independent sources can never be coherent.

Ques. - What is interference?

Sol. - Interference → When two highly coherent light waves of same frequency and amplitude travel in same direction then due to their superposition at some points intensity increases and at some points intensity decreases, this phenomenon is called as interference.

Hence, there is a modification in the intensity of light in medium in region of superposition.

If, intensity of interference pattern increases it is said constructive interference, and if intensity of interference pattern decreases it is said destructive interference.

Conditions for steady and observable interference →

1. Two light sources must be coherent.
2. The interfering waves must have nearly equal amplitudes
3. Direction of interfering waves must be along same line.
4. The seperation between two sources must be as small as possible.
5. The frequencies of two waves must be equal.

Classification of Interference – In general there are two methods of obtaining coherent sources giving rise to two different classes of interference phenomenon.

① **Division of Wave front** – When interference pattern is found by division of wave front using prisms, mirrors or lenses etc. then interference is called as division of wave front. The two wave fronts such seperated travel unequal distances and are finally brought together to produce interference.

eg. – Fresnel's Bi Prism, Lloyd's mirror, Billet's split lens, and LASER.

② **Division of Amplitude** – In this class of interference the amplitude of incoming beam is divided into two or more parts either by reflection or refraction. These divided parts travel different pathes and finally brought together to produce interference.

eg. – Michelson's interferometer, Fabry Perot interferometer

Ques. – Give the theory of Young's double slit experiment.

Sol. – **Young's Double Slit Experiment (1801)** →

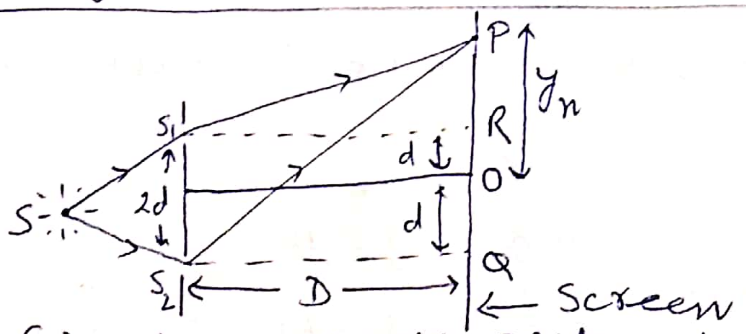


Fig. - Young's Double Slit experiment arrangement

Let S be a monochromatic source of light. S_1 & S_2 are two slits seperated by a distance 2d. S_1 & S_2 act as coherent sources. The distance of screen from coherent sources is D. At point P any n^{th} fringe takes place. Waves from S_1 & S_2 travel unequal path and meet on the screen to produce interference pattern on the screen.

① At Point 'O' - The point 'O' is equidistant from S_1 & S_2. Hence both the rays meet at point O at same time i.e. there is no path difference or phase difference between the two waves. Hence, at point 'O' we get maxima or bright fringe.

② At Point 'P' - at point 'P' both the waves travel unequal path. Hence there is a path difference & phase difference between waves S_1P & S_2P. If both the waves from S_1 & S_2 meet in same phase we get maxima or bright fringe at point P and if both the waves meet in opposite phase we get minima or dark fringe at point P.

S_1R & S_2Q are perpendiculars drawn on the Screen.

$$OP = y_n$$

$$RP = (y_n - d)$$

$$QP = (y_n + d)$$

$$S_1R = S_2Q = D$$

Path difference between S_1P & S_2P $\Delta = S_2P - S_1P$ — ①

from ΔS_2PQ we get

$$S_2P^2 = S_2Q^2 + PQ^2$$
$$= D^2 + (y_n+d)^2$$

or, $$S_2P = \left[D^2 + (y_n+d)^2\right]^{1/2}$$
$$= D\left[1 + \frac{(y_n+d)^2}{D^2}\right]^{1/2}$$
$$= D\left[1 + \frac{(y_n+d)^2}{2D^2}\right]$$ Using Binomial theorem ($\because D >> y_n+d$)

$$S_2P = D + \frac{(y_n+d)^2}{2D} \quad \text{--- (2)}$$

similarly from ΔS_1PR, we get

$$S_1P = D + \frac{(y_n-d)^2}{2D} \quad \text{--- (3)}$$

from eqn (2) & (3) Path difference

$$\Delta = S_2P - S_1P$$
$$= \left[D + \frac{(y_n+d)^2}{2D}\right] - \left[D + \frac{(y_n-d)^2}{2D}\right]$$
$$= \frac{(y_n+d)^2 - (y_n-d)^2}{2D}$$
$$\boxed{\Delta = \frac{y_n \cdot 2d}{D}} \quad \text{--- (4)}$$

(3) Condition for maxima or Bright fringe at P

For maxima or ~~int~~ bright fringe at point P, path difference must be ~~inte~~ even multiple of $\frac{\lambda}{2}$.
$\lambda \rightarrow$ wavelength of light.

i.e, $$\Delta = 2n \cdot \frac{\lambda}{2} \quad \text{--- (5)}$$
$n = 0, 1, 2, 3 \ldots$

from (4) & (5)

$$\frac{y_n \cdot 2d}{D} = 2n \cdot \frac{\lambda}{2}$$

or $$\boxed{y_n = \frac{n D \lambda}{2d}} \qquad \text{——— (6)}$$

$n = 0, 1, 2, 3 - - -$

(4) Condition for minima or dark fringe at point P –

For minima at point P, path difference must be odd multiple of $\frac{\lambda}{2}$. i.e.

$$\Delta = (2n-1)\frac{\lambda}{2} \qquad \text{——— (7)}$$

$n = 1, 2, 3, - - -$

$\therefore$ from (4) & (7),

$$\frac{y_n 2d}{D} = (2n-1)\frac{\lambda}{2}$$

or $$y_n = \frac{(2n-1)}{2}\frac{D\lambda}{2d}$$

$$\boxed{y_n = \left(n - \frac{1}{2}\right)\frac{D\lambda}{2d}} \qquad \text{——— (8)}$$

$n = 1, 2, 3, - - -$

(5) Fringe Width – The distance between any two consecutive bright or dark fringes is called fringe width. It is denoted by ω.

(a) Bright Fringe Width – Let, y_n & y_{n+1} be the distanced of two consecutive bright fringes from centre O. $\therefore$ Using eqn. (6) we get

$$y_n = n\frac{D\lambda}{2d}$$

and, $$y_{n+1} = (n+1)\frac{D\lambda}{2d}$$

$\therefore$ Bright fringe width, $\omega = y_{n+1} - y_n$

$$\omega = (n+1)\frac{D\lambda}{2d} - \frac{nD\lambda}{2d}$$

$$\boxed{\omega = \frac{D\lambda}{2d}} \quad \text{———} \quad (9)$$

(b) Dark fringe Width - Let, y_n & y_{n+1} be the distances of two consecutive dark fringes from centre O. ∴ Using eqn. (8) we get

$$y_n = (n-\tfrac{1}{2})\frac{D\lambda}{2d}$$

$$y_{n+1} = (n+\tfrac{1}{2})\frac{D\lambda}{2d}$$

∴ Dark fringe width

$$\omega = y_{n+1} - y_n$$

$$= (n+\tfrac{1}{2})\frac{D\lambda}{2d} - (n-\tfrac{1}{2})\frac{D\lambda}{2d}$$

$$\boxed{\omega = \frac{D\lambda}{2d}} \quad \text{———} \quad (10)$$

From eqns (9) & (10) it is clear that Bright fringes and dark fringes are of equal width. Further it is clear that fringe width is independent of fringe number 'n'. Hence, all fringes are of equal width.

Also, from (9) & (10)

$$\beta \propto D \quad \text{———} \quad (11)$$

and $$\beta \propto \frac{1}{2d} \quad \text{———} \quad (12)$$

Hence, to obtain good interference pattern -

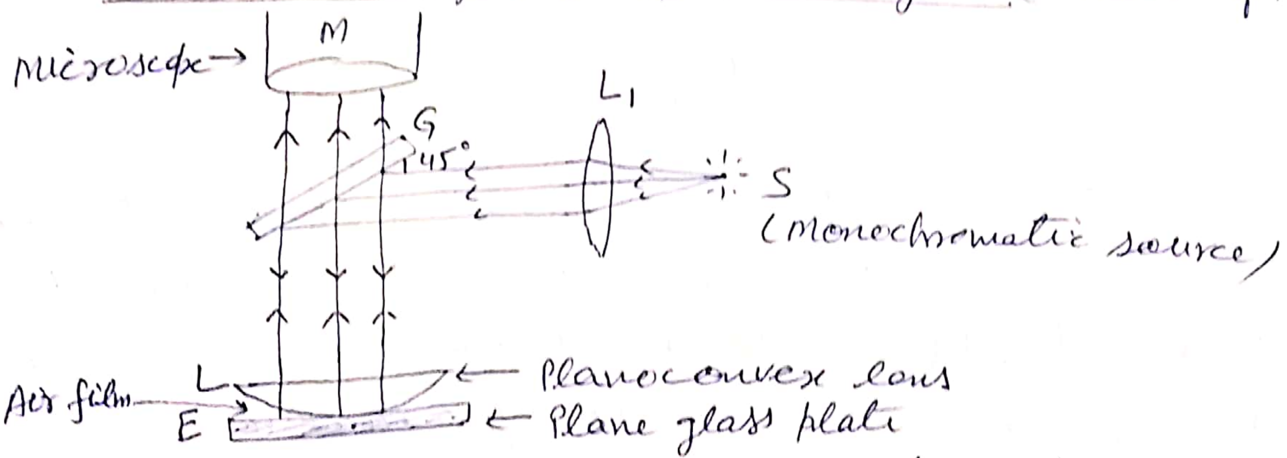
1. The distance between source & screen (D) should be more.
2. separation between coherent sources (2d) should be small.

Ques - Give the theory of Newton's rings experiment.

Sol - Newton's Rings - The phenomenon of Newton's rings is a special case of interference in an air film of variable thick-ness. When we place a plano convex lens of large focal length on plane glass plate, we get a thin air film between lower surface of lens and upper surface of glass plate. When a beam of monochromatic light is allowed to fall normally on such a film of variable thickness, we get an inner dark spot surrounded by alternatively bright and dark circular rings when seen by reflective light and vice-versa by transmitted light.

First of all Newton saw such fringes and hence are called Newton's rings.

Newton's Rings by Reflected Light (Division of Amplitude)



Let, S be a monochromatic extended source of light placed at focus of lens L_1. The horizontal parallel beam of light fall on a glass plate G at an angle of 45° and are partly reflected from it. These reflected rays fall normally on the plano convex L, placed on glass plate E. The interference rings are viewed with a low power microscope M, focussed on the air film where the rings are formed.

Theory of Newton's Rings.

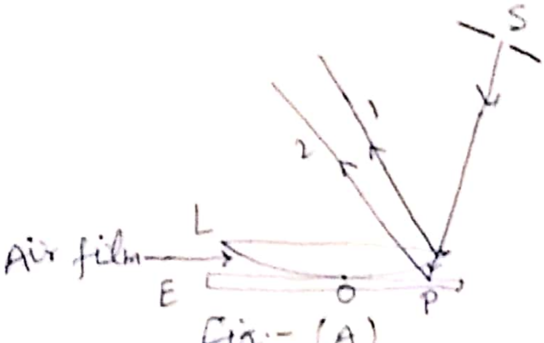


Fig.- (A)

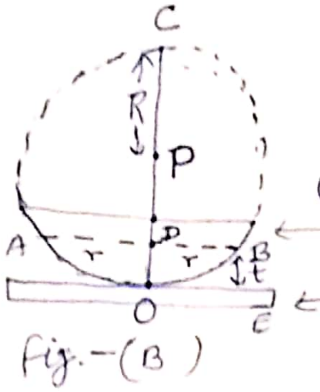


fig.-(B)

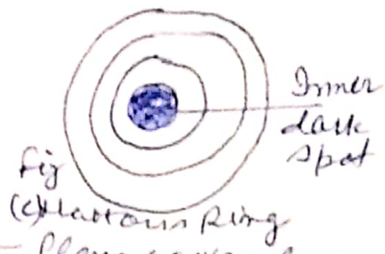


fig (c) Newton's Ring

When a ray SP from an extended monochromatic source of light falls on the lens it is reflected from upper surface (ray-1) and lower surface (ray 2) of thin air film. Reflected rays 1 & 2 reach at microscope and produce interference.

Path difference between rays 1 & 2,

$$\Delta = 2\mu t\cos(r+\theta) - \frac{\lambda}{2} \quad \text{--- (1)}$$

where, μ = refractive index of thin film (air),

t = thickness of air film

r = angle of refraction

θ = angle of wedge

Here, $r = 0^\circ$. for normal incidence, and angle of wedge $\theta \approx 0^\circ$ (very small)

$$\therefore \cos(r+\theta) \approx \cos 0^\circ = 1$$

$\therefore$ from eqn (1)

$$\Delta = 2\mu t - \frac{\lambda}{2} \quad \text{--- (2)}$$

at the point of contact,

$$t = 0$$

$$\therefore \Delta = -\frac{\lambda}{2} \text{ or } \frac{\lambda}{2} \quad \text{leaving -ve sign}$$

$$\boxed{\Delta = 1 \times \frac{\lambda}{2}} \quad \text{--- (3)}$$

obviously, $\frac{\lambda}{2}$ is odd multiple, hence the central spot of ring is dark.

The condition of maximum intensity or Bright ring -

For maxima path difference is always even multiple of $\frac{\lambda}{2}$.

Hence, $\Delta = 2m.\frac{\lambda}{2}$, —④ $m = 0,1,2,3 - - - -$

∴ from eqn ② & ④.

$$2\mu t - \frac{\lambda}{2} = 2m\frac{\lambda}{2}$$

$$\boxed{2\mu t = (2m+1)\frac{\lambda}{2}} \quad \text{—— ⑤}$$

$m = 0,1,2,--$

For dark ring or minimum intensity -

For minima path difference is always odd multiple of $\frac{\lambda}{2}$. Hence

$$\Delta = (2m-1)\frac{\lambda}{2} \quad \text{—— ⑥}$$

$m = 1,2,3 - - -$

from eqn ② & ⑥

$$2\mu t - \frac{\lambda}{2} = (2m-1)\frac{\lambda}{2}$$

$$\boxed{2\mu t = (2m)\frac{\lambda}{2}} \quad \text{—— ⑦}$$

$m = 1,2,3 - - -$

It is clear from eqns ⑤ & ⑦ that for a bright or dark ring of any particular order, t sould be constant. But here in the air film the locus of points having same thickness is a circle with its centre at the point of contact. Hence, fringes are circular rings having common centre at the point of contact.

Diameter of Newton's Rings - In fig. (B) a plano convex lens is placed on plane glass plate E with 'O' as point of contact. Let, R be the radius of curvature of curved surface of the lens and t the thickness of air film.

from the property of circle.

$$AD \times DB = CD \times OD$$

$$r \times r = (2R - t) \times t$$

$$r^2 = 2Rt - t^2$$

$r \rightarrow$ radius of ring passing through B

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But t is very small. $\therefore t^2 \approx 0$

$$\therefore r^2 = 2Rt$$

$$\boxed{t = \frac{r^2}{2R}} \qquad \text{—— (8)}$$

Diameter of Bright Rings →

for bright ring from eqn (5)

$$2\mu t = (2m+1)\frac{\lambda}{2} \qquad m = 0, 1, 2, 3.$$

Using eqn (8) we get,

$$\cancel{2}\mu \cdot \frac{r^2}{\cancel{2}R} = (2m+1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2m+1)\lambda R}{2\mu}$$

If, D_m is the diameter of m^{th} bright ring

$$\left(\frac{D_m}{2}\right)^2 = \frac{(2m+1)\lambda R}{2\mu}$$

$$\text{or } D_m^2 = (2m+1) \cdot \frac{2\lambda R}{\mu}$$

for, air, $\mu = 1$

$$\therefore D_m^2 = (2m+1) \cdot 2\lambda R \qquad \text{—— (9)}$$

for a given lens and light, λ & R are constant.

$$\therefore D_m^2 = K \cdot (2m+1)$$

$$\boxed{D_m \propto \sqrt{2m+1}} \qquad \text{—— (10)}$$

$m = 0, 1, 2, 3 ---$

Thus, diameter of bright rings is proportional to square root of odd natural numbers.

Diameter of Dark Rings - For dark ring using eqn (7) & (8)

$$2\mu \cdot \frac{r^2}{2R} = \cancel{2}m \cdot \frac{\lambda}{\cancel{2}}$$

$$r^2 = \frac{m\lambda R}{\mu}$$

for air, $\mu = 1$

$\therefore r^2 = m\lambda R$

If D_m is the diameter of mth dark ring

$$\therefore \frac{D_m^2}{4} = m\lambda R$$

$$D_m^2 = 4m\lambda R \quad \text{——— (11)}$$

$$D_m = 2\sqrt{m}\sqrt{\lambda R}$$

For given lens and light, λ & R are const.

$$\therefore \boxed{D_m \propto \sqrt{m}} \quad \text{——— (12)}$$

$m = 0, 1, 2, 3 \ldots$

Thus the diameter of dark rings are proportional to square roots of natural numbers.

✓ <u>Calculation of wavelength (λ) of source of light By Newton's Rings</u> -

(i) <u>Practically</u> - If D_n is the diameter of nth dark ring then we have, $D_n^2 = 4n\lambda R$

i.e. $$\frac{D_n^2}{n} = 4\lambda R \quad \text{——— (1)}$$

Thus, if we plot a graph between n & D_n^2 we get a straight line. The slope of this straight line gives D_n^2/n and hence λ.

(ii) <u>Theortically</u> - If D_n is the diameter of nth dark ring then we have.

$$D_n^2 = 4n\lambda R \quad \text{(for air)}$$

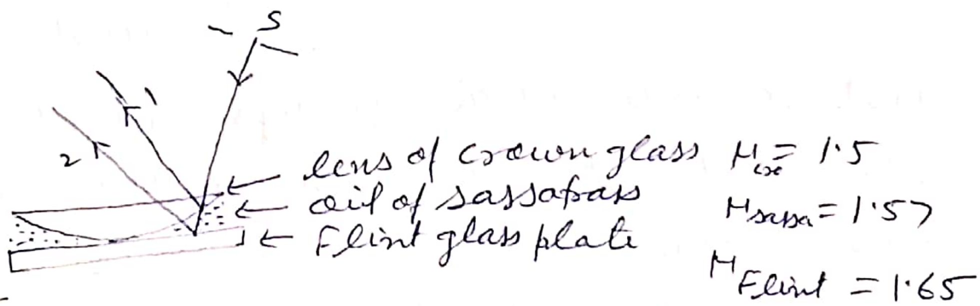
for, $(n+p)^{th}$ ring $D_{n+p}^2 = 4(n+p)\lambda R$

$$\therefore \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \quad \text{——— (2)}$$

Ques. - How can we get Newton's ring with bright centre in reflected light.

Sol. - In order to obtain Newton's ring with bright centre in reflected light we use plano convex lens of crown glass ($\mu = 1.5$), glass plate of flint glass ($\mu = 1.65$) and oil of sassafrass ($\mu = 1.57$). Now, we place a drop of oil of sassafass between lower surface of lens and upper surface of glass plate. As a result a thin layer of oil of sassafass takes place in place of air. Here, the reflection of rays 1 & 2 will be from denser to rarer medium and thus both the rays suffer a phase change of π and central spot appears bright.

Newton Rings with Bright centre



Ques. - How can we determine the refractive index of a liquid with the help of Newton's rings.?

Sol. - In order to find the refractive index of a given liquid with the help of Newton's rings we spread given liquid between lens and plate as a result a film of liquid is formed between lens and plate. Let, μ be the refractive index of liquid. For n^{th} dark ring we have

$$D_n^2 = \frac{4n\lambda R}{\mu} \qquad \text{--- (1)}$$

the diameter of $(n+p)^{th}$ ring -

$$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu} \quad — (2)$$

$$\therefore \left(D_{n+p}^2 - D_n^2\right)_{liquid} = \frac{4p\lambda R}{\mu} \quad — (3)$$

For, air, $\mu = 1$

$$\left(D_{n+p}^2 - D_n^2\right)_{air} = 4p\lambda R \quad — (4)$$

dividing eqn (4) by (3) -

$$\frac{\left(D_{n+p}^2 - D_n^2\right)_{air}}{\left(D_{n+p}^2 - D_n^2\right)_{liq.}} = 4p\lambda R \times \frac{\mu}{4p\lambda R}$$

$$\therefore \boxed{\mu = \frac{\left(D_{n+p}^2 - D_n^2\right)_{air}}{\left(D_{n+p}^2 - D_n^2\right)_{liquid}}} \quad — (5)$$

First we made all the arrangement of Newton's rings experiment and note down the diameter of dark Newton's rings in air. Now, we introduce the liquid (μ) between lens and plate and repeat the experiment to measure the diameter of Newton's rings in liquid. Eqn (5) gives us the value of refractive index of liquid.

Ques. - Describe the construction and working of the Michelson's interferometer.

Sol. - Michelson's Interferometer →

(1) Apparatus - It consists of following parts -

(i) Two plane glass plates P and Q which are inclined at an angle of 45° to the beam of light. Both are of equal thickness. Back side of plate P is semi silvered so that rays can be reflected and transmitted.

(ii) It has two plane mirrors M_1 & M_2 such that they are inclined at right angle. Mirror M_1 is fixed while M_2 is moveable.

(iii) A monochromatic source of light.

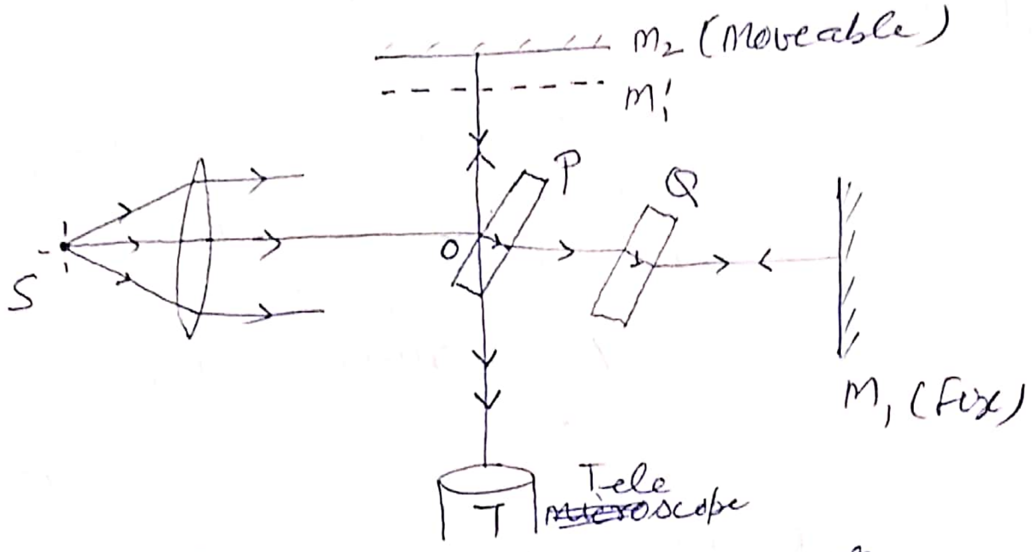


Fig. - Michelson's Interferometer

(2) Working - When a parallel beam of monochromatic light falls on plate P then some part of it is transmitted and some part is reflected. Reflected part travells towards mirror M_2 and again reflected by M_2 goes to telescope. The ~~reflected~~ ~~ray~~ transmitted ray travells towards mirror M_1 and reflected back to plate P and goes to telescope.

Since, these two rays are divided by plate P from a single ray hence these two rays are coherent and produce interference.

(3) Function of Plate Q - Plate Q is made of same material and thickness as that of plate P. The ray reflected at P has to travell twice through plate P. Due to this the ray reflected at P has to travel an extra optical path $2(\mu-1)t$. where, μ is refractive index of plate P and t is thickness of plate. Hence to equalize the path between reflected and transmitted ray we introduce plate Q. The plate Q is called compensating plate.

(4) Type of Fringes – The types of fringes in Michelson Interferometer depends upon inclination of M_1 and M_2. Let M_1' be the image of M_1 formed by reflection at the semi silvered surface of plate P such that $OM_1 = OM_1'$.

The two waves will interfere constructively or destructively according as the path difference Δ between them is even or odd multiple of $\frac{\lambda}{2}$.

Thus condition of constructive interference (maxima)

$$\Delta = 2n\frac{\lambda}{2} \qquad n = 0, 1, 2, ----$$

i.e, $2\mu t \cos r = n\lambda$

or, $2t\cos r = n\lambda$ ——— (1) (for air $\mu = 1$)

and, condition for destructive interference (minima)

$$\Delta = (2n-1)\frac{\lambda}{2} \qquad n = 1, 2, 3 --$$

i.e, $2t\cos r = (2n-1)\frac{\lambda}{2}$ ——— (2)

The shape of fringes will depend upon the inclination of M_1 & M_2. If M_1 & M_2 are exactly at right angle to each other the reflectic surfaces M_2 & M_1' are parallel to each other and hence air film between M_2 & M_1' is of constant thickness so that we get circular fringes of equal thickness inclination or Haideinger's fringes in the field of view of telescope. When the distance between M_1 and M_2 or M_2 and M_1' decreases the circular fringes shrink and vanish at the centre. A ring disappears each time when $2t$ decreases by λ. When t is further decreased a limit comes when M_2 and M_1' coinside and the path difference between two rays is zero. Now the field of view is perfectly bright. When M_2 is further moved, the fringes appear again.

If M_1 and M_2 are not exactly perpendicular a wedge shaped film is formed between M_1 and M_2' and then we get straight line fringes of equal thickness in the field of view of telescope.

Ques. - What are the uses of Michelson's Interferometer?

Sol. - Uses of Michelson's Interferometer.

① Determination of the wavelength of Monochromatic light -

The monochromatic light whose wavelength is to be determined is allowed to fall on the plate P. If 't' is the thickness (distance between M_2 and M_1') and 'n' is the order of spot obtained we have at the centre (for circular fringes)

$$2t = n\lambda \quad \text{---} (1)$$

If the mirror M_2 is moved through a distance x and the number of fringes which cross the field of view is N, then we have

$$2(t + x) = (n + N)\lambda \quad \text{---} (2)$$

subtracting ① from ② we get

$$2x = N\lambda$$

$$\text{or,} \quad \boxed{\lambda = \frac{2x}{N}} \quad \text{---} (3)$$

Thus, the value of λ can be evaluated if we measure the distance x with the micrometer screw and count the number N. By this method x can be determined to an accuracy of 10^{-4} mm and the number N can be sufficiently increased.

② To calculate difference in wavelength between two neighbouring lines - Let the source of light emit two close

wavelengths λ_1 and λ_2 (like sodium D lines). Let $\lambda_1 > \lambda_2$.
The apparatus is adjusted for circular fringes.

When x is the distance moved by the mirror, the path difference is $2x$. During this movement n fringes of λ_1 appears at the centre then $(n+1)$ will be change in order of wavelength λ_2 at the centre, so that we have

$$2x = n\lambda_1 \qquad \text{——— (1)}$$

and

$$2x = (n+1)\lambda_2 \qquad \text{——— (2)}$$

or

$$n\lambda_1 = (n+1)\lambda_2$$

or

$$n = \frac{\lambda_2}{(\lambda_1 - \lambda_2)} \qquad \text{——— (3)}$$

substituting this value of n in eqn (1) we get

$$2x = \frac{\lambda_2}{(\lambda_1 - \lambda_2)}\lambda_1$$

$$\therefore \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x}$$

as λ_1 and λ_2 are very close. $\lambda_1 \approx \lambda_2 \approx \lambda$ $\therefore \lambda_1\lambda_2 = \lambda^2$

$$\therefore \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

or

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2x}} \qquad \text{——— (4)}$$

By eqn (4) the difference in wavelength can be calculated.

(3) Determination of refractive index of thin films →

To determine the refractive index of a thin film the interferometer is adjusted to produce white light straight line fringes and telescope is focussed with its cross wire on the central dark fringe. Now the thin film with refractive index μ is introduced in the path of one of the interfering rays. As a result a path difference of $2(\mu-1)t$ is produced between two rays, where t is the thickness of thin film. If the displacement of mirror M_2 is x, we have.

$$2x = 2(\mu-1)t$$

$$\text{or,} \quad x = (\mu-1)t \qquad \text{——— (1)}$$

Thus by measuring x and t, the value of μ can be calculated.

(4) Standardization of the Meter → If the moveable mirror M_2 of Michelson Interferometer is moved through x cm and N (dark) fringes cross the field of view then,

$$x = N\frac{\lambda}{2}$$

thus, by counting N, x may be determined in terms of wavelength λ of the light waves. This is the principle of measuring the standard meter in terms of known wave-length λ of the light source.

Ques → Describe Mach-Zehnder Interferometer.

Sol. - Mach-Zehnder Interferometer → This interferometer was developed by the physicists Ludwing Mach and Ludwing Zehnder. It uses to seperate beam splitters (BS_1 & BS_2) to split and recombine the beams and has two outputs, which can be sent to photodetectors. The optical path in two arms may be nearly identical or may be different. The distribution of optical powers at the two outputs depends on the precise difference in optical arm lengths and on wavelength.

If the interferometer is well aligned, the path difference can be adjusted so that for a particular wavelength the total power goes into one of the outputs. For misaligned beams there will be some fringe patterns in both outputs and variations of path length difference affect mainly the shapes of these interference patterns. Actually in this interferometer a light beam is first split into two parts by a beam splitter and then recombined by a second beam splitter.

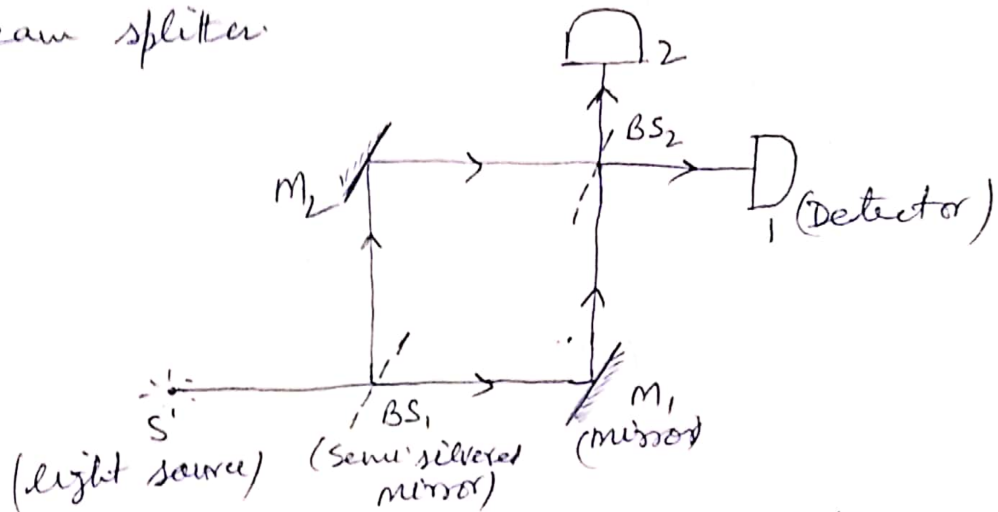


Fig: Mach-Zehnder Interferometer

Uses -

1. In determination of variation of refractive index of air with pressure
2. in determination of wavelength of light.
3. In determination of refractive index of glass plate.

Ques - Explain what is meant by diffraction of light?

Sol. - Diffraction of Light → When a light beam passes through an obstacle or aperture it deviates from its true rectilinear path or bends round the corners of obstacle, this phenomenon is called diffraction. Diffraction of light takes place only when the size of obstacle or aperture is of order of wavelength of light.

This phenomenon was discovered by Grimaldi and later explained by A.J. Fresnel.

The diffraction is caused by the interference of innumerable secondary wavelets produced by the unobstructed portion of the same wave front. Hence, diffraction is a special case of interference.

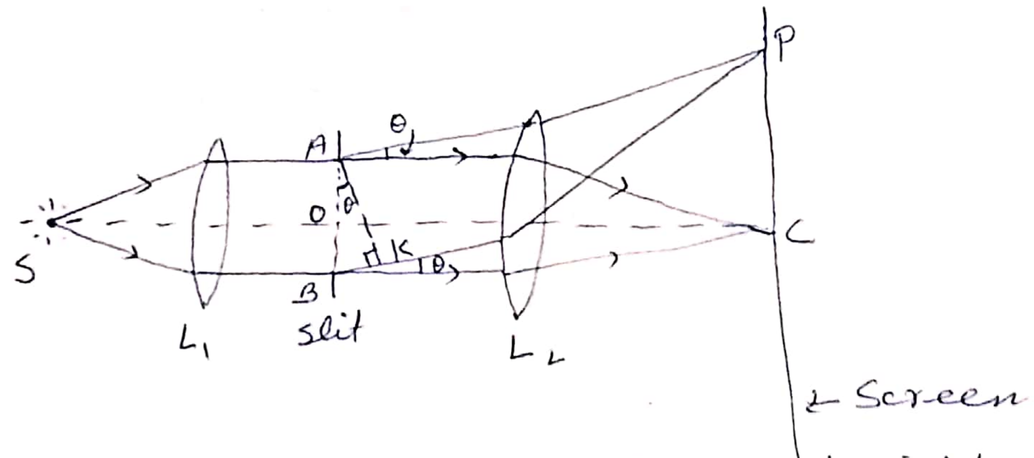
This phenomenon may be conveniently divided into two groups.

① Fresnel diffraction ② Fraunhoffer diffraction.

Ques. - Discuss Fraunhofer diffraction due to single slit or a circular aperture.

Sol. - Fraunhofer Diffraction → In fraunhofer diffraction the source of light and screen are effectively at infinite distance from the diffracting element. This condition is achieved by rendering the incident and diffracted beams parallel by using a convex lens.

Diffraction at Single Slit -



Let, S be a monochromatic source of light of wavelength λ. L_1 is a collimating lens placed at a distance equal to focal length of lens from S. The diffracted light is focussed by ~~at~~ another lens L. AB is slit. Slit width is a. This diffraction pattern consists of a bright central band having alternate dark and bright bands of decreasing intensity on both the sides.

Let, AK be perpendicular to BK. The path difference between rays originating from points A & B is given by

$$\Delta = BK = AB \sin\theta$$

$$\Delta = a \sin\theta$$

$\therefore$ Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path diff

$$\phi = \frac{2\pi}{\lambda} a \sin\theta$$

The phase difference between two consecutive pts.

$$\delta = \frac{1}{n} \cdot \frac{2\pi}{\lambda} a \sin\theta \quad \text{—— (1)}$$

But, resultant amplitude at P is given by

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \frac{\delta}{2}}$$

$$= \frac{a \sin\left(\frac{n}{2} \cdot \frac{1}{n} \frac{2\pi}{\lambda} a \sin\theta\right)}{\sin\left(\frac{1}{2} \cdot \frac{1}{n} \frac{2\pi}{\lambda} a \sin\theta\right)}$$

$$= \frac{a \sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a \sin\theta}{n\lambda}\right)}$$

Let us substitute

$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

$$\therefore R = \frac{a \sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$$= \frac{a \sin\alpha}{\alpha/n}$$

[$\because$ n is very large so α/n is very small $\therefore \sin\frac{\alpha}{n} = \frac{\alpha}{n}$]

$$R = na \frac{\sin\alpha}{\alpha}$$

$$R = \frac{A \sin \alpha}{\alpha}$$

$A = na$ = amplitude when all vibrations are in same phase.

Intensity at P

$$I = R^2 = \frac{A^2 \sin^2 \alpha}{\alpha^2} \quad \text{—— (2)}$$

For maxima or minima.

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha} \frac{A^2 \sin^2 \alpha}{\alpha^2} = 0$$

or, $$\frac{2A^2 \sin \alpha}{\alpha} \left[\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} \right] = 0$$

either (i) $\frac{\sin \alpha}{\alpha} = 0$ or/if $\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$

i.e., either (i) $\frac{\sin \alpha}{\alpha} = 0$ or (ii) $\alpha = \tan \alpha$ —— (3)

Directions for minimum Intensity.

When $\frac{\sin \alpha}{\alpha} = 0$, it is clear from eqn (2) that intensity is zero. Thus for minimum intensity

$$\frac{\sin \alpha}{\alpha} = 0$$

$$\Rightarrow \sin \alpha = 0$$

or $\alpha = \pm m\pi \qquad m = 1, 2, 3, \ldots$

putting the value of α

$$\frac{\pi a \sin \theta}{\lambda} = \pm m\pi$$

or $a \sin \theta = \pm m\lambda$ —— (4)

$m = 1, 2, 3 \ldots$

eqn (4) gives the directions of first, second, third --- minima by putting $m = 1, 2, 3$ ---- respectively, at $m = 0$ we get maximum intensity.

Directions for Maximum Intensity -

For maximum intensity.

$$\alpha = \tan\alpha$$

This eqn can be solved graphically by plotting curves.

$$y = \alpha \text{ and } y = \tan\alpha$$

The eqn $y = \alpha$ represents a straight line passing through origin and making an angle of 45°.

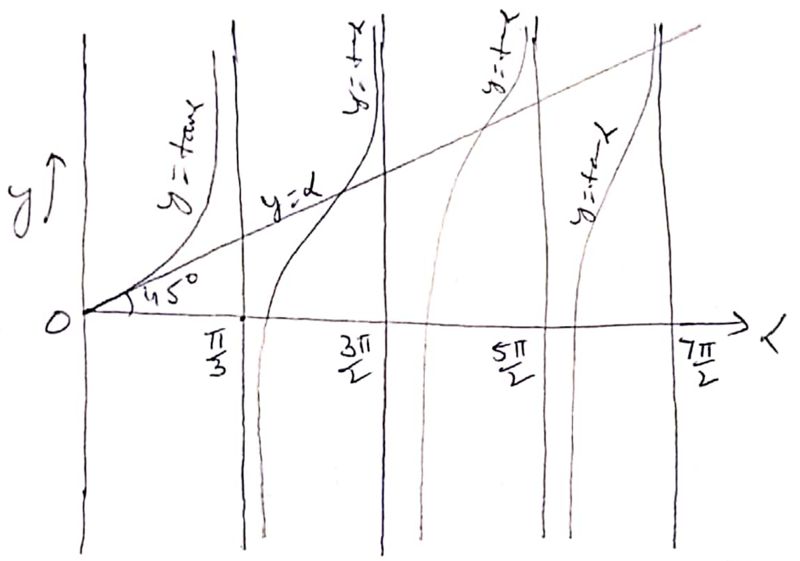


Fig. - Curves for $\alpha = \tan\alpha$

The first value of α is zero while remaining values are approximately $\frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

The intensity of central principal maxima is

$$I_0 = A^2 \frac{\sin^2\alpha}{\alpha^2}$$

$$I_0 = A^2 \quad \text{—} (5)$$

The intensity of first secondary maxima,

$$I_1 = A^2 \frac{(\sin\frac{3\pi}{2})^2}{(3\pi/2)^2} = A^2 \cdot \frac{4}{9\pi^2} \quad \text{—} (6)$$

The intensity of second secondary maxima

$$I_2 = A^2 \frac{(\sin\frac{5\pi}{2})^2}{(5\pi/2)^2} = A^2 \cdot \frac{4}{25\pi^2} \quad \text{—} (7)$$

similarly $I_3 = A^2 \cdot \frac{4}{49\pi^2}$ — (8)

Thus the relative intensities of successive maxima are

$$I_0 : I_1 : I_2 : I_3 \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \ldots$$

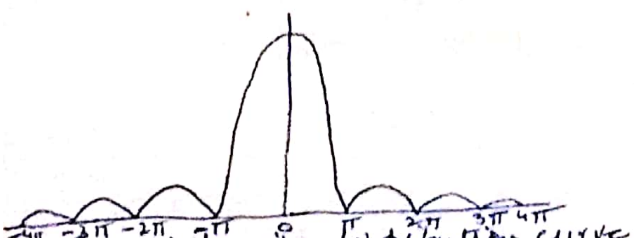


Fig. Intensity distribution curve

Thus diffraction pattern consists of bright central maximum surrounded alternatively by minima of zero intensity and feeble secondary maxima of rapidly decreasing intensities.

Ques. - Discuss Rayleigh criterion for limit of resolution.

Sol. - Resolving Power - The ability of an optical instrument to just resolve the images of two nearby point sources is called its resolving power.

Rayleigh Criterion for Resolution → According to Rayleigh's criterion, the two point sources or two spectral lines of equal intensity are just resolved by an optical instrument when the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of other.

Let's consider the intensity distribution curves of two wavelengths λ and $\lambda + d\lambda$. If $d\lambda$ is sufficiently large, the central maxima due to two wavelengths are quite seperate (fig 1) and the two spectral lines appear well resolved.

However, $d\lambda$ will have a limiting value for which the angular seperation between their maxima is such that the central maximum of one coincides with first minimum of the other and vice-versa (fig. 2)

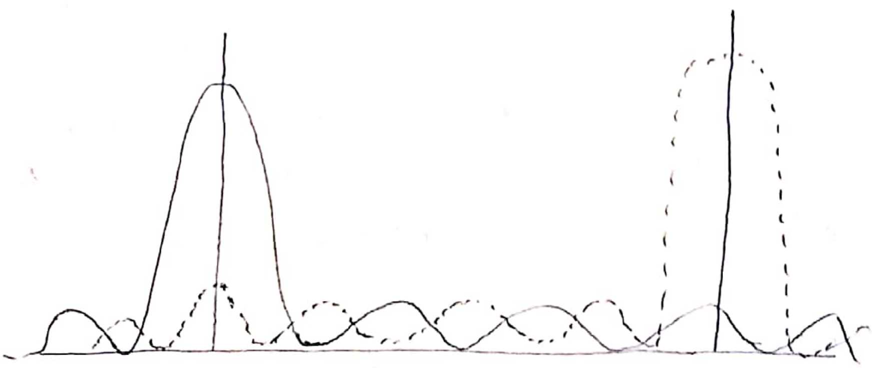


Fig 1

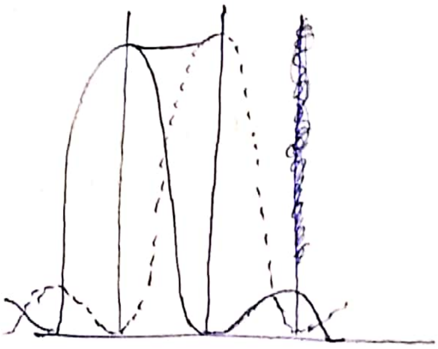


Fig - 2

The intensity distribution in grating or prism spectra is of the form

$$I = I_{max} \frac{\sin^2\alpha}{\alpha^2} \qquad \text{for first minima } \alpha = \pi$$

At the middle of two maxima the intensity (Fig 2) due to each is given by putting $\alpha = \frac{\pi}{2}$, Hence total intensity at middle of maxima (Fig. 2) is given by

$$I_{middle} = 2 I_{max} \frac{\sin^2 \pi/2}{(\pi/2)^2} = \frac{8}{\pi^2} I_{max}.$$

$$\boxed{\therefore \quad \frac{I_{middle}}{I_{max}} = \frac{8}{\pi^2} = 0.81 \text{ (approx)}}$$

Thus rayleigh criterion may be stated as the two spectral lines are just resolved if the intensity at the dip in the middle is $\frac{8}{\pi^2}$ times the intensity at either of the maxima.

However, if $d\lambda$ is smaller than the limiting value satisfying Rayleigh's criterion, the two spectral lines can't be resolved.

Ques. - Define Grating and Resolving power of grating.

Sol. - Grating or Plane Transmission Grating - An arrangement consisting of a large number of parallel slits of equal width and seperated from one another by equal opaque spaces is called a diffraction grating.

It may be made by ruling a large number of fine equidistant and parallel lines with a diamond point on an optically plane glass plate. The ruled widths are opaque to the light while the space between any two lines is transparent. Such a grating is called Plane Transmission Grating.

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